# PREDICTIVE MIMO BEAM FORMING IN THE CASE OF PHYSICAL PATH MOVING IN MULTIPATH TRANSMISSION ENVIRONMENT BY USING TAYLOR SERIES 

# BỨC XẠ MIMO DỰ ĐOÁN TRONG TRƯỜNG HỢP ĐƯỜNG VẬT LÝ DI CHUYẺN TRONG MÔI TRƯỞNG ĐA ĐƯỜNG SỬ DỤNG CHUÕ̃I TAYLOR 

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#### Abstract

: Taylor series is useful mathematical formula in many applications, even in the wireless communication. It is used in some papers to create converged algorithms to find the location of mobile, the attacked sensor nodes, etc... However, the paper uses the Taylor series to predict the transmit beam vector as a function of time through a limited observations of MIMO channels at the receiver in the multipath environment having the obstacles in a rotation around the transmitter. The simulation shows if using beam vector at any time using value of the proposed function of beam that can make higher capacity (bits/s/Hz) compared using SVD (Singular Value Decomposition) at the beginning of moving receiver.


## Key words:

Taylor series, MIMO, beam prediction, channel capacity.

## Tóm tắt:

Chuỗi Taylor là một công thức toán học hữu ích trong nhiều ứng dụng, thậm chí trong truyền thông vô tuyến. Nó được dùng cho một số bài báo dùng tạo các thuật toán hội tụ để tìm ra vị trí chính xác của di động, các nút cảm biến bị tấn công... Tuy nhiên, bài báo này sử dụng chuỗi Taylor để dự đoán bức xạ phát như một hàm thời gian thông qua một số lần quan sát kênh truyền tại máy thu trong môi trường đa đường khi có chướng ngại vật di chuyển tròn quanh trạm phát. Mô phỏng chứng minh nếu dùng vector bức xạ tại bất cứ giá trị nào trong hàm thời gian cải tiến trên, dung lượng kênh truyền (bit/s/Hz) cao hơn việc chỉ sử dụng truyền thống vector bức xạ dùng phân tích giá trị riêng SVD tại thời điểm máy thu bắt đầu di chuyển.

## Từ khóa:

Chuỗi Taylor, MIMO, dự đoán bức xạ, dung lượng kênh truyền.

## 1. INTRODUCTION

In [1], [2], they describes MIMO channel

[^0]where the scatterers are static for broadband mobile or massive MIMO, but in reality, some scatterers may move like air blocks, autos, motorcycles, etc... When the scatterers move, the time-domain signal vector received by the mobile:
$\mathbf{y}_{R}(t)=\mathbf{H}(t) \mathbf{s}_{T}(t)+\mathbf{n}(t)$
where $\mathrm{s}_{T}(t)$ is the time- varying transmit signal vector.
$\mathrm{H}(t)$ is the $N \times M$ channel matrix where each entry $h_{n m}(t)$, is a composite time varying channel response between the $m$ th transmit element and the $n$th receive element at the receiver. It can be determined by [3]:
$h_{n m}(t)=\sum_{l=l}^{L} \alpha_{l} e^{-j \theta_{l}}$ $l=1$
$e^{-j \kappa\left((m-1) \sin \varphi_{l} s T^{+}+(n-1) \sin \theta_{l} s R\right)_{e} j \kappa \cos \theta_{l} v t}$ where $\varphi_{l}, \theta_{l}$ are the transmit and the receive angles of the $l$ th physical path, correspondingly, the transmit angles are functions of time due to the motion of scatterers and the receiver; $\kappa=\frac{2 \pi}{\lambda}$ is the wave number where $\lambda$ is the wavelength of the carrier signal and $\alpha_{l}$ is the composite complex valued $l$ th propagation path strength, defined in [3].

The SVD (Singular Value Decomposition) is often applied to form the beams at the transmitter. If channel matrix is known by the receiver, it will use the SVD to find the eigenvectors and the eigenvalues by using the analysis below [3]:

$$
\begin{equation*}
\mathbf{H}^{H}(t)=\mathbf{Z} \boldsymbol{\Sigma} V^{H} \tag{3}
\end{equation*}
$$

It is assumed that there are $L$ physical paths between the transmitter and the receiver, therefore matrices of
eigenvectors $\mathbf{Z}, \mathbf{V}$ has sizes of $M \times L$ and $N \times L$, matrix of eigenvalues $\boldsymbol{\Sigma}$ has size of $L \times L$. Matrix $\mathbf{Z}$ has $L$ columns $\mathbf{z}_{l}, l=1: L$, called eigenvectors which the receiver feeds back to the transmitter. The transmitter creates beam eigenvectors $\mathbf{u}_{l}, l=1: L$ to increase the channel capacity, based on:
$\mathbf{u}_{l}=\mathbf{z}_{l}^{H}$


Figure 1. The multipath environment where a scatterer 1 moves in a circle

## 2. TAYLOR SERIES

In mathematics, a Taylor series is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point [4]. Based on characteristics of Taylor series, any signal can be determined through its higher deviation. It can be described as below:

$$
\begin{align*}
& f(x)=\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}=f(a)+\frac{f^{\prime}(a)}{1!}(x-a)+\ldots \\
& \frac{f^{\prime \prime}(a)}{2!}(x-a)^{2}+\frac{f^{\prime \prime \prime}(a)}{3!}(x-a)^{3}+\ldots \tag{5}
\end{align*}
$$

Some papers [5], [6] use to create converged algorithms to finds the location of mobile, the attacked sensor nodes, etc... However, the paper uses Taylor series to predict the transmit beam vector as a function of time through a limited observations of MIMO channels at the receiver in the multipath environment having the obstacles in rotation around the transmitter. When physical path changes, the beam vector has to be changed direction to track on this movement of the path. If the $2^{\text {nd }}$ path changed gradually with a constant velocity in a rotation around the base station, beam vector $\mathbf{u}_{2}(t)$ should be rotated the same velocity. Other beams vectors $\mathbf{u}_{2, i}, i=1: K \mathrm{i}=1$ to K are assumed relating to original beam vector $\mathbf{u}_{2}(t)$ as its derivatives with the order of 0 to $\mathrm{K}-1$, where K is the times the receiver observes the channel matrix. Therefore, after K times of observations, the transmitter has K eigenvectors $\mathbf{u}_{2, i}$ that are fed back from the receiver in the new method, it forms $\mathbf{u}_{2}(t)$ and will uses this beam for further time (in a long term). The receiver stops feed back the eigenvectors to the transmitter. This is different to the SVD which requires the instantaneous update the eigenvectors. This proposal can be proved exactly for increasing by the simulation presented in Section 3.

## 3. THE COMPARISON WITH THE USE OF THE BEAM VECTOR AT THE BEGINNING OF MOVING THE RECEIVER

The simulations have been conducted to
show the relationship between vectors $\mathbf{u}_{2, i}, i=1: K$ of the matrix $\mathbf{U}$ (applying the SVD to matrix $\mathbf{H}(t)$ ) and how to predict the beam. Here, we present the MIMO two-path model in which there are 4 antenna elements at both the ends of the model and only one moving physical path. The signal departs from the transmitter at the beginning angle of $315^{\circ}$ (beam 2 in figure $2, \mathbf{u}_{2}(t)$ ) then the path moves anticlockwise with a constant angular speed. The signal also arrives to the receiver at the constant angle of $120^{\circ}$ (considered far-field to the receiver). The carrier wavelength is defined as 1 (m). Inter- element spacing at both the transmitter and the receiver are 0.5 (m). The proposed covariance matrix is built by the receiver using $K=8$ observations with the rate at 1 per second to extract the vectors $\mathbf{u}_{2, i}, i=1: K$. The new discovery is illustrated in figures 3 (the path moves with a speed of $15\left({ }^{0} / s\right)$ ) and $4\left(2\left({ }^{0} / s\right)\right)$ wherein we see, at the convex points of $i$ th array factor, values of the $(i+1)$ th array factor are concave or convex and vice verse. Based on a Taylor series expansion, the future transmit vector $\mathbf{u}_{2}(t)$ can be described as a function of time, through the vectors $\mathbf{u}_{2, i}, i=1: K$ :
$\mathbf{u}_{2}(t)=\mathbf{u}_{2,1}+t \mathbf{u}_{2,2}+\frac{1}{2} t^{2} \mathbf{u}_{2,3}+\ldots$
$+\frac{1}{K!} t^{K-1} \mathbf{u}_{2, K}$
This prediction can inform and lead to
predicted the transmitter know and form the optimum beam pattern at a future time $t$ then can maintain the accepted channel capacity for a longer time, for example, for the model in Figure 1 comparing with the beam vector extracted from the SVD of the channel matrix.


Figure 2. Two beams are simulated at the beginning of moving the receiver



Figure 3. Beam 2 is simulated at 8 times of moving the scatterer 2 with velocity of $15^{\circ} / \mathrm{s}$


Figure 4. Beam 2 is simulated at 8 times of moving the scatterer 2 with velocity of $2^{\circ} / \mathrm{s}$


Figure 5. Channel capacities using beam vectors $u_{1}$ at the time of $1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 10 \mathrm{~s}$ (predicted) and 15 s (predicted) compared use of $\mathrm{u}_{2}$ at the beginning of moving the receiver ( 0 s )

Based on figure 3 and 4, we consider the other beam vectors at 8 times of observations as the derivatives of $\mathbf{u}_{1}$ and can apply Taylor series to generalise the
beam vector $\mathbf{u}_{2}(t)$ as a function of time. This helps the transmitter to determine the beam vector for the $2^{\text {nd }}$ path in a long term.

The channel capacity can be given by the beam vector taken at any time. In figure 5 , times to determine are $1,2,3,4,10$ and 15 s . The capacity can be improved when not using Taylor series and using only $\mathbf{u}_{2}(t)$ at the time of moving the receiver $t=0$, especially good at the further times.

## 4. CONCLUSION

The paper has used Taylor series to predict the beam vector along with time as a funtion. The environment has some physical paths in which a physical path moving a circle around the transmitter. The paper shows if the transmitter uses any value of the proposed beam vector take a specific time, the channel capacity can be higher than the case just use of SVD of channel matrix at the beginning the receiver moves.

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## Biography:



Tran Hoai Trung was born in 1976. He got Bachelor degree in University of Transport and Communications (UTC) in 1997 and hold the post of lecturer at the University. He then got a Master degree from Hanoi University of Science and Technology (HUST) in 2000. In the period 2003 to 2008, he had concentrated on researching in the field of Telecommunication engineering and got his PhD at University of Technology, Sydney (UTS) in Australia. He is currently lecturer at the UTC. His main research interests are digital signal processing (DSP), applied information theory, radio propagation, MIMO antenna techniques and advanced wireless transceiver design.


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