ANALYSING ULTIMATE STRENGTH OF OPEN BOX GIRDERs UNDER BENDING AND TORQUE MOMENT SIMULTANEOUSLY

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ABSTRACT

In this paper, the nonlinear finite element method is employed to predict the ultimate strength of open box girders model under combined loads of bending and torsion. The primary aim of this study is to investigate the ultimate strength characteristics of the open box girders model under sagging bending moment and torque simultaneously. Results of theoretical and numerical analyses show that the bending moment and torque loads have different influences on the structural ultimate strength.

Keywords: ultimate strength, nonlinear finite element, open box girders, sagging bending moment, torque.

1. Introduction

In analysis and design of ship structure, the ultimate strength analysis is an essential stage, which usually gives an assessment result of the structural safety condition. A ship hull structure is very complicated three-dimensional thin-wall structure. When a finite element analysis is performed with the actual object of a ship based on the influence of material nonlinearity and geometric nonlinearity, the calculational cost would be considerable and time-consuming. Therefore, simplified model is regularly adopted to reduce workload and to improve research efficiency. In the structural aspect, the box girder is similar to the hull, as both of them are constructed by shell plate, related frame and other support structures. As a result, when studying the ultimate strength of hull, the box girder is often used as a research object. This paper is not an exception, a simple box girder model is used to calculate and estimate the ultimate strength analysis under combined load. The numerical results of this study provide invaluable reference for the design of hull and box girder structures.
obtained from the present study can be used as a base for accounting the ultimate strength of the actual ship model.

Nishihara [1] built up four box girder model: single bottom tanker, double bottom tanker, bulk carrier, container carrier, used ultimate strength calculation formula and experimental results to calculate and analyzed the ultimate strength of single skin tanker model. The author tested four box girder model to determine the ultimate strength of sagging and hogging bending moment.


Paik et al (2009a and 2009b) [3, 4] used nonlinear finite element to calculate ultimate strength of plate structure and stiffened-plate under the effect of vertical pressure. The research object is outer bottom plate and stiffened-plate structures of 100,000 ton.

Shi Gui-jie et al (2013a and 2013a) [5, 6] proposed a simple model for estimating the residual ultimate strength of open box girders with crack damage under single load and combined loads, using the numerical results obtained after analyze the ultimate strength of open box girders with crack damage under pure torque, compressive force, bending moment and combined loads.

In this paper, a typical open box girder model as a bulk carrier model will be taken as the research object using a commercial. The aim of the study is to investigate the ultimate strength characteristics of the open box girders model under combined loads. Based on the numerical results obtained a graph for the relationship between ultimate torque and ultimate bending moment is proposed.

2. Nonlinear finite element analysis of the box girder

2.1. Geometric and Material properties

In this paper, an open box girder model (as shown in Fig. 1) will be taken as the calculation object for research. The dimension and material properties of open box girder model are shown in Table 1.

<table>
<thead>
<tr>
<th>Stiffened Plate</th>
<th>Dimension (mm)</th>
<th>$\sigma_y$(MPa)</th>
<th>E(GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top plate</td>
<td>$t_p$=3.0</td>
<td>290</td>
<td>210</td>
</tr>
<tr>
<td>Bottom plate</td>
<td>$t_p$=3.0</td>
<td>290</td>
<td>210</td>
</tr>
<tr>
<td>Sides shell</td>
<td>$t_p$=3.0</td>
<td>290</td>
<td>210</td>
</tr>
<tr>
<td>Bottom stiffeners</td>
<td>50x3.0</td>
<td>290</td>
<td>210</td>
</tr>
<tr>
<td>Side of stiffener</td>
<td>50x3.0</td>
<td>290</td>
<td>210</td>
</tr>
</tbody>
</table>
Length of stiffener: $L = 540\text{mm}$; breadth of box girder $B=720\text{mm}$; height of box girder $H=720\text{mm}$.

2.2. Finite element model

The research object has a section long of 540mm. The middle section of 540mm in three-span model of 1+1+1 is taken as the study object [1, 5, 6, 12]. Moreover, both ends of the section are protracted for 540mm (as shown in Fig. 2), so that boundary condition may be exerted on the protected section of both ends to eliminate the influence of boundary condition on calculation result. In addition, in order to ensure damage of core section occurs before the protracted sections, the structure of protracted sections is reinforced. The thickness of plate is denoted as $t=5\text{mm}$, while the thickness of core section is set as $t=3.0\text{mm}$. In this paper, S4R shell element in FEA program was used for plates and stiffeners of box girder (IACS, 2012, Paik, J. K et al., 2008b) [8, 11]. Fig.2 shows the finite element model of the box girder model.
2.3. **Loads and Boundary Conditions**

On the two lateral faces of box girder model, a master node constraint is applied to define boundary condition. Slave nodes constraint controls the displacement and the angle (Liu Bin and Wu Wei Guo, 2013) [9]. So that, it is necessary to set corresponding boundary condition at master node. As the cross section of open box girder model is centrally-symmetric structure, master nodes are hereby deployed in the center of both end faces of the box girder. Meanwhile, slave nodes refer to all nodes along the border of the end face, as shown in Fig. 3.

For hull structure, the external loads mainly include two categories:

- Overall loads, including overall bending moment and torque...
- Local loads, including cargo pressure, cargo inertia pressure, hydrostatic pressure, hydrodynamic pressure, etc.

In this paper, the ultimate strength of open box girder structure under sagging bending moment and torque loads are also taken into account the above two categories of loads in this paper.

2.4. **Nonlinear finite element mesh modeling**

Fig. 4 shows the nonlinear finite element model for analyzing the ultimate strength of the Nishihara open box girder (bulk carrier model). Four mesh sizes are chosen in this paper, and the ideal open box girder model is used to account the limit bending moment of these four meshes to compare the results.

From Fig. 5 and table 2, the maximum deviation of the ultimate strength of the box girder of four different elements models under bending moment is 4.14%, which means the influence of mesh size on the ultimate strength bending moment accuracy of the box girder is not so remarkable. But in fact, the calculational model with smaller mesh spend a longer time.

This paper aims to investigate the factors which influence the ultimate strength, but not refer to the working efficiency. So the model with mesh size 4 is used in the following analysis.

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of grids</th>
<th>Computed result of (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal</td>
<td>Longitudinal</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>
In this paper, Arc-length method from nonlinear finite element calculation approach is adopted to perform calculation (Paik, J. K et al., 2008a) [10]. In order to test the reliability of the calculation method, the ultimate strength of bulk carrier model under pure bending condition is calculated. Then, the result is compared with test result. Besides, an ideal model (without initial deflection) and defective model (with initial deflection) are calculated separately and compared to assess the influence of initial defect. Plates and stiffened plates members are used in the open box girder models. For the present study, the initial deflection of plating and stiffener web are determined by empirical formula (Paik, J. K et al., 2009a and 2009b) [3, 4]. The membrane stress distribution with initial deflection of open box girder model is shown in Fig. 6.

When calculating the ultimate strength under pure bending condition, the selected boundary condition is the left master node constrains displacement along X, Y and Z directions, as well as rotation angle along Y and Z directions. The right master node is deployed to constrain displacement along X and Y directions, as well as rotation angle along Y and Z directions. In actual analysis, bending moments along direction, with
equal magnitude and opposite direction, are separately exerted on master nodes at both ends. Arc length method is applied to perform the calculation until the structure fails.

Deformed shapes and von Mises stress distributions of open box girder model structure at the ultimate strength under sagging bending moment are shown in Fig. 7. The relation between bending moment and Angle is shown in Fig.8

![Fig. 6. Membrane stress distribution with initial deflection (a) Top of model, (b) Bottom of model](image1)

![Fig. 7. Von Mises stress distributions of the bulk carrier model: a) Ideal model; b) Initial deflection model](image2)

![Fig. 8. Bending moment - rotation curves](image3)

![Fig. 9. Experimental value and calculation results of sagging bending moment](image4)

Fig 9 show the Nishihara experimental value and calculation results of open box girder model. From Fig. 7, Fig. 8 and Fig. 9, it is shown that the calculation iteration paths of ideal model and initial deflection model are basically the same before reaching the ultimate strength. This indicates that the initial deflection leads to structural damage under even quite small curvature, which reduces the ultimate bearing capacity of structure. When initial deflection is considered, the calculation results would be consistent to the test values. From this calculation results, nonlinear finite element method leads to high precision when being applied to calculate the ultimate strength of sagging bending moment. The calculation results and experimental value of ultimate strength of sagging bending moment are shown in Table 3.
Table 3. Calculation results of ultimate strength of sagging bending moment

<table>
<thead>
<tr>
<th>Model</th>
<th>Experimental value (N.m)</th>
<th>Calculation value based on nonlinear finite element method (N.m)</th>
<th>Deviation from experimental value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal model</td>
<td>526000</td>
<td>615143</td>
<td>14.50%</td>
</tr>
<tr>
<td>Initial deflection model</td>
<td>550315</td>
<td>4.42%</td>
<td></td>
</tr>
</tbody>
</table>

From the calculation results, nonlinear finite element method leads to high precision when being used to calculate the ultimate strength of structure. Especially, after introducing initial deflection, there is no much difference between the calculation result and the test result (4.42%).

The reasons for this result may be as follows:

(1) Welding residual stress is not considered;
(2) There is a difference between initial deflection shape added via buckling mode and the test condition of actual model. However, from the calculation results, these possible factors only lead to quite limited influence, and it is quite reliable to use the above nonlinear finite element method to calculate the ultimate strength of structure.

2.6. Ultimate strength of open box girder under combined bending moment and torque

Calculation of ultimate strength of the open box girder under pure bending condition in above section has demonstrated the reliability of nonlinear finite element method when being applied to calculate structural ultimate strength. In order to predict the ultimate strength of the box girder structure under the combined loads of bending moment and torque, the Nishihara - bulk carrier model under combination of sagging bending moment and torque with different proportions is simulated in this paper. Boundary conditions are applied as follows: the left master node constrains displacement along X, Y, Z directions and rotation angle along Y direction; the right master node is deployed to constrain displacement along X, Y directions and rotation angle along Y direction.

Proportional relation between $M_x$ and $M_z$ includes the below:

$M_x : M_z = 0.0:1.0, 0.1:0.9, 0.2:0.8, 0.3:0.7, 0.4:0.6, 0.5:0.5, 0.6:0.4, 0.7:0.3, 0.8:0.2, 0.9:0.1, 1.0:0.0$. In which, $M_x : M_z = 0:1.0$ refers to pure torque condition, and that the rotation angle of master nodes at both ends along X direction shall be constrained. $M_x : M_z = 1:0.0$ refers to pure pure bending condition, in which, the rotation angle of master nodes at both ends along X direction shall be constrained. The calculation results are shown in Fig. 9.

Where:

- $M_{UX}$ - Ultimate strength of sagging bending moment under combined load of bending moment and torque.
- $M_{UZ}$ - Ultimate strength of torque under combined load of bending moment and torque.
and

\[
\frac{M_x}{M_z}
\]

(b)

\[
\frac{M_x}{M_z}
\]

(c)

\[
\frac{M_x}{M_z}
\]

(d)

\[
\frac{M_x}{M_z}
\]

(e)

\[
\frac{M_x}{M_z}
\]

(f)

\[
\frac{M_x}{M_z}
\]
\( M_x / M_z = 0.3 : 0.7 \)

\( M_x / M_z = 0.4 : 0.6 \)

\( M_x / M_z = 0.5 : 0.5 \)
\[ \text{Moment (Nm)} \]

**Diagram (m)**

\[ \frac{M_x}{M_z} = 0.6:0.4 \]

**Diagram (n)**

\[ \frac{M_x}{M_z} = 0.6:0.4 \]

**Diagram (o)**

\[ \frac{M_x}{M_z} = 0.7:0.3 \]

**Diagram (p)**

\[ \frac{M_x}{M_z} = 0.7:0.3 \]

**Diagram (q)**

\[ \frac{M_x}{M_z} = 0.8:0.2 \]

**Diagram (r)**

\[ \frac{M_x}{M_z} = 0.8:0.2 \]
Fig. 10. (a, c, e, g, i, k, m, o, q, s): Rotation - Bending Moment curve under different calculation conditions. (b, d, f, h, j, l, n, p, r, t): Rotation - Torque curve under different calculation conditions

Table 4. Calculation results of combined effect of sagging bending moment and torque

<table>
<thead>
<tr>
<th>Load ratio</th>
<th>Calculated ultimate strength</th>
<th>Load ratio</th>
<th>Calculated ultimate strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M_x : M_z</td>
<td>M_UX</td>
<td>M_UZ</td>
</tr>
<tr>
<td>0.0:1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1:0.9</td>
<td>117134</td>
<td>491351.4</td>
<td>0.6:0.4</td>
</tr>
<tr>
<td>0.2:0.8</td>
<td>221177</td>
<td>452316.2</td>
<td>0.7:0.3</td>
</tr>
<tr>
<td>0.3:0.7</td>
<td>314135</td>
<td>403911.7</td>
<td>0.8:0.2</td>
</tr>
<tr>
<td>0.4:0.6</td>
<td>382121.3</td>
<td>364437.8</td>
<td>0.9:0.1</td>
</tr>
<tr>
<td>0.5:0.5</td>
<td>427003.2</td>
<td>319039.6</td>
<td>1.0:0.0</td>
</tr>
</tbody>
</table>

Fig. 10 shows Rotation - Bending Moment curve and Torque curve under different calculation conditions. The left column Fig. 10(a, c, e, g, i, k, m, o, q, s) refers to rotation - bending moment curve under different calculation conditions. In this case, the x axis shows the rotation, the y axis shows the M_UX, while the right column shows rotation - torque curve under different calculation conditions. In this case, the x axis shows the rotation, the y axis shows the M_UZ. According to the peak value in the above curves, we may be able to figure out ultimate sagging bending moment and torque under different conditions, as shown in Table 4 and Fig. 11.
In Fig. 11, the x axis shows the ultimate strength of sagging bending moment under bending moment and torque ($M_{UX}$); the y axis shows the ultimate strength of torque under bending moment and torque ($M_{UZ}$).

![Graph showing interaction relationships between sagging bending moment and torque](image)

**Fig. 11. Ultimate strength interaction relationships between sagging bending moment**

The calculation values of ultimate strength of sagging bending moment and torque under combined load are basically consistent to their proportion in initial load, i.e. higher proportion of $M_x$ or $M_z$ in initial load leads to higher calculation value of ultimate sagging bending moment $M_{UX}$ or ultimate torque $M_{UZ}$.

Interaction relationships between ultimate strength of sagging bending moment and torque shows in Eq. (1).

$$\left(\frac{M_{ux}}{M_{ux,0}}\right)^{1.725} + \left(\frac{M_{uz}}{M_{uz,0}}\right)^{1.725} = 1$$

(1)

Where:

- $M_{ux}$ - Ultimate strength of torque under combined load of bending moment and torque
- $M_{UX}$ - Ultimate strength of torque under pure torque
- $M_{uz}$ - Ultimate strength of sagging bending moment under combined load of bending moment and torque
- $M_{UZ}$ - Ultimate strength of sagging bending moment under pure bending moment

3. **Conclusion**

In this paper, the ultimate strength of a model of open box girder under combined load is studied numerically. Major external loads considered include bending moment and torque. Through analysis on calculation results, following conclusions are drawn:

Nonlinear finite element method leads to high precision when being applied to calculate the ultimate strength of structure. Especially if initial deflection is considered, the calculation results would be consistent to the experimental value. It is important to study the ultimate limit state of ship structural.
Build the interaction relationships between ultimate strength of sagging bending moment and torque of open box girder under sagging bending moment and torque simultaneously. A simple formula proposed in Eq. (1) was used to calculated the relationship between ultimate torque and ultimate bending moment.

It is shown that sagging bending moment and torque may lead to different influences on the ultimate strength of structure. As for this, in order to assess the ultimate strength of ship hull more accurately, it is necessary to comprehensively consider the effect of sagging bending moment and torque loads when calculating the ultimate strength of ship hull.

REFERENCES
2. IACS (2012), Common Structural Rules for Bulk Carriers.

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