MULTI-LAYER MOVING PLATE METHOD FOR DYNAMIC ANALYSIS OF PAVEMENT STRUCTURES SUBJECTED TO MOVING LOADS

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ABSTRACT

This paper presents a new approach, namely multi-layer moving plate method (MMPM), for the dynamic analysis of pavement structures subjected to moving load. The pavement resting on multi-layer foundation is modeled as a two-layer plate connected by a spring-damper system resting on a viscoelastic foundation. This model gives a accurately pavement structure model so that the dynamic responses of the surface slab and the base can be obtained. The governing equations as well as the plate element mass, damping and stiffness matrices are formulated in a convected coordinate with the idea of attaching its origin to the applied point of the moving load. The proposed method simply treats the moving load as 'stationary' at the node of the plate to avoid the updating of the location of moving load due to change of contact points on the plate. Numerical examples related to the dynamic analysis of the pavement structure subjected to a moving load are conducted to investigate the effects of various parameters such as concrete slab thickness, base thickness, foundation stiffness and the load's velocity on dynamic responses of the pavement structure.

Keywords: Multi-layer moving plate method; pavement structure; multi-layer foundation; moving load; interaction.

1. Introduction

The dynamic interactions between pavement structure and moving vehicles such as cars, trucks and aircraft have attracted much research attention in the last two decades. The results of these studies can be employed in many branches of modern transportation engineering such as the design of track/road beds, highway and runway pavements. Analytical methods were used in earlier research works to study the dynamic responses of the pavement structure. Gbadeyan and Oni (1992) investigated the dynamic responses of rectangular plate resting on elastic Pasternak foundation traversed by an arbitrary number of moving concentrated masses by using double Fourier sine integral transformation. Kim and Reosset (1998) studied the steady state response of infinite plate on an elastic foundation subjected to constant amplitude or harmonic moving loads, in which, the double and triple Fourier transformations are used to approximate the moving constant and moving harmonic loads.

The effects of parameters such as load's velocity, load frequency, and internal damping on the responses of the plate were examined. Fryba (1999) investigated the dynamical characteristics of an infinite or finite plate subjected to moving load for various boundary conditions. Huang and Thambiratnam (2001, 2002) developed a procedure with use the finite strip method to treat the responses of rectangular plate structures resting on an elastic foundations. Sun (2003, 2005) established a closed form solution by using Fourier transformation to derive the analytical dynamic solution of a Kirchhoff plate on a viscoelastic foundation to harmonic loads. Javad *et al.* (2013) employed the Eigenfunction Expansion Method (EEM) to investigate the vibration of Mindlin elastic plate under moving mass excitation.

The above mentioned works treat the pavement structure as a plate resting on a viscoelastic foundation model subjected to moving load. These approaches may be cumbersome when the foundation is modeled by multi-layer foundation. Hence, their use for dynamic analysis of pavement resting on multi-layer foundation is rather limited.

In practice, the finite element method (FEM) has been more widely used to solve the dynamic responses of pavement structure subjected to moving load. Yoshida and Weaver (1971) investigated the dynamic response of simply supported beams and plates to moving force and moving mass load. Wu *et al*. (1987) used FEM to analyze the response of flat plate structures and to study the effects of eccentricity, span length, acceleration and initial velocity of moving load. Zaman *et al.* (1991) investigated the dynamic response of a thick plate on viscoelastic foundation to moving load by taking into account the transverse shear deformation as well as bending of the slab. Pan and Atluri (1995) analyzed the dynamic response of finite sized elastic runways subjected to moving loads using a couple FEM/BEM approach. Li *et al.* (2013) studied the dynamic response of a rectangular plate on viscoelastic foundation under moving load with varying velocity. Zhang *et al.* (2013) investigated the dynamic response of highway subgrade under moving heavy trucks in cold regions. In this study, the 3D dynamic interaction model of heavy truck–pavement–subgrade is employed to calculate heavy truck vibration induced by road roughness and the dynamic response of the subgrade. Wu *et al*. (2014) built a three-dimensional finite element model (3D FEM) using ABAQUS to analyze the dynamic response of a concrete pavement structure with an asphalt isolating layer under moving loads. In FEM studies, a global coordinate system fixed in space is normally defined to form the structure matrices. In such an approach, as the load moves from one finite element into the next, the load vector has been updated at every time step of the solution procedure. Furthermore, some boundary conditions have been introduced artificially to truncate the infinitely long plate at the edge of the plate.

To overcome the complication encountered by FEM, Krenk *et al.* (1999) proposed the use of FEM in convected coordinates to obtain the response of an elastic half-space subject to a moving load. The key advantage enjoyed by this approach is its ability to overcome the problem due to the moving load travelling over a finite domain. Andersen *et al*. (2001) gave an FEM formulation for the problem of a beam on a Kelvin foundation subject to a harmonic moving load. Koh *et al.* (2003) adopted the idea of convected coordinates for solving train-track problems, and named the numerical algorithm as moving element method (MEM). The method was subsequently applied to the analysis of in-plane dynamic response of annular disk (Koh *et al.* 2006) and moving load on continuum (Koh *et al.* 2007). Xu *et al.* (2009) extended the one-dimensional MEM proposed by Koh *et al*. (2003) to two dimensional problem in which the vehicle moves on an infinite Kirchoff plate supported by Kelvin foundation. Recently, Ang *et al*. (2014) applied the MEM to investigate the "jumping wheel" phenomenon in high-speed train motion at constant velocity over a transition region where there is a sudden change of foundation stiffness. The effects of various key parameters such as speed of train, degree of track irregularity and degree of change of foundation stiffness at the transition region were examined. Tran *et al*. (2014) analyzed the dynamic response of high-speed rail traveling at non-uniform speed using moving element method. The linear and nonlinear wheel-rail contact models were examined in this study. On the same thread, Tran *et al.* (2016a, 2016b, 2017) employed the MEM method to investigate the dynamic response of high-speed rail experiencing heavy braking and abrupt braking. Most recently, Tran *et al.* (2017) examined the vertical dynamic response of high-speed rails during sudden deceleration. The effects of the wheel sliding, initial train deceleration, initial train speed and the severity of the railhead roughness on the dynamic response of the HSR were investigated. However, as the author's knowlegde there are not researches on dynamic responses of pavement structure resting on multi-layer foundation under moving load using moving element method.

In this study, a new approach, namely multi-layer moving plate method (MMPM), is proposed to investigate the dynamic responses of pavement structure resting on multi-layer foundation subjected to moving load. The pavement structure resting on multi-layer foundation is modeled as a two-layer plate connected by a spring-damper system and resting on a viscoelastic foundation. In which, the first layer plate is the surface slab of the pavement and the second layer plate is the base of the pavement. This model give a accurately pavement structure model so that the dynamic responses of the surface slab and the base can be obtained. The governing equations as well as plate element mass, damping and stiffness matrices are formulated in a relative coodinate system attached to the moving load. The main advantage is to simply treat the moving load as 'stationary' at the node of the plate to avoid the updating of the location of moving load due to change of contact points on the plate. To verify the accuracy of this method, the maximum displacement of the pavement structure under static load is calculated and compared with the solution of the ABAQUS 3D finite element model. Next, numerical examples related to the dynamic analysis of the pavement structure subjected to a moving load are conducted to investigate the effects of various parameters such as concrete slab thickness, base thickness, foundation stiffness and the load's velocity on dynamic responses of the pavement structure.

2. Formulation and methodology

2.1. Pavement on Multi-Layer Foundation Model

The pavement on multi-layer foundation, including slab, isolating layer, base, subbase and subgrade (Wu *et al.*, 2014) is considered as shown in Figure 1. The concrete slab and the base are modeled as two rectangular plates connected by the spring k_c and dashpot c_c system, which is the model of the asphalt isolating layer. The viscoelastic foundation are presented by the spring k_g and dashpot c_g. The pavement is subjected to a load (vehicle) moving with velocity V on the surface.

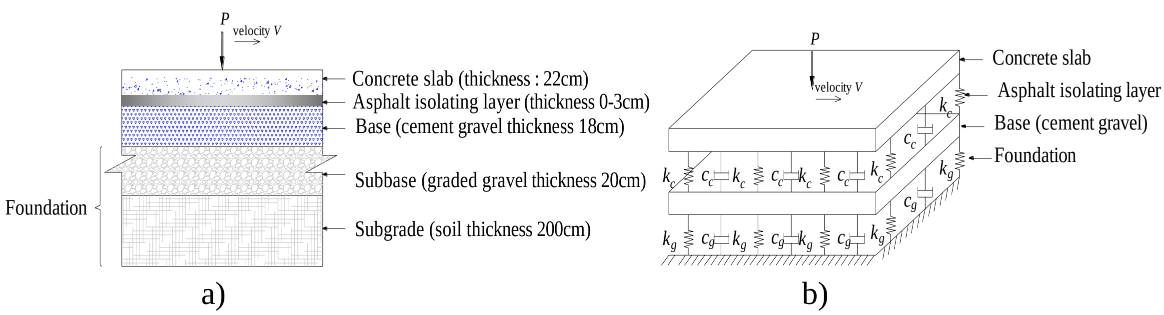


Figure 1. a) Concrete pavement structure; b) Concrete structure model.

Consider the plates under bending deflection. The middle (neutral) surfaces of two plates are chosen as the reference planes. Let w_c, w_g be the deflections and $\boldsymbol{\beta}_c = \left[\beta_{xc} \quad \beta_{yc} \right]^{\mathrm{T}}, \boldsymbol{\beta}_g = \left[\beta_{xg} \quad \beta_{yg} \right]^{\mathrm{T}}$ be the vectors of rotations of the concrete plate and the base plate, respectively. In which, $\beta_{xc}, \beta_{yc}, \beta_{xg}, \beta_{yg}$ are the rotations of the middle plane of the concrete plate and the base plate around $y-$axis and $x-$axis, respectively.

The unknown vector of three independent field variables at any point in the middle surfaces of the concrete plate and the base plate can be written as

$$\mathbf{u}_c = \left[w_c \quad \beta_{xc} \quad \beta_{yc} \right]^{\mathrm{T}} \tag{1}$$

$$\mathbf{u}_g = \left[w_g \quad \beta_{xg} \quad \beta_{yg} \right]^{\mathrm{T}} \tag{2}$$

The curvature of the deflected plate $\boldsymbol{\kappa}_{bc}, \boldsymbol{\kappa}_{bg}$ and the shear strains $\boldsymbol{\gamma}_{sc}, \boldsymbol{\gamma}_{sg}$ of the concrete plate and the base plate are defined, respectively, as

$$\boldsymbol{\kappa}_{bc} = \mathbf{L}_d \boldsymbol{\beta}_c \tag{3}$$

$$\boldsymbol{\kappa}_{bg} = \mathbf{L}_d \boldsymbol{\beta}_g \tag{4}$$

$$\boldsymbol{\gamma}_{sc} = \nabla w_c + \boldsymbol{\beta}_c \tag{5}$$

$$\boldsymbol{\gamma}_{sg} = \nabla w_g + \boldsymbol{\beta}_g \tag{6}$$

where $\nabla = \left[\partial / \partial x \quad \partial / \partial y \right]^T$, and $\mathbf{L}_d$ is a differential operator matrix defined by

$$\mathbf{L}_d = \begin{bmatrix} \partial / \partial x & 0 \\ 0 & \partial / \partial y \\ \partial / \partial y & \partial / \partial x \end{bmatrix} \tag{7}$$

The weak-form Galerkin for the dynamic response of the concrete plate is given by

$$\begin{aligned} & \int_{\Omega_c} \delta \boldsymbol{\kappa}_{bc}^{\mathrm{T}} \mathbf{D}_{bc} \boldsymbol{\kappa}_{bc} d\Omega + \int_{\Omega_c} \delta \boldsymbol{\gamma}_{sc}^{\mathrm{T}} \mathbf{D}_{sc} \boldsymbol{\gamma}_{sc} d\Omega \\ & + \int_{\Omega_c} \delta \mathbf{u}_c^{\mathrm{T}} \mathbf{m}_c \ddot{\mathbf{u}}_c d\Omega + \int_{\Omega_c} \delta w_c^{\mathrm{T}} k_c (w_c - w_g) d\Omega \\ & + \int_{\Omega_c} \delta w_c^T c_c (\dot{w}_c - \dot{w}_g) d\Omega = \int_{\Omega_c} \delta \mathbf{u}_c^T \mathbf{b}_c d\Omega \end{aligned} \tag{8}$$

The weak-form Galerkin for the dynamic response of the base plate is given by

$$\begin{aligned} & \int_{\Omega_g} \delta \boldsymbol{\kappa}_{bg}^{\mathrm{T}} \mathbf{D}_{bg} \boldsymbol{\kappa}_{bg} d\Omega_g + \int_{\Omega_g} \delta \boldsymbol{\gamma}_{sg}^{\mathrm{T}} \mathbf{D}_{sg} \boldsymbol{\gamma}_{sg} d\Omega_g \\ & + \int_{\Omega_g} \delta \mathbf{u}_g^{\mathrm{T}} \mathbf{m}_g \ddot{\mathbf{u}}_g d\Omega_g - \int_{\Omega_g} \delta w_g^{\mathrm{T}} k_c (w_c - w_g) d\Omega_g \\ & - \int_{\Omega_g} \delta w_g^T c_c (\dot{w}_c - \dot{w}_g) d\Omega_g + \int_{\Omega_g} \delta w_g^{\mathrm{T}} k_g w_g d\Omega_g \\ & + \int_{\Omega_g} \delta w_g^T c_c \dot{w}_g d\Omega_g = 0 \end{aligned} \tag{9}$$

where $\mathbf{b}_c = \left[P \quad 0 \quad 0 \right]^{\mathrm{T}}$ is the load vector applied on the concrete plate; $\mathbf{m}_c$ and $\mathbf{m}_g$ are mass matrices of the concrete plate and base plate containing the mass density of the material ρ_c, ρ_g and the thickness h_c, h_g as

$$\mathbf{m}_c = \rho_c \begin{bmatrix} h_c & 0 & 0 \\ 0 & \dfrac{h_c^3}{12} & 0 \\ 0 & 0 & \dfrac{h_c^3}{12} \end{bmatrix}; \quad \mathbf{m}_g = \rho_g \begin{bmatrix} h_g & 0 & 0 \\ 0 & \dfrac{h_g^3}{12} & 0 \\ 0 & 0 & \dfrac{h_g^3}{12} \end{bmatrix} \tag{10}$$

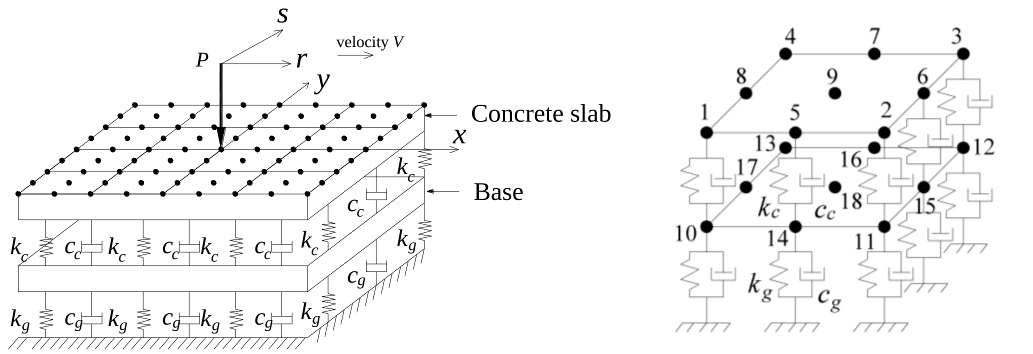
and $\mathbf{D}_{bc}$, $\mathbf{D}_{bg}$ and $\mathbf{D}_{sc}$, $\mathbf{D}_{sg}$ are the material matrices related to the bending and shear deformation of the concrete plate and base plate, respectively, as follows

$$\mathbf{D}_{bc} = \frac{E_c h_c^3}{\left(1 - \nu_c^2\right)} \begin{bmatrix} 1 & \nu_c & 0 \\ \nu_c & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu_c}{2} \end{bmatrix}; \mathbf{D}_{bg} = \frac{E_g h_g^3}{\left(1 - \nu_g^2\right)} \begin{bmatrix} 1 & \nu_g & 0 \\ \nu_g & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu_g}{2} \end{bmatrix} \tag{11}$$

$$\mathbf{D}_{sc} = \frac{k E_c h_c}{2\left(1 + \nu_c\right)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \mathbf{D}_{sg} = \frac{k E_g h_g}{2\left(1 + \nu_g\right)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{12}$$

2.2. Multi-layer moving plate method (MMPM)

In this section, a new approach, namely multi-layer moving plate method, is proposed to study dynamic response of pavement structure resting on multi-layer foundation. In this method, the multi-layer moving plate elements are formulated in a convected coordinate system attached to the moving load, instead of a fixed coordinate system. In the model, the concrete slab and the base plate are considered as two layer plates and discretized into multi-layer moving plate elements, as shown in Figure 2.



a) b)

Figure 2. a) Discretization of the pavement structure model into multi-layer moving plate elements; b) A typical multi-layer moving plate element with 54 DOFs.

The quadrilateral nine-node $(Q9)$ element of the serendipity family is employed in this study. The node displacement vector can be defined for the multi-layer moving plate element as

$$\mathbf{d} = \begin{bmatrix} w_1 & \beta_{x1} & \beta_{y1} & \dots & w_{18} & \beta_{x18} & \beta_{y18} \end{bmatrix}^T \quad (13)$$

where w_i, β_{xi}, β_{yi} $(i = 1, 2, ..., 18)$ represent vertical displacements and rotations for node i.

Using the shape function, the displacement vectors of the concrete plate and the base plate can be expressed as

$$\mathbf{u}_c = \mathbf{N}_c \mathbf{d} \quad (14)$$

$$\mathbf{u}_g = \mathbf{N}_g \mathbf{d} \quad (15)$$

with

$$\mathbf{N}_c = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \dots & N_9 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \dots & 0 & N_9 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & \dots & 0 & 0 & N_9 & \dots & 0 & 0 & 0 \end{bmatrix}_{3x54} \quad (16)$$

$$\mathbf{N}_g = \begin{bmatrix} 0 & 0 & 0 & \dots & N_1 & 0 & 0 & N_2 & 0 & 0 & \dots & N_9 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & N_1 & 0 & 0 & N_2 & 0 & \dots & 0 & N_9 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & N_1 & 0 & 0 & N_2 & \dots & 0 & 0 & N_9 \end{bmatrix}_{3x54} \quad (17)$$

and the vertical displacements of the concrete plate and the base plate can be expresses as

$$w_c = \mathbf{N}_{wc} \mathbf{d} \quad (18)$$

$$w_g = \mathbf{N}_{wg} \mathbf{d} \quad (19)$$

where $\mathbf{N}_{wc}$, $\mathbf{N}_{wg}$ are the matrices containing the displacement of shape function.

$$\mathbf{N}_{wc} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \dots & N_9 & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix}_{1x54} \quad (20)$$

$$\mathbf{N}_{wg} = \begin{bmatrix} 0 & 0 & 0 & \dots & N_1 & 0 & 0 & N_2 & 0 & 0 & \dots & N_9 & 0 & 0 \end{bmatrix}_{1x54} \quad (21)$$

The bending and shear strains of the concrete plate can be expressed in the matrix forms as

$$\boldsymbol{\kappa}_{bc} = \mathbf{B}_{bc} \mathbf{d}, \ \boldsymbol{\gamma}_{sc} = \mathbf{B}_{sc} \mathbf{d} \quad (22)$$

in which

$$\mathbf{B}_{bc} = \begin{bmatrix} 0 & \frac{\partial N_1}{\partial x} & 0 & \dots & 0 & \frac{\partial N_9}{\partial x} & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_1}{\partial y} & \dots & 0 & 0 & \frac{\partial N_9}{\partial y} & \dots & 0 & 0 & 0 \\ 0 & \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \dots & 0 & \frac{\partial N_9}{\partial y} & \frac{\partial N_9}{\partial x} & \dots & 0 & 0 & 0 \end{bmatrix}_{3x54} \quad (23)$$

$$\mathbf{B}_{sc} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & N_1 & 0 & \dots & \frac{\partial N_9}{\partial x} & N_9 & 0 & \dots & 0 & 0 & 0 \\ \frac{\partial N_1}{\partial y} & 0 & N_1 & \dots & \frac{\partial N_9}{\partial y} & 0 & N_9 & \dots & 0 & 0 & 0 \end{bmatrix}_{2x54} \quad (24)$$

and the bending and shear strains of the base plate can be expressed in the matrix forms as

$$\boldsymbol{\kappa}_{bg} = \mathbf{B}_{bg} \mathbf{d}, \ \boldsymbol{\gamma}_{sg} = \mathbf{B}_{sg} \mathbf{d} \quad (25)$$

in which

$$\mathbf{B}_{bg} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & \frac{\partial N_1}{\partial x} & 0 & \dots & 0 & \frac{\partial N_9}{\partial x} & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \frac{\partial N_1}{\partial y} & \dots & 0 & 0 & \frac{\partial N_9}{\partial y} \\ 0 & 0 & 0 & \dots & 0 & \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \dots & 0 & \frac{\partial N_9}{\partial y} & \frac{\partial N_9}{\partial x} \end{bmatrix}_{3x54} \quad (26)$$

$$\mathbf{B}_{sg} = \begin{bmatrix} 0 & 0 & 0 & \dots & \frac{\partial N_1}{\partial x} & N_1 & 0 & \dots & \frac{\partial N_9}{\partial x} & N_9 & 0 \\ 0 & 0 & 0 & \dots & \frac{\partial N_1}{\partial y} & 0 & N_1 & \dots & \frac{\partial N_9}{\partial y} & 0 & N_9 \end{bmatrix}_{2x54} \quad (27)$$

The MMPM method adopts the idea of attaching the origin of the spatial coordinates system to the applied point of the moving load. Fig. 2 shows a travelling coordinate (r, s) moving at the same speed with the moving load. The relationship between the moving coordinate (r, s) and the fixed coordinate (x, y) is given by

$$r = x - Vt \\ s = y \tag{28}$$

With simple transformation, the govening equations calculated in the moving coordinate (r, s) for the concrete plate in Eq. (8) can be rewritten as

$$\mathbf{M}_{ec}\ddot{\mathbf{d}} + \mathbf{C}_{ec}\dot{\mathbf{d}} + \mathbf{K}_{ec}\mathbf{d} = \mathbf{F}_{ec} \tag{29}$$

in which

$$\mathbf{M}_{ec} = \mathbf{m}_c \int_{\Omega_{ec}} \mathbf{N}_c^T \mathbf{N}_c d\Omega_{ec} \tag{30}$$

$$\mathbf{C}_{ec} = \int_{\Omega_{ec}} \left(-2\mathbf{m}_c V \mathbf{N}_c^T \mathbf{N}_{c,r} + c_c \mathbf{N}_{wc}^T \mathbf{N}_{wc} - c_c \mathbf{N}_{wc}^T \mathbf{N}_{wg}\right) \mathrm{d}\Omega_{ec} \tag{31}$$

$$\mathbf{K}_{ec} = \int_{\Omega_{ec}} \mathbf{B}_{bc}^T \mathbf{D}_{bc} \mathbf{B}_{bc} \mathrm{d}\Omega_{ec} + \int_{\Omega_{ec}} \mathbf{B}_{sc}^T \mathbf{D}_{sc} \mathbf{B}_{sc} \mathrm{d}\Omega_{ec} + \int_{\Omega_{ec}} \begin{pmatrix} \mathbf{m}_c V^2 \mathbf{N}_c^T \mathbf{N}_{c,rr} + k_c \mathbf{N}_{wc}^T \mathbf{N}_{wc} - k_c \mathbf{N}_{wc}^T \mathbf{N}_{wg} \\ -c_c V \mathbf{N}_{wc}^T \mathbf{N}_{wc,r} + c_c V \mathbf{N}_{wc}^T \mathbf{N}_{wg,r} \end{pmatrix} \mathrm{d}\Omega_{ec} \tag{32}$$

$$\mathbf{F}_{ec} = \int_{\Omega_{ec}} \mathbf{N}_c^T \mathbf{b} \mathrm{d}\Omega_{ec} \tag{33}$$

The govening equations calculated in the moving coordinate (r, s) for the base plate in Eq. (9) can be rewritten as

$$\mathbf{M}_{eg}\ddot{\mathbf{d}} + \mathbf{C}_{eg}\dot{\mathbf{d}} + \mathbf{K}_{eg}\mathbf{d} = \mathbf{F}_{eg} \tag{34}$$

in which

$$\mathbf{M}_{eg} = \mathbf{m}_g \int_{\Omega_{eg}} \mathbf{N}_g^T \mathbf{N}_g d\Omega_{eg} \tag{35}$$

$$\mathbf{C}_{eg} = \int_{\Omega_{eg}} \begin{pmatrix} -2\mathbf{m}_g V \mathbf{N}_g^T \mathbf{N}_{g,r} + c_g \mathbf{N}_{wg}^T \mathbf{N}_{wg} \\ -c_c \mathbf{N}_{wg}^T \mathbf{N}_{wc} + c_c \mathbf{N}_{wg}^T \mathbf{N}_{wg} \end{pmatrix} \mathrm{d}\Omega_{eg} \tag{36}$$

$$\mathbf{K}_{eg} = \int_{\Omega_{eg}} \mathbf{B}_{bg}^T \mathbf{D}_{bg} \mathbf{B}_{bg} \mathrm{d}\Omega_{eg} + \int_{\Omega_{eg}} \mathbf{B}_{sg}^T \mathbf{D}_{sg} \mathbf{B}_{sg} \mathrm{d}\Omega_{eg} + \int_{\Omega_{eg}} \begin{pmatrix} \mathbf{m}_g V^2 \mathbf{N}_g^T \mathbf{N}_{g,rr} - c_g V \mathbf{N}_{wg}^T \mathbf{N}_{wg,r} \\ +k_g \mathbf{N}_{wg}^T \mathbf{N}_{wg} - k_c \mathbf{N}_{wg}^T \mathbf{N}_{wc} + k_c \mathbf{N}_{wg}^T \mathbf{N}_{wg} \\ +c_c V \mathbf{N}_{wg}^T \mathbf{N}_{wc,r} - c_c V \mathbf{N}_{wg}^T \mathbf{N}_{wg,r} \end{pmatrix} \mathrm{d}\Omega_{eg} \tag{37}$$

$$\mathbf{F}_{eg} = 0 \tag{38}$$

Finally, the mass, damping and stiffness mactrices for a typical multi-layer moving plate element can be obtained as

$$\mathbf{M}_e = \mathbf{M}_{ec} + \mathbf{M}_{eg} \tag{39}$$

$$\mathbf{C}_e = \mathbf{C}_{ec} + \mathbf{C}_{eg} \tag{40}$$

$$\mathbf{K}_e = \mathbf{K}_{ec} + \mathbf{K}_{eg} \tag{41}$$

$$\mathbf{F}_e = \mathbf{F}_{ec} + \mathbf{F}_{eg} \tag{42}$$

Assembling all element matrices gives the equation of motion of the pavement structure resting on multi-layer foundation as:

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{C}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{F} \tag{43}$$

where $\ddot{\mathbf{d}}$, $\dot{\mathbf{d}}$ and $\mathbf{d}$ denote the global acceleration, velocity and displacement vectors of the nodal, respectively; $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ the global mass, damping and stiffness matrices, respectively; and $\mathbf{F}$ the global load vector. The above dynamic equation can be solved by any direct integration methods such as Newmark-β method.

3. Numerical results

In this section, several numerical examples are carried out in order to investigate the dynamic responses of pavement structures resting on multi-layer foundation subjected to moving load. Firstly, the displacement of the pavement under static load is investigated for verifying the accuracy of the proposed method. Then numerical examples related to the dynamic analysis of the pavement subjected to moving load are conducted to determine the effects of concrete slab thickness, base thickness, foundation stiffness and the load's velocity on the dynamic responses of the pavement.

Static analysis

For the purpose of verification, the concrete pavement structure model is used the same with the model of Wu et al. (2014) as shown in Figure 3. The size of concrete pavement slab is 4.5mx3.75m and the slab thickness is 0.22m. The thickness of base plate (cement stabilized gravel) is 0.18m. The material parameters of the concrete slab and the base (cement gravel) are shown in Table 1. The asphalt isolating layer is modeled by the spring k_c and dashpot c_c system and the viscoelastic foundation is modeled by the spring k_g and dashpot c_g system. The values of these stiffness and damping coefficients are presented in Table 2. The pavement structure subjected to a static load P=100000N at the center of the pavement and the plate is discretized into 20×20 elements.

a)

b)

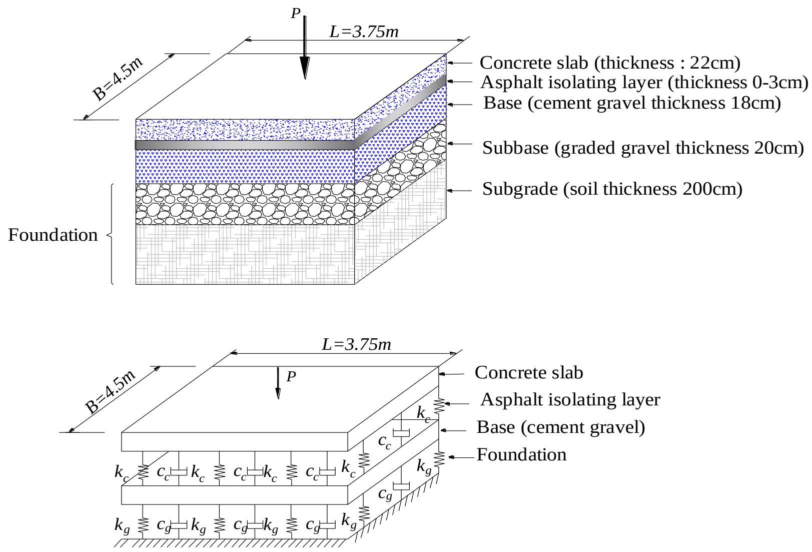


Figure 3. a) Concrete pavement structure and b) Concrete pavement model.

Table 1

Material parameters

Pavement structure	Modulus of elasticity E_c (N/m^2)	Poisson's ratio ν_c	Density ρ_c (kg/m^3)
Concrete slab	$3.1\text{x}10^{10}$	0.15	2400
Base (cement gravel)	$1.5\text{x}10^{9}$	0.25	2300

Table 2

The stiffness and damping coefficients of the connecting system of two plates and the stiffness and damping coefficients of the foundation

Pavement structure	Stiffness coefficient (N/m^3)	Damping coefficient (Ns/m^3)
Connection	$k_c = 1.72\text{x}10^8$	$c_c = 1.75\text{x}10^6$
Foundation	$k_g = 1.52\text{x}10^7$	$c_g = 3.79\text{x}10^5$

Table 3 presents the maximum displacement at the center of the concrete slab under static load. The solutions obtained in this study are compared with the result of Wu *et al.* (2014) using the ABAQUS 3D finite element model. The comparision shows that these results are strong agreement.

Table 3

Maximum displacement at the center of concrete slab under static load

Method	Maximum displacement (mm)
ABAQUS 3D finite element model (Wu *et al.*, 2014)	0.3562
MMPM (This study)	0.3561
Percentage difference (%)	0.03%

4. Dynamic analysis

After verifying the accuracy of the proposed method, the effects of various parameters on the dynamic responses of the pavement structure subjected to moving load are investigated in this section. A rectangular pavement structure having length $L = 20\,m$, width $B = 10\,m$ is considered. The material properties of the pavement structure are employed as in example 3.1. A load $P = 100000\,N$ moves along the longitudinal middle line of the plate with velocity $V = 40km/h$. In this analysis, the plate is discretized into 20×10 moving elements of size $1m \times 1m$. The equations of motion are solved using Newmark's constant acceleration method employing a time-step of 0.0025s. Note that this configuration of the MEM mesh and time-step size have been decided based on the outcome of a convergence study.

Firstly, the effects of the concrete slab thickness and the base thickness on the maximum displacement of the pavement are investigated. Figure 4 and Figure 5 show the variation of maximum displacements of concrete slab and base against the change of concrete slab thickness and base thickness, respectively. Figure 6 and Figure 7 show the deflected shapes of the along longitudinal mid line of the concrete pavement slab and the base plate with different concrete pavement slab thickness. Figure 8 and Figure 9 show the deflected shapes of the along longitudinal mid line of the concrete pavement slab and the base plate with different base thickness. The general trend is that when the concrete slab thickness or the base thickness increases, the maximum displacements of both two layers decrease. Figure 4 shows that the the concrete slab thickness has more strong effect on the displacement than the base thickness. However, when the concrete slab thickness increases more than 0.8m, the displacement decreases very slightly.

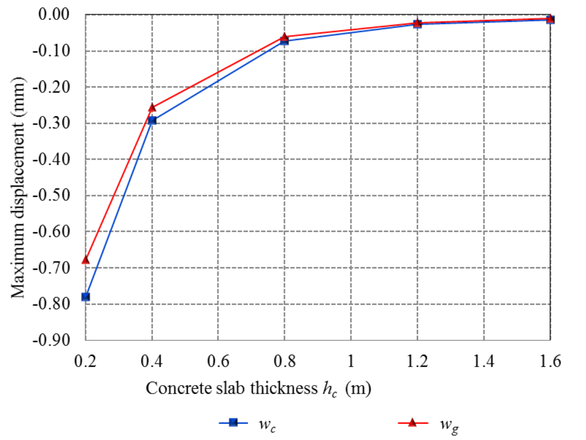


Figure 4. The variation of the maximum displacement of the concrete slab and the base against the change of the concrete slab thickness

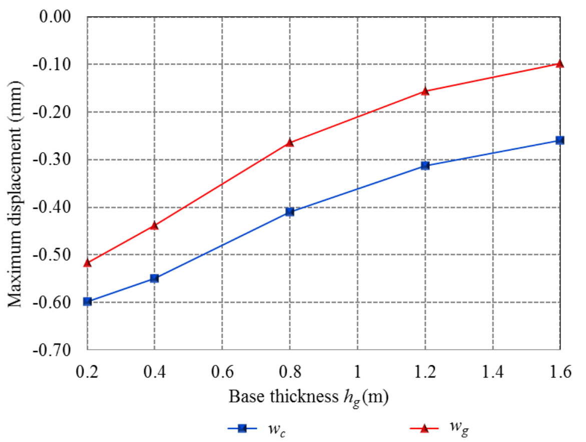


Figure 5. The variation of the maximum displacement of the concrete slab and the base against the change of the base thickness

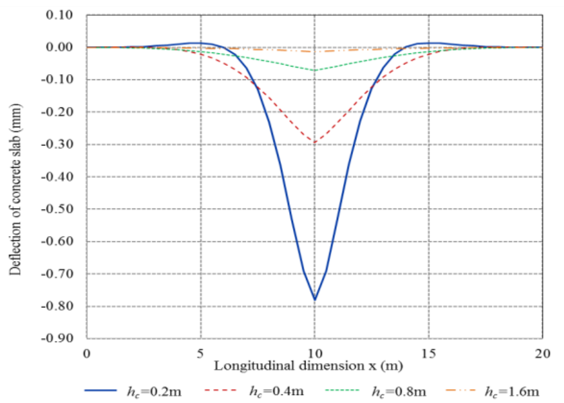


Figure 6. The deflected shape of the concrete slab along longitudinal mid line of the plate for different concrete slab thickness

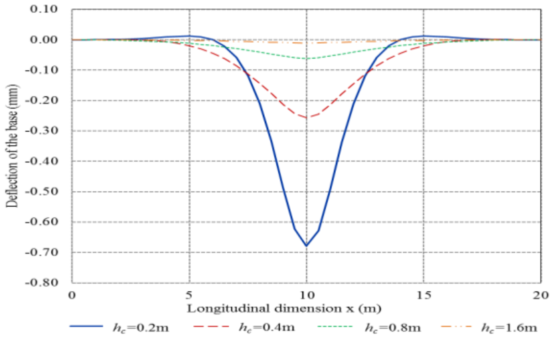


Figure 7. The deflected shape of the base plate along longitudinal mid line of the plate for different concrete slab thickness

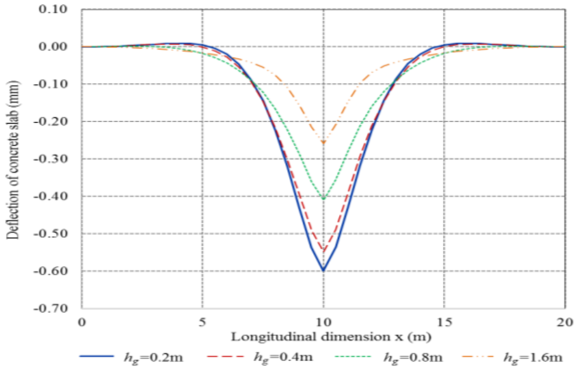


Figure 8. The deflected shape of the concrete slab along longitudinal mid line of the plate for different base thickness

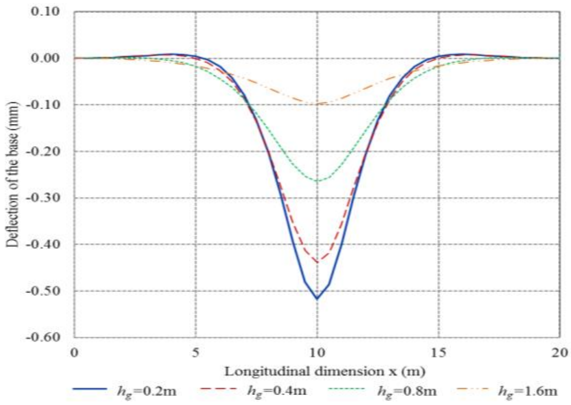


Figure 9. The deflected shape of the base plate along longitudinal mid line of the plate for different base thickness

Secondly, the effects of the foundation stiffness on the maximum displacement of the pavement structure are investigated. Figure 10 shows the variation of maximum displacement of the concrete slab and base plate when the foundation stiffness increases from 1.52x10^{7}N/m^3 to 10x1.52x10^{7}N/m^3. Figure 10 shows that the maximum displacements of both the concrete slab and the base plate decrease gradually as the foundation stiffness increases. It can be inferred that the increment of foundation stiffness can reduce the damage of the pavement structure.

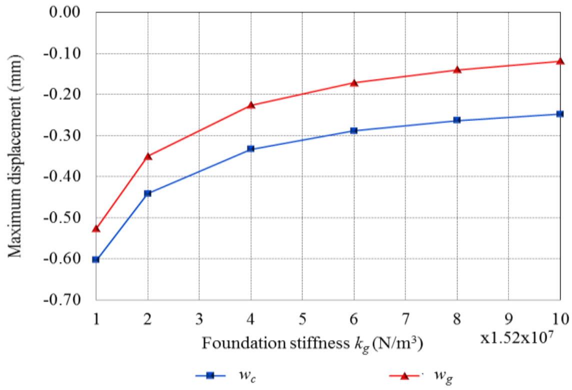


Figure 10. The variation of the maximum displacement of the concrete slab and the base plate against the change of foundation stiffness.

Finally, the effects of the load's velocity on the maximum displacement of the pavement structure are investigated. Figure 11 and Figure 12 shows the variation of maximum displacements of the concrete slab and the base against the increment of the load’s velocity. Figure 11 shows that when the load's velocity increases, the the maximum displacement of the concrete slab decreases. By contrast, when the load's velocity increases, the the maximum displacement of the base increases. It can be inferred that when the load moves with high velocity, the base is affected stronger than the concrete pavement on the top.

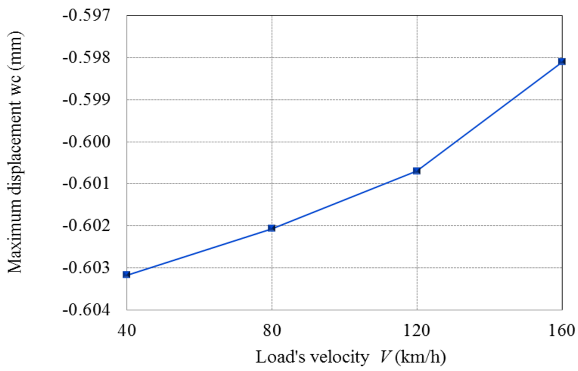


Figure 11. The variation of the maximum displacement of the concrete slab against the increment of load’s velocity

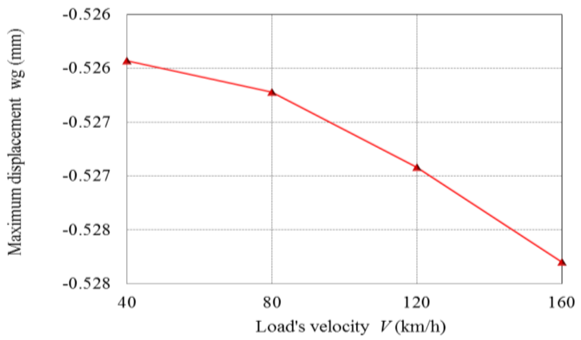


Figure 12. The variation of the maximum displacement of the base against the increment of load's velocity

5. Conclusion

In this paper, a new approach, namely multi-layer moving plate method (MMPM), has been suggested to investigate the dynamic responses of the pavement structure resting on multi-layer foundation subjected to moving load. The pavement structure resting on multi-layer foundation is modeled as a two-layer plate connected by a spring-damper system and resting on a viscoelastic foundation. This model gives a accurately pavement structure model so that the dynamic responses of the surface slab and the base can be obtained. The governing equations as well as the multi-layer plate element mass, damping and stiffness matrices formulated in a relative coodinate system attached to the moving load are proposed. Through the numerical examples, some concluding remarks can be drawn as follows:

- The proposed method converts the moving load problem into an equivalent static problem which can be solved more efficiently than solving dynamic equation. Thus, this method can overcome the difficulty encountered by finite element method.
- This method gives a accurately pavement structure model so that the dynamic responses of the surface slab and the base can be obtained.
- The accuracy of the method is illustrated by comparing the results obtained in this study with those of the ABAQUS 3D finite element model.
- The general trend is that when the concrete slab thickness, the base thickness and the foundation stiffness increase, the maximum displacements of both two layers decrease. However, when the concrete slab thickness increases more than 0.8m, the displacement decreases very slightly. In addition, as the load moves with high velocity, the base is affected stronger than the concrete pavement on the top■

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References

Gbadeyan, J. A., Oni, S. T. (1992). Dynamic response to moving concentrated masses of elastic plates on a non-Winkler elastic foundation. *Journal of Sound and Vibration*, *154*(2), 343-358.

Kim S. M., Roesset, J. M. (1998). Moving loads on a plate on elastic foundation, *Journal of Engineering Mechanics*, *124*, 1010–1017.

Fryba, L. (1999). *Vibration of solid and structures under moving loads*, Thomas Telford, London.

Huang, M. H., Thambiratnam, D. P. (2001). Deflection response of plate on winkler foundation to moving accelerated loads, *Engineering Structures*, *23*, 1134-1141.

Huang, M. H., Thambiratnam, D. P. (2002). Dynamic response of plates on elastic foundation to moving loads. *Journal of Engineering mechanics*, *128*(9), 1016-1022.

Sun, L. (2003). Dynamic response of Kirchhoff plate on viscoelastic foundation to harmonic circular loads. *Journal of Applied Mechanics*, *70*, 595-600.

Sun, L. (2005). Dynamic of plate generated by Moving Harmonic Loads. *Journal of Applied Mechanics*, *72*, 772-777.

Javad, A. V., Ali, N., Mohammad, D. R., Mohsen, H. E. (2013). Vibration analysis of Mindlin elastic plate under moving mass excitation by eigenfunction expansion method. *Thin-Walled Structures*, *62*, 53–64.

Yoshida, D. M., Weaver, W. (1971). Finite element analysis of beams and plates with moving load. *International Association for Bridge and Structural Engineering*, *31*, 179-195.

Wu, J. S., Lee, M. L., Lai, T. S. (1987). The dynamic analysis of a flat plate under a moving load by finite element method. *International Journal for Numerical Methods in Engineering*, *124*, 1010-1017.

Zaman, M., Taheri, M. R., Alvappallai, A. (1991). Dynamic response of thick plate on elastic foundation to moving loads. *International Journal for Numerical and Analytical Methods in Geomechanics*, *15*, 627-647.

Pan, G., Atluri, S. N. (1995). Dynamic response of finite sized elastic runways subjected to moving loads: a couple BEM/FEM approach. *International Journal for Numerical Methods in Engineering*, *38*, 3143-3167.

Li, M., Quian, T., Zhong Y., Zong, H. (2013). Dynamic response of rectangular plate subjected to moving load with variable velocity. *Journal of Engineering Mechanics*, *4*, 1943-7889.

Zhang, F., Feng, D. C., Ling, X. Z., Li, Q. L (2013). Numerical simulation of dynamic response of subgrade under moving heavy truck in cold regions. *Sciences in cold and arid regions*, *5*, 0468-0477.

Wu, J., Liang, J., Adhikari, S., (2014). Dynamic response of concrete pavement structure with asphalt isolating layer under moving load. *Journal of Traffic and Transportation Engineering*, *1*, 439-447.

Krenk, S., Kellezi, L., Nielsen, S. R. K., Kirkegaard, P. H. (1999). Finite elements and transmitting boundary conditions for moving loads. *Proceedings of the 4th European Conference on Structural Dynamics, Eurodyn '99, Praha, June 7-1*, *1*, 447-452.

Andersen, L., Nielsen, S. R. K., Kirkegaard, P. H. (2001). Finite element modelling of infinite Euler beams on Kelvin foundations exposed to moving loads in convected co-ordinates. *Journal of Sound and Vibration*, *241*(4), 587-604.

Koh, C. G., Ong, J. S. Y., Chua, D. K. H., Feng, J. (2003). Moving element method for train-track dynamics. *International Journal for Numerical Methods in Engineering*, *56*, 1549–1567.

Koh, C. G., Sze, P. P., Deng T. T. (2006). Numerical and analytical methods for in-plane dynamic response of annular disk. *International Journal of Solids and Structures*, *43*, 112-131.

Koh, C. G., Chiew, G. H., Lim, C. C. (2007). A numerical method for moving load on continuum. *Journal of Sound and Vibration*, *300*, 126-138.

Xu, W. T., Lin, J. H., Zhang, Y. H., Kennedy, D., Williams, F. W. (2009). 2D moving element method for random vibration analysis of vehicles on Kirchoff plate with Kelvin foundation. *Latin American Journal of Solids and Structures*, *6*, 169-183.

Ang, K. K., Jian, D., Tran, M. T., Luong, V. H. (2014). Analysis of high-speed rail accounting for jumping wheel phenomenon. *International Journal of Computational Methods*, *11*, 1343007.

Tran, M. T., Ang, K. K., Luong, V. H. (2014). Vertical dynamic response of non-uniform motion of high-speed rails. *Journal of Sound and Vibration*, *333*, 5427-5442.

Tran, M. T., Ang, K. K., Luong, V. H. (2016a). Dynamic response of high-speed rails due to heavy braking. *Journal of Rail and Rapid Transit*, http://dx.doi.org/10.1177/0954409716639997.

Tran, M. T., Ang, K. K., Luong, V. H. (2016b). Multiple-rail high speed train subject to braking. *Internaltional Journal of Structural Stability and Dynamics*, DOI: http://dx.doi.org/10.1142/S021945541 7500717.

Tran, M. T., Ang, K. K., Luong, V. H. (2017). Dynamic response of high-speed rails subject to abrupt braking, Vehicle System Dynamics. *International Journal of Vehicle Mechanics and Mobility*, (Accepted).

Tran, M. T., Ang, K. K., Luong, V. H. (2017). Vertical dynamic response of high-speed rails during sudden deceleration. *International Journal of Computational Methods*, *14*(1), 1750014 (24 pages), DOI: 10.1142/S0219876217500141