

3D numerical investigation of functionally graded concrete

Nguyen Ngoc Hau^{1*}, Le Thanh Cuong²

¹University of Science and Technology, The University of Danang, Danang City, Vietnam

²Ho Chi Minh City Open University, Ho Chi Minh City, Vietnam

*Corresponding author: nnhau@dut.udn.vn

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ABSTRACT

In the design of engineering structures, concrete is assumed to have homogeneously mechanical properties. However, in reality, the concrete material is usually not uniform with respect to the height of structures because segregation mostly occurs during construction. The higher density is presented at the lower part of the structure due to a higher proportion of aggregate and vice versa. Moreover, the disparity of creep, shrinkage, hydration heat, and curing method also leads to the difference in the quality of concrete. Therefore, it is necessary to study the responses of a concrete element with various mechanical properties. In this study, the functional elastic modulus of concrete material (FGC) is investigated numerically for its effects on the static responses of the element under axial compression. Conventional Finite Element Method (FEM) and Meshless method are used to model the structure. The reliability of the numerical model is validated by comparing the obtained results with experimental data.

1. Introduction

The conventional concrete structures are cast according to engineering designs, which follow a standard procedure, whereas the uniform distribution of concrete properties is considered throughout the entire structure. Nevertheless, concrete is a complex material, caused mainly by compaction methods, especially in the cases of high water-cement ratio. In this process, the heavier aggregates tend to sink down while the paste tends to float up, which is known as segregation (Ozyildirim & Lane, 2003). As a result, the mechanical properties of concrete are varied with respect to structure dimensions (height or width). These properties can be elastic modulus, compressive strength, mass density, and Poisson's ratio (Ramu & Mohanty, 2014). The properties of concrete are not only randomly created during construction but also intentionally made to optimize the working condition of the structure. For example, in design, the stress in the tension fiber of a reinforced concrete beam is transferred to reinforcement. Therefore, it is more economical if the lower-strength concrete is used in this area. Otherwise, the higher compressive strength concrete should be used in the compressive portion of the beam.

Gan, Aylie, and Pratama (2015) experimentally studied heterogeneous concrete as a functionally graded material. They made a cylindrical element with mechanical properties varying across its height. The compressive responses of the tested elements were successfully obtained. Hidayat, Purwanto, Puspowardojo, and Aziz (2015) investigated the real properties of a relatively high concrete element, which is cast in-place. A series of different experiments were carried out on plain concrete specimens and reinforced beams of a building. These specimens were normally cast with the design compressive strength of 60MPa. The authors concluded that the compressive

strength was proportionally increased to the bottom of the structures. Guo, Lin, Qin, Xu, and Dong (2022) numerically studied the tension behavior of fully-graded concrete specimens under uniaxial load. The mesoscopic models were investigated by using the phase-field method. The crack propagation was the target of their study, considering the effects of step time, material parameters, and mesostructures. Additionally, the authors modeled the aggregates with polygonal shapes. The results of stress-strain curves and crack paths were compared to experimental results with high agreement. Yazhini and Chithra (2022) studied the performance of Functionally Graded Concrete (FGC) pipes reinforced by steel and basalt fiber. A particular 0.75% dosage of fiber reinforcement was presented as optimum for FGC in terms of compressive strength, flexural strength, and split strength. The comparison of the strength and crack behavior of functionally graded concrete pipes under three-edge bearing made of steel and basalt fiber were also presented. They showed that steel fiber was good for concrete mechanical strength, while basalt fiber increased the post-crack strength.

In this study, the static responses of the three-dimensional cylinders under uniaxial compression were numerically investigated. The specimens were experimentally tested by Gan et al. (2015), which have the elastic modulus varies along the height. The conventional Finite Element Method (FEM) and meshless methods are used to model the structures. The material functions with power relation and exponential relation were studied for their best performance.

2. Formulations of material and numerical methods

2.1. Functionally graded concrete

In this study, the most common functions of graded material are considered to verify if they reflect the behavior of real concrete specimens. The exponential law and power-law of elastic modulus with respect to the height of the specimen are expressed as follows (Sayyad & Ghugal, 2019):

2.1.1. Power-law distribution

The power-law distribution of material gradation is used widely in the analysis of functionally graded material (Figure 1a). It is formulated based on the rule of mixtures, which is the distribution of volume fraction of composite material varying across FG elements' dimensions. The formulation of concrete property, which is elastic modulus in this study, is given as follows:

$$P(z) = P_t V_t(z) + P_b V_b(z) \quad (1)$$

Where the volume fraction functions (V_t , V_b) of the materials at the top and bottom layers, respectively, are as follows:

$$V_t(z) + V_c(z) = 1 \quad (2)$$

$$V_c(z) = \left(0.5 + \frac{z}{h}\right)^p, \text{ for } z \in \left[-\frac{h}{2}, \frac{h}{2}\right] \quad (3)$$

Where p is the power coefficient in the z direction (the height of the specimen).

The young modulus at an interest point $\mathbf{x}(z)$ can be re-written as:

$$E_c(z) = E_{cb} \left[1 + \frac{E_{ct} - E_{cb}}{E_{cb}} \left(\frac{z}{h}\right)^p\right] \quad (4)$$

2.1.2. Exponential-law distribution:

The mechanical properties of functionally graded concrete specimens distribute according to an exponential law as follows:

$$E_c(z) = E_{cb} \exp \left[\ln \left(\frac{E_{ct}}{E_{cb}} \right) \left(\frac{z}{h} \right)^p \right], \text{ for } z \in \left[-\frac{h}{2}, \frac{h}{2} \right] \quad (5)$$

In this study, the various distributions of concrete material will be studied. Additionally, the shapes of material functions are changed by using different constants, $p = [0.2, 0.5, 1, 2, 5]$ as illustrated in Figure 1.

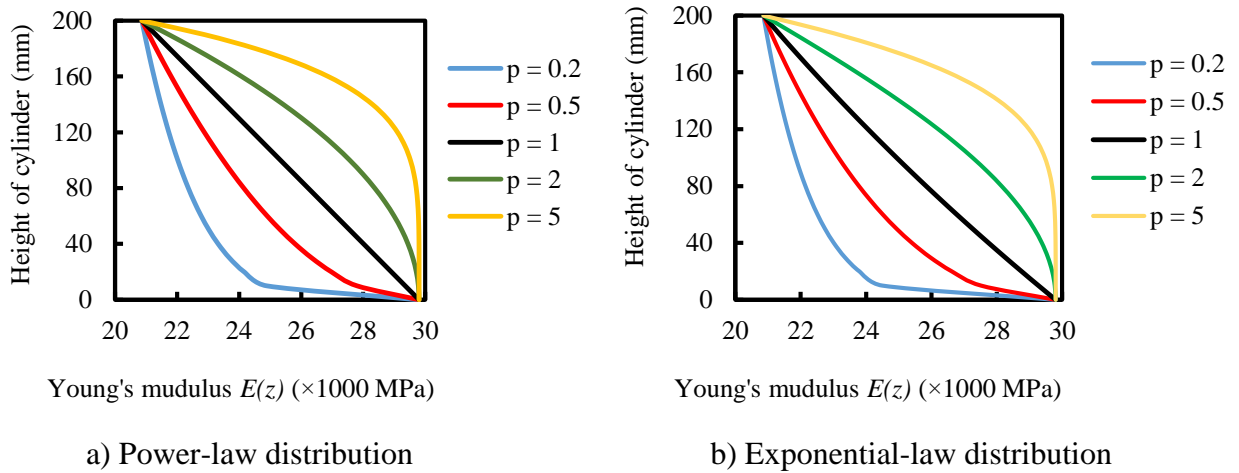


Figure 1. Distribution of Young's modulus of a concrete cylinder, which has $E_{ct} = 20865\text{MPa}$ and $E_{cb} = 29798\text{MPa}$

2.2. Meshless method formulations

In this analysis, a typical meshless method with a set of discretized nodes is used to solve the Galerkin weak form of static problems. The moving least square approximation is applied to interpolate the problem domain as shape functions. The mentioned meshless method is also called as the element-free Galerkin method (EFGM), which is used extensively in many solid mechanics problems. The moving least square shape functions are constructed to interpolate with a support domain, which has a spherical shape in this study. These support domains overlap each other to define the connectivity of discretised points of the problems domain.

Considering a spherical support domain Ω_s containing N nodes (Figure 2). The MLS approximation $u^h(\mathbf{x})$ over domain Ω_s is given as

$$u^h(\mathbf{x}) = \sum_{i=1}^N \phi_i(\mathbf{x}) u_i \quad (6)$$

where

$$\phi_i(\mathbf{x}) = \sum_{j=1}^m p_j(\mathbf{x}) [\mathbf{A}^{-1}(\mathbf{x}) \mathbf{B}(\mathbf{x})]_{ji} \quad (7)$$

and

$$w_i(\mathbf{x}) = \begin{cases} 1 - 6 \left(\frac{d_i}{r_i}\right)^2 + 8 \left(\frac{d_i}{r_i}\right)^3 - 3 \left(\frac{d_i}{r_i}\right)^4 & 0 \leq d_i \leq r_i \\ 0 & d_i \geq r_i \end{cases} \quad (8)$$

with $d_i = |\mathbf{x}_i - \mathbf{x}|$ is the distance between \mathbf{x}_i and \mathbf{x} , r_i is the radius of the support domain.

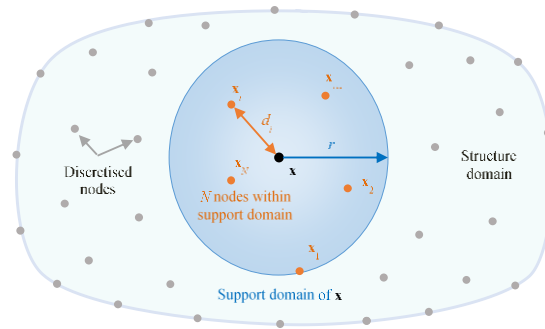
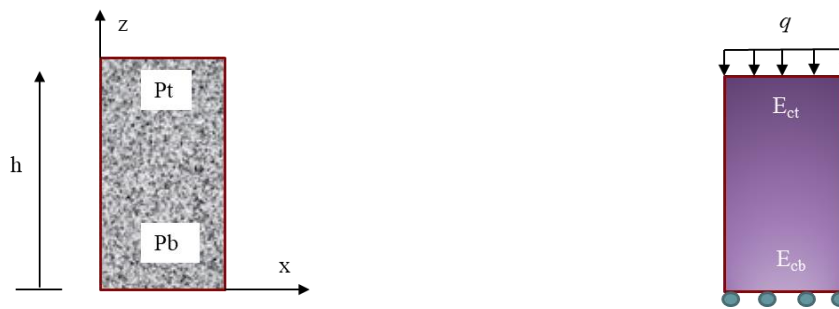


Figure 2. A spherical support domain within the problems domain

3. Functionally graded concrete cylinder

In this study, the three-dimensional cylindrical specimen, which was experimentally tested by Gan et al. (2015), is investigated (Figure 3). The concrete design compressive strength is varied from 20MPa at the top to 60MPa at the bottom of the cylinder. The corresponding Young's modulus are $E_{ct} = 20865\text{MPa}$, and $E_{cb} = 29798\text{MPa}$ and Poisson's ratio $\nu_t = 0.266$ and $\nu_b = 0.231$. The mechanical properties functionally vary between the two ends of the specimen. The cylinder has a diameter of 100mm and a height of 200mm.



a) Sketch of the FG concrete cylinder

b) The specimen under compression

Figure 3. Illustration of the functionally graded concrete cylinder under axial compression

4. Numerical analysis

The elastic responses of the functionally graded cylinder are studied. The structure is supported at the bottom and subjected to uniform load q at the top. The conventional FEM and EFGM are used to model the elastostatics test. The compressive load, $q = 10.18\text{MPa}$, is applied to limit the responses of specimens in the elastic zone. The load value is taken from the experiment of Gan et al. (2015). The results will be compared to experimental data to verify the feasibility and accuracy of the developed models.

Figure 4 shows the convergence trends of vertical displacement at $h = 175\text{mm}$, which are approximated by using conventional FEM with tetrahedral elements and the Meshfree method. Five different meshes with the number of nodes varying from around 1,100 to 11,900 are investigated. As can be seen from the figure, the last mesh (11,900 nodes) produces results

that can be considered convergence. Therefore, this mesh is applied for further investigation of the problem.

Figure 5 and Figure 6 present the strain results with respect to the height h of the cylindrical specimen. The concrete material distributes according to power law functions with various values of constant p . Similarly, Figure 7 and Figure 8 are the strain of FG concrete specimen with the exponential-law distribution of material. Those figures show that power-law distribution produces better results than exponential-law. Moreover, the conventional FEM gives smooth results along the height of the cylinder. Nevertheless, the strain obtained from the Meshless method is more accurate than that of FEM.

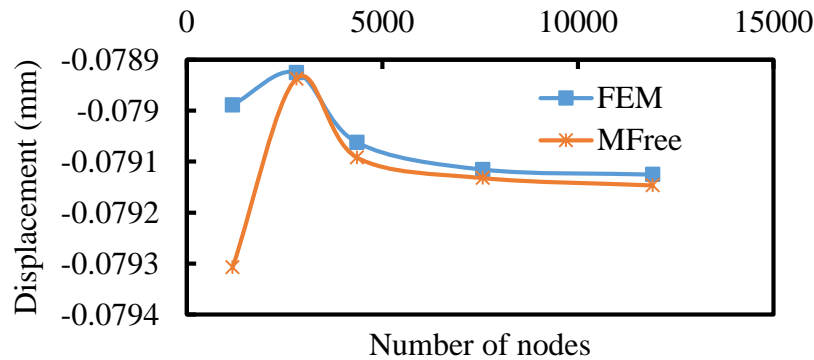


Figure 4. Convergence of vertical displacement obtained from standard FEM and Meshfree methods

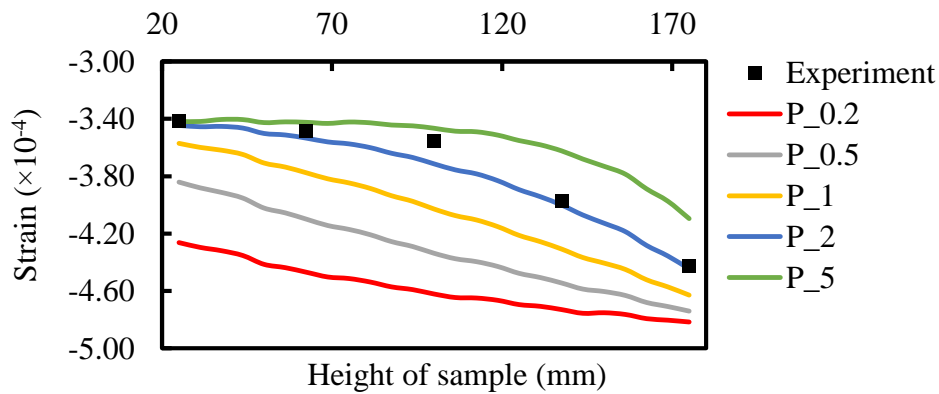


Figure 5. Results from standard FEM with power functional material

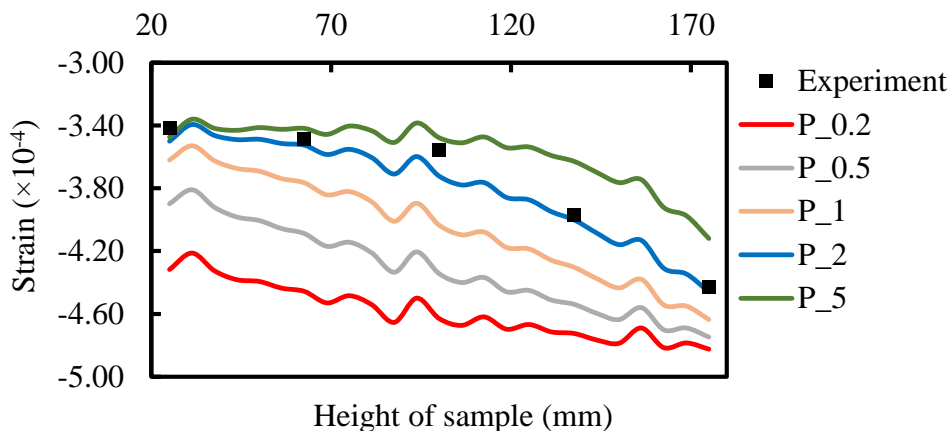


Figure 6. Results from Meshfree with power functional material

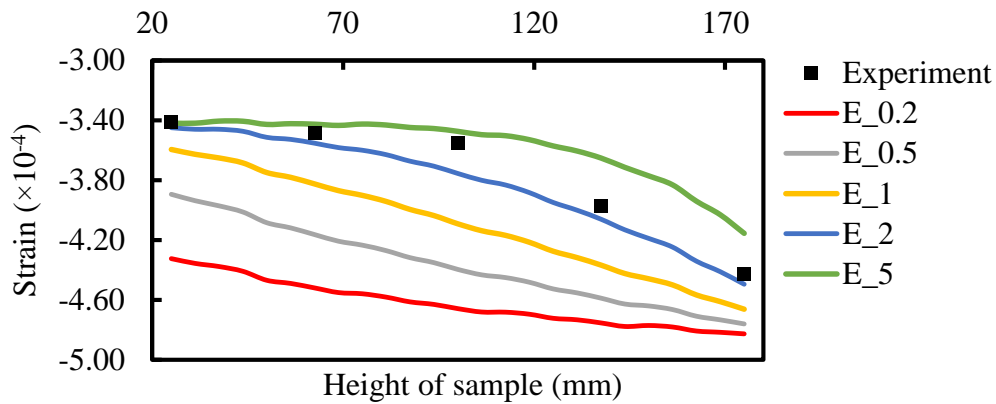


Figure 7. Results from standard FEM with exponential functional material

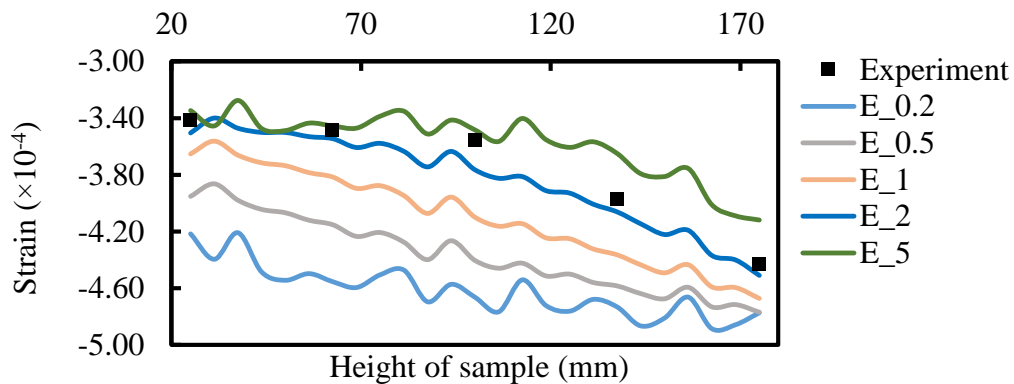


Figure 8. Results from Meshfree with exponential functional material

5. Conclusion

In this study, the functionally graded concrete specimen was numerically investigated. The material was assumed to distribute following power-law and exponential-law functions. Various constants p of the distribution functions were studied to find the best one for modeling the structure. The elastostatics strain of the FG concrete specimen under axial compression load was obtained. The accuracy of the results was compared to those from the experimental test. The specimen was modeled by using the conventional FEM and the Meshless methods. The following conclusions were derived:

- The conventional FEM can produce a smoother strain along the height of the sample compared to the Meshless method. However, the accuracy of results from the Meshless method is better.
- The model with a power-law distribution of concrete material with power constant $p = 2$ presents the most accurate results.

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