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SUBOPTIMAL GUARANTEED COST CONTROL OF UNCERTAIN 2D DISCRETE-TIME SYSTEMS WITH MULTIPLICATIVE STOCHASTIC NOISES

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Abstract. The problem of guaranteed cost control in this paper for a class of two-dimensional (2D) systems is described by the Roesser model with multiplicative stochastic noises. A convex optimization problem with linear matrix inequality constraints is formulated to show the guaranteed cost controller which minimizes the guaranteed cost of the closed-loop uncertain systems.
Keywords: stochastic 2D systems, Roesser model, uncertain systems, guaranteed cost control.

1. Introduction

Various dynamical systems control engineering are determined by the information propagation which occurs in each of the two independent directions. Such models are typically described by (2D) systems. For instance, 2D systems have found various applications in areas such as iterative learning control, gas absorption, thermal processes, and digital filtering [1, 2, 3]. Thus, due to a wide range of applications, the theory of 2D systems has received significant research attention in recent years.

In addition, exogenous disturbances are also unavoidably encountered in engineering systems because of the inaccuracy of data processing, measurement errors, or linear approximations [4, 5]. Various results concerning the analysis and control of dynamical systems involving certain types of additive stochastic noises have been reported recently [6]. However, the aforementioned works are not applicable to multiplicative stochastic noisy systems (MSNSs) due to the nature of the model themselves. Multiplicative noises exist in many practical models such as medical ultrasound, synthetic aperture radar and tomography images [7]. Technically, in MSNSs, stochastic signals get multiplied into relevant system states. This makes the analysis and design of MSNSs more complicated and challenging.

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Moreover, the guaranteed cost control technique for 2D uncertain systems aim to design a controller such that the closed-loop system is asymptotically almost sure stable and the closed-loop cost function value is not more than a specified upper bound for all admissible uncertainties. Such research for 2D MSNSs have received very little research attention and existing results concerning stability analysis and synthesis guaranteed cost control of 2D systems are quite scarce. Recently, a few results have been obtained for the guaranteed cost control of 2D discrete uncertain systems. The optimal guaranteed cost control problem for 2-D discrete uncertain systems is an important problem [8]. However, to the best of the author's knowledge, the optimal guaranteed cost control problem for 2-D discrete uncertain systems is represented by the Roesser model with multiplicative stochastic noises. This paper, therefore, addresses the optimal guaranteed cost control problem for 2D discrete uncertain systems described by the Roesser model with norm-bounded uncertainties.

2. Model description

We consider the uncertain 2D discrete-time linear systems described by the Roesser model

$$\begin{aligned}\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} &= (A+\Delta A_{ij})x(i,j) + (B+\Delta B_{ij})u(i,j) \\ &+ \sum_{s=1}^{d}\Big((\hat{A}_s+\Delta\hat{A}_s)x(i,j) + (\hat{B}_s+\Delta\hat{B}_s)u(i,j)\Big)\beta^s_{ij}, \end{aligned} \tag{2.1}$$

where $x(i,j) = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \in \mathbb{R}^n$, $A, \hat{A}_s \in \mathbb{R}^{n\times n}$; $B, \hat{B}_s \in \mathbb{R}^{n\times m}, s = \overline{1,d}$ are known constant matrices. The matrices $\Delta A_{ij}, \Delta B_{ij}, \Delta\hat{A}_s, \Delta\hat{B}_s, s = \overline{1,d}$ represent parameter uncertainties satisfying the following conditions

$$\begin{aligned} [\Delta A_{ij} \quad \Delta B_{ij}] &= LF_{ij}[M \quad N], \\ [\Delta\hat{A}_s \quad \Delta\hat{B}_s] &= LF_{ij}[\hat{M}_s \quad \hat{N}_s],\ s = \overline{1,d}. \end{aligned}$$

$L \in \mathbb{R}^{n\times p},\ M, \hat{M}_s \in \mathbb{R}^{q\times n};\ N, \hat{N}_s \in \mathbb{R}^{q\times m}, s = \overline{1,d}$ are known matrices of uncertainty and $F_{ij} \in \mathbb{R}^{p\times q}$ is an unknown matrix representing parameter uncertainty satisfying $F^T_{ij}F_{ij} \leqslant I$. Moreover, $\beta_{ij} = col\{\beta^1_{ij}, \ldots, \beta^d_{ij}\},\ \beta^s_{ij}\ (s = \overline{1,d})$ are scalar valued white noises on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, which are independent variables with zero-mean and satisfy

$$\mathbb{E}\beta^k_{ij}\beta^l_{ij} = \delta_{kl} \quad k, l = 1, 2 \ldots, d. \tag{2.2}$$

We define the following infinite-horizon quadratic cost function associated with system (4.1) as a form:

$$J = \mathbb{E}\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} x^T(i,j)Qx(i,j) + u^T(i,j)Ru(i,j)\right), \tag{2.3}$$

where $Q \in \mathbb{S}_n^+,\ R \in \mathbb{S}_m^+$ are given matrices representing weights of the cost function, $x(i,j) = [x^{hT}(i,j) \quad x^{vT}(i,j)]^T$ denotes the state vector.

In addition, without loss of generality, we can assume that initial conditions of system (2.1) are arbitrary but belong to the set

$$\{x^h(0,i) = X_h F_1,\ x^v(i,0) = X_v F_2, F_k^T F_k \leqslant 1, k = 1,2\}, \tag{2.4}$$

where X_h, X_v are given matrices and F_1, F_2 are unknown vectors.

3. Problem formulation and Preliminaries

Definition 3.1. *System* (2.1) *with initial condition* (2.4) *is said to be asymptotically almost sure stable if*

$$\mathbb{P}\{\lim_{i+j\to\infty} ||x(i,j)|| = 0\} = 1.$$

Definition 3.2. *System* (2.1) *with initial condition* (2.4) *is said to be guaranteed cost stabilizable if there exists a control law* $u^*(i,j)$ *and a positive scalar* J^* *such that the closed-loop system* (2.1) *is asymptotically almost sure stable and the cost function* (2.3) *satisfies* $J \leqslant J^*$*, for all admissible uncertainties.* J^* *is said to be a guaranteed cost value and* $u^*(i,j)$ *is a guaranteed cost control law for the system* (2.1).

Lemma 3.1. *(conditional expectation properties) [9] Let* X *and* Y *be integrable random variables defined on a probability space* $(\Omega, \mathcal{F}, \mathbb{P})$ *and let* $\mathcal{A}$ *be a sub-*σ*-algebra of* $\mathcal{F}$*. Then*

(i) $\mathbb{E}(aX + bY|\mathcal{A}) = a\mathbb{E}(X|\mathcal{A}) + b\mathbb{E}(Y|\mathcal{A}),\ a,b \in \mathbb{R};$
(ii) $\mathbb{E}(\mathbb{E}(X|\mathcal{A})) = \mathbb{E}X;$
(iii) If X *is independent of* $\mathcal{A}$*, then* $\mathbb{E}(X|\mathcal{A}) = \mathbb{E}X;$
(iv) $\mathbb{E}(X|\mathcal{A}) = X$ *and* $\mathbb{E}(XY|\mathcal{A}) = X\mathbb{E}(Y|\mathcal{A})$ *if* X *is* $\mathcal{A}-$*measurable;*
(v) *If* $X \leqslant Y$ *then* $\mathbb{E}(X|\mathcal{A}) \leqslant \mathbb{E}(Y|\mathcal{A})$.
(vi) *If* $\mathcal{A}_1 \subset \mathcal{A}_2 \subset \mathcal{F}$ *then* $\mathbb{E}(\mathbb{E}(X|\mathcal{A}_1)|\mathcal{A}_2) = \mathbb{E}(\mathbb{E}(X|\mathcal{A}_2)|\mathcal{A}_1) = \mathbb{E}(X|\mathcal{A}_1)$.

All the equalities and inequalities above shown hold almost surely.

Lemma 3.2. *(Schur complement)[10] Given matrices* A, B, C *with appropriate dimensions where* $A = A^T$*, and* $C = C^T > 0$*, then* $A + BC^{-1}B^T < 0$ *if and only if* $\begin{bmatrix} A & B \\ B^T & -C \end{bmatrix} < 0$ *or* $\begin{bmatrix} -C & B^T \\ B & A \end{bmatrix} < 0.$

Lemma 3.3. *Let* $G \in \mathbb{R}^{m\times n},\ L \in \mathbb{R}^{m\times p},\ E \in \mathbb{R}^{q\times n}$ *and* $Q = Q^T \in \mathbb{R}^{n\times n}$*. Assume that there exits a positive definite matrix P and a scalar* $\varepsilon > 0$ *such that*

$$\begin{bmatrix} -P^{-1} + \varepsilon LL^T & G \\ G^T & \varepsilon^{-1}E^T E + Q \end{bmatrix} < 0. \tag{3.1}$$

Then

$$(G + LFE)^T P(G + LFE) + Q < 0, \tag{3.2}$$

for all F satisfying $F^T F < I$.

Proof. Since $P > 0$ and (3.1), we have $\begin{bmatrix} I & \sqrt{\varepsilon}L^T \\ \sqrt{\varepsilon}L & P^{-1} \end{bmatrix} > 0$, which implies

$$I - \varepsilon L^T PL > 0.$$

We denote

$$\Gamma = (\varepsilon^{-1}I - L^T PL)^{-\frac{1}{2}} L^T PG - (\varepsilon^{-1}I - L^T PL)^{\frac{1}{2}} FE.$$

Applying the fact that $\Gamma^T\Gamma \geqslant 0$, we get

$$\begin{aligned} G^T PLFE + E^T F^T L^T PG \leqslant G^T PL(\varepsilon^{-1}I - L^T PL)^{-1} L^T PG \\ +E^T F^T (\varepsilon^{-1}I - L^T PL)FE. \end{aligned} \tag{3.3}$$

Note that for any matrices B, C and a scalar λ, we have

$$(\lambda I + BC)^{-1}B = B(\lambda I + CB)^{-1}. \tag{3.4}$$

Utilizing (3.4) and condition $F^T F < I$, it follows from (3.3) that

$$\begin{aligned} (G + LFE)^T P(G + LFE) \leqslant & G^T (PL(\varepsilon^{-1}I - L^T PL)^{-1} L^T P + P)G + \varepsilon^{-1}E^T F^T FE \\ \leqslant & G^T (PLL^T P(\varepsilon^{-1}I - LL^T P)^{-1} + P)G + \varepsilon^{-1}E^T E \\ = & G^T (\varepsilon PLL^T (P^{-1} - \varepsilon LL^T)^{-1} + P)G + \varepsilon^{-1}E^T E. \end{aligned} \tag{3.5}$$

Since $\varepsilon PLL^T (P^{-1} - \varepsilon LL^T)^{-1} + P = (P^{-1} - \varepsilon LL^T)^{-1}$, then we have

$$(G + LFE)^T P(G + LFE) \leqslant G^T (P^{-1} - \varepsilon LL^T)^{-1} G + \varepsilon^{-1}E^T E. \tag{3.6}$$

It follows from Schur complement and (3.1) that

$$G^T (P^{-1} - \varepsilon LL^T)^{-1} G + \varepsilon^{-1}E^T E + Q < 0. \tag{3.7}$$

Combining with (3.6), we obtain (3.2). □

4. Main results

4.1. Stability analysis of 2D uncertain stochatic systems

We design a static-state feedback control law $u(i,j) = Kx(i,j)$ such that the closed-loop system of (2.1) is given as

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A^c_{ij}x(i,j) + \sum_{s=1}^{d} \hat{A}^{cs}_{ij}x(i,j)\beta^s_{ij}, \tag{4.1}$$

where $A^c_{ij} = A + \Delta A_{ij} + (B + \Delta B_{ij})K,\ \hat{A}^{cs}_{ij} = \hat{A}_s + \Delta\hat{A}_s + (\hat{B}_s + \Delta\hat{B}_s)K,\ s = \overline{1,d}$ is asymptotically almost sure stable and for all admissible uncertainties, the cost function (2.3) satisfies $J \leqslant J^*$, where J^* is some specified constant.

Theorem 4.1. *Assume that there exists a matrix $P = diag(P_h, P_v) \in \mathbb{S}^+_n$ satisfying the following conditions*

$$\mu_{ij} := (A^c_{ij})^T PA^c_{ij} + \sum_{s=1}^{d}(\hat{A}^{cs}_{ij})^T P(\hat{A}^s_{ij}) - P + Q + K^T RK < 0, \tag{4.2}$$

then the closed-loop system (4.1) *is asymptotically stable under controller* $u(i,j) = Kx(i,j)$ *and* $J \leqslant J^*$*, with* $J^* = T_1\lambda_{\max}(X_h^T P_h X_h) + T_2\lambda_{\max}(X_v^T P_v X_v)$.

Proof. Consider the following 2D-Lyapunov functional

$$V(x(i,j)) = x^{hT}(i,j)P_h x^h(i,j) + x^{vT}(i,j)P_v x^v(i,j). \tag{4.3}$$

Denote

$$l(x(i,j),u(i,j)) = x^T(i,j)Qx(i,j) + u^T(i,j)Ru(i,j),$$

and

$$\begin{aligned}
\Delta V(i,j) &= \mathbb{E}(V^1(x^h(i+1,j)) + V^2(x^v(i,j+1))|\mathcal{F}_{i+j}) - V(x(i,j)) \\
&= \mathbb{E}\Big(\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}^T \text{diag}(P_h,P_v) \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} |\mathcal{F}_{i+j}\Big) - V(x(i,j)) \\
&= \mathbb{E}\Big((A^c_{ij}x(i,j) + \sum_{s=1}^{d} \hat{A}^{cs}_{ij}x(i,j)\beta^s_{ij})^T P(A^c_{ij}x(i,j) \\
&\quad + \sum_{s=1}^{d} \hat{A}^{cs}_{ij}x(i,j)\beta^s_{ij})\big|\mathcal{F}_{i+j}\Big) - x^T(i,j)Px(i,j),
\end{aligned} \tag{4.4}$$

where $\mathcal{F}_{i+j}$ is the σ-algebra generated by $\{\beta_{kl} : (k,l) \in \Omega_{i+j-1}\}$ and, for each positive integer κ, $\Omega_\kappa = \{(k,l) \in \mathbb{N}^2_0 : k + l \le \kappa\}$, that is,

$$\mathcal{F}_{i+j} = \sigma\{\beta_{kl} : k + l \le i + j - 1, (k,l) \in \mathbb{N}^2_0\}, \quad (i,j) \in \mathbb{N}^2_0,$$

and $\mathcal{F}_0 = \{\emptyset, \Omega\}$, where $\emptyset$ denotes the empty set and Ω is the sample space. Obviously, $\mathcal{F}_{k-1} \subset \mathcal{F}_k$ for any $k \in \mathbb{N}$. Recall that β_{ij} is a double sequence of $\mathbb{R}^d$-valued random variables defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Without loss of generality, we assume $\mathcal{F} = \sigma\{\cup_{k=0}^{\infty}\mathcal{F}_k\}$. It is easy to see that the state trajectory $x(i,j)$ of system (4.1) is $\mathcal{F}_{i+j}$-adapted.

By the properties of $\mathbb{R}^d$-valued random variable β_{ij} and applying Lemma 3.1 we obtain from (4.4) that

$$\Delta V(i,j) = x^T(i,j)\big((A^c_{ij})^T P A^c_{ij} + \sum_{s=1}^{d}(\hat{A}^{cs}_{ij})^T P \hat{A}^{cs}_{ij} - P\big)x(i,j). \tag{4.5}$$

Therefore,

$$\begin{aligned}\Delta V(i,j) + l(x(i,j), u(i,j)) &= \Delta V(i,j) + x^T(i,j)(Q + K^T R K)x(i,j)\\ &= x^T(i,j)\mu_{ij}x(i,j).\end{aligned}$$

Under condition (4.2), we have

$$\Delta V(i,j) + l(x(i,j), u(i,j)) < 0, \tag{4.6}$$

which implies

$$\Delta V(i,j) \leqslant -\lambda_{\min}(Q + K^T R K)||x(i,j)||^2.$$

Utilizing Theorem 1 in [11] we obtain

$$\lim_{i+j\to\infty} W(i,j) = 0 \quad a.s,$$

so

$$\lim_{i+j\to\infty} ||x(i,j)|| = 0 \quad a.s.$$

Hence, the closed-loop system (4.1) is asymptotically stable.
Moreover, by summing up both sides of (4.6) for any positive integers $r_1,\ r_2$, we get

$$\begin{aligned}&\sum_{i=0}^{r_1}\sum_{j=0}^{r_2}\Big\{\mathbb{E}\Big(V^1(x^h(i,j)) + V^2(x^v(i,j))\big|\mathcal{F}_{i+j}\Big) - V(x(i,j)) \\ &+ x^T(i,j)(Q + K^T R K)x(i,j)\Big\} < 0.\end{aligned} \tag{4.7}$$

Taking expectation both sides of (4.7), we obtain

$$\begin{aligned}\mathbb{E}\Big(\sum_{i=0}^{r_1}\sum_{j=0}^{r_2} x^T(i,j)(Q + K^T R K)x(i,j)\Big) &\leqslant \sum_{i=0}^{r_1}\mathbb{E}(V^2(x^v(i,0))) \\ &+ \sum_{j=0}^{r_2}\mathbb{E}(V^1(x^h(0,j))).\end{aligned} \tag{4.8}$$

Let $r_1 \to \infty,\ r_2 \to \infty$, it follows from (4.8) that

$$J \leqslant T_1\lambda_{\max}(X_h^T P_h X_h) + T_2\lambda_{\max}(X_v^T P_v X_v).$$

□

Remark 4.1. *Note that LMI* (4.2) *can be rewritten as*

$$\begin{aligned}&(A+BK+LF_{ij}(M+NK))^T P(A+BK+LF_{ij}(M+NK))\\&+\sum_{s=1}^{d}(\hat{A}_s+\hat{B}_sK+LF_{ij}(\hat{M}_s+\hat{N}_sK))^T P(\hat{A}_s+\hat{B}_sK+LF_{ij}(\hat{M}_s+\hat{N}_sK))\\&-P+Q+K^TRK<0.\end{aligned} \tag{4.9}$$

We denote

$$\mathcal{A}=col\{A+BK,\hat{A}_1+\hat{B}_1K,\dots,\hat{A}_d+\hat{B}_dK\},\quad \mathcal{L}=\underbrace{diag\{L,\dots,L\}}_{d+1\ blocks},$$

$$\mathcal{F}_{ij}=\underbrace{diag\{F_{ij},\dots F_{ij}\}}_{d+1\ blocks},\quad \mathcal{P}=diag\underbrace{\{P,\dots,P\}}_{d+1\ blocks},$$

$$\mathcal{M}=col\{M+NK,\hat{M}_1+\hat{N}_1K,\dots,\hat{M}_d+\hat{N}_dK\}.$$

It follows from (4.2) *that*

$$(\mathcal{A}+\mathcal{L}\mathcal{F}_{ij}\mathcal{M})^T\mathcal{P}(\mathcal{A}+\mathcal{L}\mathcal{F}_{ij}\mathcal{M})-P+Q+K^TRK<0. \tag{4.10}$$

Applying Lemma 3.3, condition (4.10) *holds if the following one does*

$$\begin{bmatrix}-\mathcal{P}^{-1}+\varepsilon\mathcal{L}\mathcal{L}^T & \mathcal{A}\\ \mathcal{A}^T & \varepsilon^{-1}\mathcal{M}^T\mathcal{M}-P+Q+K^TRK\end{bmatrix}<0. \tag{4.11}$$

Therefore, assume that condition (4.11)*, which does not depend on i, j, holds then system* (4.1) *is guaranteed cost stabilizable.*

4.2. Controller design

Let us denote the following block matrices

$$\overline{M}=YM^T+U^TN^T;\ \overline{B}_s=A_sY+B_sU;\ \overline{N}_s=Y\hat{M}_s^T+U^T\hat{N}_s^T,\ s=\overline{1,d}.$$

$$\Lambda=diag\{-Y+\epsilon LL^T,\dots,-Y+\epsilon LL^T\};\quad \mathcal{T}=\begin{bmatrix}\overline{A}\\ \vdots\\ \overline{B}_d\end{bmatrix};$$

$$\mathcal{Y}_1=\begin{bmatrix}Q^{\frac{T}{2}}Y\\ R^{\frac{T}{2}}U\end{bmatrix};\quad \mathcal{Y}_2=\begin{bmatrix}\overline{M}^T\\ \overline{N}_1^T\\ \vdots\\ \overline{N}_d^T\end{bmatrix};\quad \mathcal{H}_1=\begin{bmatrix}YQ^{\frac{1}{2}} & U^TR^{\frac{1}{2}}\end{bmatrix};$$

$$\mathcal{H}_2=\begin{bmatrix}\overline{M} & \overline{N}_1 & \dots & \overline{N}_d\end{bmatrix};\quad \mathcal{U}=\begin{bmatrix}\overline{A}^T & \cdots & \overline{B}_d^T\end{bmatrix};\quad \overline{A}=AY+BU.$$

Theorem 4.2. *Consider system* (2.1) *with initial conditions* (2.4) *and cost function* (2.3)*, system* (2.1) *is guaranteed cost stabilizable if there exist a positive scalar* ε*, a matrix* $Y = diag\{Y_h, Y_v\} \in \mathbb{S}_n^+$ *and a* $m \times n$ *matrix U such that the following LMI is feasible*

$$\left[\begin{array}{c|ccc} \Lambda & \mathcal{T} & 0 & 0 \\ \hline \mathcal{U} & -Y & \mathcal{H}_1 & \mathcal{H}_2 \\ 0 & \mathcal{Y}_1 & -I & 0 \\ 0 & \mathcal{Y}_2 & 0 & -\epsilon I \end{array}\right] < 0, \tag{4.12}$$

A control law is given by $K = UY^{-1}$.
Moreover, a guaranteed cost value is given by

$$J^* = T_1 \lambda_{\max}(X_h^T Y_h^{-1} X_h) + T_2 \lambda_{\max}(X_v^T Y_v^{-1} X_v). \tag{4.13}$$

Proof. The condition (4.11) can be rewritten as

$$\left[\begin{array}{ccc|c} -P^{-1} + \varepsilon LL^T & \cdots & 0 & A + BK \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & -P^{-1} + \varepsilon LL^T & \hat{A}_d + \hat{B}_d K \\ \hline (A+BK)^T & \cdots & (\hat{A}_d + \hat{B}_d K)^T & -P \end{array}\right] + \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \varepsilon^{-1}\mathcal{M}^T\mathcal{M} + Q + K^T RK \end{array}\right] < 0, \tag{4.14}$$

By premultiplying and postmultiplying LMI (4.14) by the matrix
$diag\{\underbrace{I_n, \ldots, I_n}_{d+1 \text{ times}}, P^{-1}\}$, we have

$$\left[\begin{array}{ccc|c} -Y + \varepsilon LL^T & \cdots & 0 & (A + BK)Y \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & -Y + \varepsilon LL^T & (\hat{A}_d + \hat{B}_d K)Y \\ \hline Y(A+BK)^T & \cdots & Y(\hat{A}_d + \hat{B}_d K)^T & -Y^{-1} \end{array}\right] + \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \mathcal{V} \end{array}\right] < 0$$

where $Y = P^{-1}$, $\mathcal{V} = \varepsilon^{-1} Y(M + NK)^T (M + NK)Y + \sum_{s=1}^{d} \varepsilon^{-1} Y(\hat{M}_s + \hat{N}_s K)^T (\hat{M}_s + \hat{N}_s K)Y + YQY + YK^T RKY$. Applying Schur complement and by (4.2.), we get LMI (4.12). Moreover, from Theorem 4.1, it is trivial to show (4.13). □

Proposition 4.1. *Consider system* (2.1) *with initial conditions* (2.4) *and cost function*

(2.3)*, if the following optimization problem*

$$\textit{minimize } T_1\alpha + T_2\beta \tag{4.15}$$

$$\textit{subject to} \begin{cases} (i). \quad (4.12), \\ (ii). \begin{bmatrix} -\alpha I_{n_1} & X_h^T \\ X_h & -Y_h \end{bmatrix} < 0, \\ \\ (iii). \begin{bmatrix} -\beta I_{n_2} & X_v^T \\ X_v & -Y_v \end{bmatrix} < 0 \end{cases} \tag{4.16}$$

has a feasible solution $\alpha > 0, \beta > 0, 0 < Y = diag\{Y_h, Y_v\} \in \mathbb{R}^{n\times n}, U \in \mathbb{R}^{m\times n}$*, then the control law defined by* $u(i,j) = Kx(i,j)$ *are the suboptimal guaranteed cost control law, where* $K = UY^{-1}$*.*

Proof. By Theorem 4.2, the control law $K = UY^{-1}$ which is constructed in terms of any feasible solution ε, U and Y is a guaranteed cost controller of system (2.1). To obtain the optimum value of upper bound of guaranteed cost, the term $\lambda_{\max}(X_h^T Y_h^{-1} X_h)$ is changed to $\lambda_{\max}(X_h^T Y_h^{-1} X_h) < \alpha \Leftrightarrow X_h^T Y_h^{-1} X_h < \alpha I_{n_1}$ and the term $\lambda_{\max}(X_v^T Y_v^{-1} X_v)$ is changed to $\lambda_{\max}(X_v^T Y_v^{-1} X_v) < \beta \Leftrightarrow X_v^T Y_v^{-1} X_v < \beta I_{n_2}$. Applying the Schur complement, we have conditions (ii) and (iii). Therefore, the minimization of $T_1\alpha + T_2\beta$ implies the minimization of the upper bound of guaranteed cost J in (4.13). □

5. Conclusions

A solution to the suboptimal guaranteed cost control the problem for the uncertain 2D discrete Roesser with multiplicative stochastic noises has been presented. The sufficient condition for the existence of a guaranteed cost controller. A convex optimization the problem has been formulated to select the optimal guaranteed cost controller which minimizes the upper bound on the closed-loop cost function.

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