Review of empirical relationships between inlet cross-section and tidal prism

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Abstract: Although other engineers had considered the relationship between tidal prism and inlet cross-sectional area before, it is O'Brien who is usually credited for deriving the familiar relationship $A = aP^m$, where A is the cross-sectional area (relative to mean sea level) and P is the spring tidal prism. The coefficients a and m vary from entrance to entrance; however O'Brien (1969) showed that for 28 US entrances, $a=4.69\ 10^{-4}$ and $m=0.85$ are best-fit values applicable to all entrances when P is measured in cubic meters (m^3) and A in square meters (m^2).

Many different suggestions have been made in literature about the rationale behind the above relation. We explore here some theoretical elaborations inspired by this literature. The most popular elaboration is to express A in terms of the so named "stability shear stress", which in the approaches of Gerritsen (1990) and Friedrichs (1995) is defined as the shear stress needed to maintain a zero net transport gradient along the channel, which is assumed close to the critical shear stress for sediment motion. However, one may claim as posed by Kraus (1998) that this shear stress is not necessarily equal to the critical shear stress, since a certain transport capacity is necessary to remove the sediment deposited by the littoral or alongshore drift.

This contribution compares some theoretical elaborations both qualitatively and quantitatively, confront these with US and Dutch empirical data and discuss their pros and cons for practical applications.

1. Introduction

Although other engineers had considered the relationship between tidal prism and inlet cross-sectional area before, it is usually O'Brien who is usually credited for deriving the familiar relationship:

$$A = aP^m \tag{1}$$

where A is the cross-sectional area (relative to mean sea level) and P is the spring tidal prism. The coefficients a and m vary from entrance to entrance; however O'Brien (1969) showed that for 28 US entrances, $a=4.69\ 10^{-4}$ and $m=0.85$ are best-fit values applicable to all entrances when P is measured in cubic meters (m^3) and A in square meters (m^2). Note that the coefficient a is not dimensionless and that the dimension depends on the power m; therefore we will consistently base the value of a on SI units.

For a sinusoidal variation of the flow discharge at the tidal frequency, P is related to the mean discharge $<Q>$ over flood or ebb flow duration by:

$$P = \frac{T < Q >}{2} \tag{2}$$

where T is the tidal period. Combining equations 1 and 2 yields:

$$A = a\left(\frac{T}{2}\right)^m < Q >^m \tag{3}$$

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For illustrative purposes, taking $T=44,700\ s$ for a semi-diurnal tide and O'Brien's values for a and m, we obtain:

$$A = 2.30 < Q >^{0.85} \tag{4}$$

which is similar to (as noted by Powell et al., 2006):

$$A_r = 1.51 < Q_r >^{0.83} \tag{5}$$

where A_r is the river cross-sectional area and Q_r is the river discharge. Equation 5, known as the regime equation, was empirically derived for several non-tidal rivers in the US by Blench (1961). The transition between river-dominated flow and tide-dominated flow depends on the ratio of $<Q>/Q_r$. The influence of the river on tidal flow becomes minor when $<Q>/Q_r >= 20$.

Many authors have attempted to apply Equation 1 to groups of entrances from the same or similar regions and have suggested, sometimes based on a theoretical derivation, many different values for the exponent m, resulting in many different values for a. However, scrutinizing the literature for the US and the Dutch entrances reveals that very good results are obtained for $m=1$ and best-fitting a. We will address theoretical approaches later.

For 8 out of the 28 entrances without jetties O'Brien (1969) showed that a best-fit results in $a=1.08\ 10^{-4}$ and $m=1$. Powell et al (2006) reanalysed 28 Florida Atlantic Coast and 39 Florida Gulf Coast entrances recommending (neglecting the artificial Port Canaveral Bay entrance) $a=6.25\ 10^{-5}$ and $m=1$ with a correlation $r^2=0.91$ (Figure 1). Rakhorst (2007) finds for the inlets in the Western Wadden Sea $a=5.65\ 10^{-5}$ and $m=1$ with a correlation $r^2=0.97$, while for the inlets in the Eastern Wadden Sea this author finds $a=7.75\ 10^{-5}$ and $m=1$ with a correlation $r^2=0.99$. Whereas O'Brien and Powell use the largest value of the mean ebb- and flood discharge during spring tide, Rakhorst uses the largest value of the mean ebb- and flood discharge. Before Rakhorst, Eysink (1990) recommended for the Dutch Wadden Sea $a=7.0\ 10^{-5}$ and $m=1$ based on the mean tidal prism.

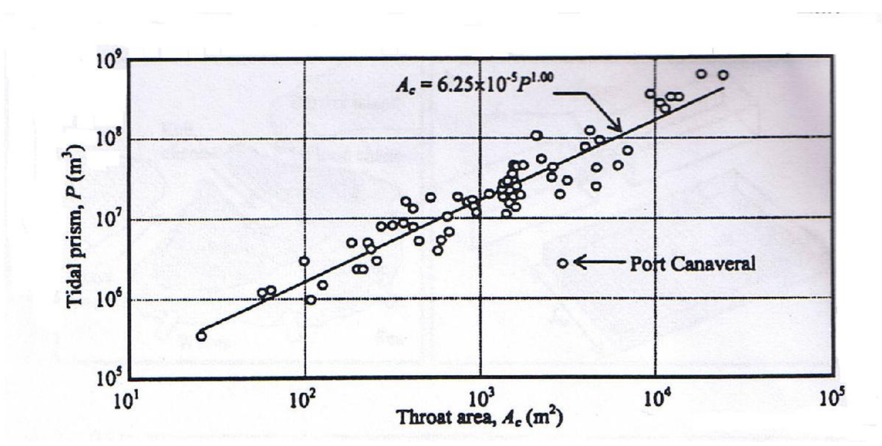

Figure 1 Throat inlet cross-section as a function of the tidal prism after Powel et al. (2006)

In summary, we observe that for a typical type of entrances, i.e. near-natural entrances, micro to meso tidal energy, medium wave energy, no fresh water discharge, and good sediment availability the exponent $m=1$ seems appropriate and the a-values are fairly close (see table 1).

Table 1 Comparison of findings for comparable inlet situations

Author(s)	a	m	Tidal prism	Location
O'Brien (1969)	$1.08\ 10^{-4}$	1	Mean spring	8 non-jettied entrances US
Powell et al (2006)	$6.25\ 10^{-5}$	1	Mean spring	66 Florida entrances
Eysink (1990)	$7.0\ 10^{-5}$	1	Mean tide	Dutch Wadden Sea entrances
Rakhorst (2007)	$5.65\ 10^{-5}$	1	Maximum of mean ebb or flood	Dutch Western Wadden Sea entrances
Rakhorst (2007)	$7.75\ 10^{-5}$	1	Maximum of mean ebb or flood	Dutch Eastern Wadden Sea entrances
Vd Kreeke & Haring (1979)	$8.2\ 10^{-5}$	1	?	Dutch Zeeland entrances

2. Generality of empirical relationships between inlet cross-section and tidal prism

An interesting discussion on the effect of jettied or protected entrances on the area prism relation is given by Kraus (1998): "*Based on data from a limited number of locations along the coast of California, LeConte (1905) arrived at the linear equation A=aP. The value of the empirical coefficient a was about 34% larger for inner harbour entrances (restricted longshore sediment transport) than for unprotected entrances (unrestricted longshore transport). LeConte's observation indicates that the same tidal prism on a coast with restricted longshore transport can maintain a larger equilibrium channel area than on a coast with less restricted or greater longshore sediment transport"*. Kraus (1998) then summarizes the findings by Jarrett (1976) who reanalysed all earlier US work since O'Brien (1931). Jarrett's objectives were to determine if inlets on all three US coasts (Atlantic, Gulf and Pacific) follow the same area prism relationship and if inlet stabilization altered that relation. Besides that the above finding of LeConte was confirmed, also an influence of the strength of the longshore drift was found. Smaller littoral drift resulted in larger equilibrium cross-sectional areas.

From the above findings the physical mechanisms that play a role in the apparently possible existence of a dynamic equilibrium cross-sectional area become clear. The tidal prism produces a current in the inlet entrance that provides a transport capacity of the inlet flow sufficient to clear the entrance from any sediment deposited by littoral or longshore transport. This concept has appeared throughout the literature since LeConte (1905). Again we quote Kraus (1998): *"In particular, Byrne et al. (1980), Riedel and Gourlay (1980), and Hume and Herdendorf (1990) studied inlet channel stability on sheltered coasts and demonstrated that larger values of the empirical coefficient a and smaller values of m apply to coasts with limited littoral transport. Quoting Riedel and Gourlay, "In contrast (to exposed coasts), for sheltered inlets the littoral drift rate is small and, consequently, a much smaller volume of material needs to be moved out of the entrance in each tidal cycle." The aforementioned studies also indicate that the mean-maximum velocity (mean over the cross-section of the maximum at spring tide) required to maintain stability of the inlet channel is less (reaching approximately one-third less) than the typical 1 m/s (Bruun and Gerritsen 1960, O'Brien 1969) required to maintain a channel on an exposed coast"*. The less protected and the higher the littoral drift the smaller the equilibrium cross-section of the entrance or inlet throat and, vice versa, the more protected and the smaller the drift the larger the equilibrium cross-section. In the light of the flow velocity magnitudes mentioned above, viz. *1 m/s* for unprotected versus *0.7 m/s* for protected, a shear stress reduction of *50%* is possible in protected cases, which corresponds in order of magnitude with a *34%* higher value of the coefficient *a* mentioned by LeConte above.

3. Theoretical elaboration

Many different suggestions have been made in literature about the rationale behind Equation 1. We explore here some theoretical elaborations inspired by this literature and refer to Van de Kreeke (2004) for a more comprehensive overview. The most popular elaboration is to express *A* in terms of the so named "stability shear stress", which in the approaches of Gerritsen (1990) and Friedrichs (1995) is defined as the shear stress needed to maintain a zero net transport gradient along the channel, which is assumed close to the critical shear stress for sediment motion. However, one may claim that this shear stress it is not necessarily equal to the critical shear stress, since a certain transport capacity is necessary to remove the sediment deposited by the littoral or alongshore drift, as follows from the above discussion by Kraus.

Our first theoretical elaboration is based on a simplified Gerritsen type of approach. We assume that an estimate can be derived of the characteristic shear velocity induced by the discharge due to the tidal prism, and that this shear stress needs to sustain a certain Shields value to produce a certain transport capacity in the entrance. In the literature two different approaches exist to derive the characteristic shear velocity, one is based on the representative maximum tide discharge Q_{max}, the other on the representative mean discharge $<Q>$, respectively:

$$Q_{\max} = \frac{P\pi}{T} \tag{6}$$

And

$$<Q>= \frac{2P}{T} \tag{7}$$

where *P* is the tidal prism and *T* the tidal period. The characteristic flow velocity is thus (realizing $Q=Au$) respectively:

$$u_{\max} = \frac{\pi P}{AT} \tag{8}$$

Or

$$u = \frac{2P}{AT} \tag{9}$$

so that these two values differ by $\pi/2$. Being interested in order-of-magnitude estimates we ignore this difference and arbitrarily continue with Equations 7 and 9.

The characteristic or representative shear stress can be estimated using the Chezy approximation:

$$\tau_0 = \rho_w g \frac{u^2}{C^2} \tag{10}$$

where *C* is the dimensional Chezy coefficient ($m^{1/2}/s$), and we assume that to first-order-of-magnitude *C* is constant, say $50\ m^{1/2}/s$.

We now introduce the commonly suggested concept that this representative shear stress needs to attain a value at least of the magnitude of the critical shear stress for sediment motion, say the Shields value $\Theta_c=0.02$. However, as argued before we need to have a transport capacity to clear the entrance from deposition, so it is more likely that its value is higher. Note that literature suggests that for $\Theta_s=0.2$ we are at the initiation of full transport capacity. Hence, in the general situation of littoral or alongshore drift the representative shear stress needs to at least equal the Shields capacity value (using Equations 7, 9, 10 and Shields):

$$\tau_0 = \rho_w g \frac{u^2}{C^2} = \rho_w g \left(\frac{2P}{CAT} \right)^2 = \Theta_s (\rho_s - \rho_w) g D_{50} \tag{11}$$

Some rearranging gives us the following expression for A:

$$A = \left(\frac{4\rho_w g}{\Theta_s (\rho_s - \rho_w) g D_{50} C^2 T^2} \right)^{1/2} P \tag{12}$$

Now let us evaluate the coefficient a with a D_{50} of $300\ \mu m$ and the other values as suggested above, we get $a=0.9\ 10^{-4}$, which is right in the range of the empirical values found for unprotected inlets. We will discuss the implications and applicability of Equation 12 later. First, we discuss attempts in the literature to introduce further parameters to extend Equation 12.

The largest group of researchers that tried to extend the above theoretical derivation claims that the friction, represented by C, is not a constant, but a function of the characteristic depth, h, which is physically defendable. One way of introducing this (cf. Friedrichs, 1995) is to express the Chezy coefficient in terms of Manning:

$$C = \frac{h^{1/6}}{n} \tag{13}$$

where n according to Manning is in the range $0.03 - 0.04$. Introducing Equation 13 in Equation 12 yields:

$$A = \left(\frac{4\rho_w g n^2}{\Theta_s (\rho_s - \rho_w) g D_{50} T^2} \right)^{1/2} h^{-1/6} P \tag{14}$$

An evaluation of the coefficient a again with a D_{50} of $300\ \mu m$ and Manning value of 0.035 yields $a=1.57\ 10^{-4}$.

Different powers of h in Equation 14 have been suggested based on similar considerations or by using empirical findings, such as between the characteristic value for u and depth. Rakhorst (2007) for instance uses:

$$A = a h^{-2/5} P \tag{15}$$

which this author applied to the same dataset of the Western Waddensea. The correlation improved from $r^2=0.97$ to $r^2=0.98$. We comment that this is only a marginal improvement of the correlation. It does not seem justified in the light of the possible variation in the other parameters that constitute a to introduce another field variable in the relation between the cross-sectional area A and the tidal prism P, even though it is physically defendable.

Most of the suggestions in literature for a theoretical derivation are falling under the above two approaches. However, the approach of Kraus (1998) is somewhat different. This author uses the characteristic or representative shear stress of equation 10 to derive a transport magnitude that is equalled to the alongshore littoral or alongshore drift, which is physically defendable. As a result two more field variables are introduced in the coefficient a, viz. the alongshore drift and the cross-sectional width of the entrance. Similar to the discussion on introducing h it is argued that it does not seem practical to have two more field variables in the relation between the cross-sectional area A and the tidal prism P, even though it is physically defendable.

4. Conclusions

From the above discussion it is clear that we find Equation 12 the most attractive to express a relation between the entrance cross-section A and the tidal prism P. We suggest absorbing the effect of any other field variables (which are hard to measure anyway) such as the alongshore drift and the degree of protection in the parameter Θ_s. By varying Θ_s between 0.1 and 0.3 the coefficient a varies $1/2\sqrt{2}$, e.g. 0.1 for sheltered or small alongshore drift situations and 0.3 for unprotected large alongshore drift situations. This yields a +/- variation of *30%* in the cross-sectional area of tidal entrances. Depending on the sediment characteristics D_{50} and the smoothness coefficient C may be changed as well. Finally we note that an interesting quantitative difference should arise in case the tide is not semi-diurnal but diurnal. We are not aware of any data to confirm this.

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