

**A HYDRODYNAMIC/SEDIMENT MODEL  
FOR SIMULATING BEDLOAD SEDIMENT IN THE RIVER**

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**Abstract:** *This paper proposes a hydrodynamic/sediment model for simulating bedload sediment through a river section. Firstly, flow characteristics such as depth-averaged velocity were obtained by solving the governing equation, which is derived from the Reynolds equations and allows for taking into account the gravity, bed shear stress, turbulent diffusion force and secondary flow in calculations. Four criteria (i.e. root mean square error, mean absolute error, Nash-Sutcliffe efficiency, and correlation coefficient) were used to assess the quality of computed results, revealing that a good agreement between simulations and observations was obtained at the studied river section. Secondly, nine relations for determining bedload sediment transport rate and bedload sediment discharge were considered and compared to identify suitable ones. The results showed that the relation proposed by Camenen and Larson (2005) proved to be well adopted, providing even better results than the others. Finally, future modeling efforts and wide-ranging applications of the model were discussed.*

**Keywords:** bedload sediment, sediment rate, hydrodynamic/sediment model, Danube river.

### **1. INTRODUCTION**

Sediments are inherent components of riverine waters, which are transported under the form of suspended and bedload sediments. Suspended sediment normally consists of fine-grained materials and relates to water quality, pollution, and aquatic ecology. On the contrary, bedload sediment consists of coarser-grained materials that can be sliding, rolling, and saltating over short distances in region close to the riverbed. The bedload sediment often occurs during episodic events such as floods. Bedload sediment usually involves bed evolution or morphological changes, and thus the navigation and flood mitigation infrastructure. Therefore, bedload sediment needs to be quantitatively assessed in order to (i) determine accurately bedload sediment transport rate, (ii) predict

bedload sediment discharge, and (iii) deal with potential changes of the bed.

Estimation of bedload sediment transport rate is often calculated by using classical bedload sediment transport formulas or relations. Because of limitations of bedload sediment measurements and the complexity of bedload sediment transport processes, all existing classical bedload sediment transport relations are empirical or semi-empirical, suggesting that large discrepancies may exist among these relations when they are applied in real applications (Wu et al., 2000). Evaluation of their performance in the real situations is thus very important for identifying the suitable ones.

Moreover, only mean velocity or mean bed shear stress over a cross-section of the river is considered in classical bedload sediment transport relations. The complexity of local hydraulic characteristics, which are resulted from the effects of various factors such as

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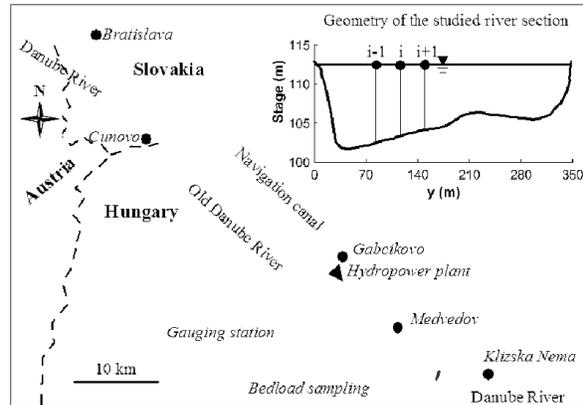
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topography, secondary flow, and transverse transfer of momentum in the river section, are not taken into account (Pham Van, 2016). Therefore, significant efforts related to representation of the complexity of local hydraulic characteristics and bedload sediment estimations are still needed to (i) improve the accuracy of calculations, (ii) reduce as much as possible the uncertainty of estimation of bedload sediment, and (iii) study transport processes of bedload sediment.

A large number of numerical models ranging from one-dimension (1D) to three-dimension (3D) have been developed for studying flow and sediment transport processes. The use of numerical simulation has become an essential tool in the discipline, complementing the other analysis tools of experiment and theory. This is because the rapid developments in numerical methods and the rapid advances in computer technology. There have been intensive numerical studies on flow and bedload sediment transport. However, in terms of practical point of view, simple numerical models are still needed and remain useful predictive tools even today.

The objectives of the present study are twofold. Firstly, the study aims at presenting a proposed model consisting of hydrodynamic and sediment modules that can be used for simulating the bedload sediment through the river section. Secondly, the study also aims at (i) accurately representing the measurement data by using the model and (ii) identifying suitable relations to compute the bedload sediment transport rate and bedload sediment discharge. The computed cross-section averaged velocity, water discharge, bed shear stress, bedload sediment rate, and bedload sediment discharge are compared to the measurement data conducted at a section of the Danube River.

## 2. MEASUREMENT DATA



*Fig.1. Schematic illustration of the Danube River, with the studied cross-section*

The measurement data of flow and bedload sediment conducted through a section of the Danube River (Camenen et al., 2011) are employed under the present consideration. The river section is located in the meandering part of Danube River approximately of about 70 km downstream from Bratislava, Slovakia (Fig.1). The slope of the river is 0.0004. Bedload sediments were collected through the river section using a basket-type bedload sampler, with a mesh size of 3 mm. Seventy-one field campaigns were performed during the period from 2000 to 2002. In each campaign, bedload samples were measured at six vertical locations across the study section. The samples were weighted and sieved to identify the grain size characteristics of the bedload sediment. At each vertical location, bedload sample was measured from 2 to 5 minutes depending on the local flow conditions and bedload transport intensity, and it was repeated ten times to derive an averaged value in order to reduce the error resulting from temporal fluctuation. The median grain-size ( $d_{50}$ ) from the bedload samples is 9 mm. The water depth and flow velocity were also measured at each vertical bedload sampling location, revealing that the water discharge in the channel section varies between 970 and 4750 m<sup>3</sup>/s in the field campaign period.

### 3. NUMERICAL MODEL

#### 3.1. Hydrodynamic module

Flow characteristics in the cross-section are determined by solving Eq. 1, in which the gravity, bed shear stress, turbulent diffusion force, and secondary flow are taken into account in order to allow for accurate simulations of distribution of flow velocity as well as of water discharge (Ervin et al., 2000).

$$\rho g H S_x - B_g \tau_b + \frac{\partial}{\partial y} \left( \rho H \nu_t \frac{\partial U}{\partial y} \right) - \frac{\partial}{\partial y} (\rho K H U^2) = 0 \quad (1)$$

where  $\rho$  is the water density ( $\text{kg/m}^3$ ),  $g$  is the gravitational acceleration,  $H$  is the water depth (m),  $S_x$  is the bed slope in the streamwise direction,  $B_g = \sqrt{1 + S_x^2 + S_y^2}$  is the geometrical factor, in which  $S_y$  is the bed slope in the lateral direction,  $\tau_b$  is the bed shear stress,  $U$  is the depth-averaged streamwise velocity (m/s),  $y$  denotes the lateral direction,  $\nu_t$  is the eddy viscosity ( $\text{m}^2/\text{s}$ ), and  $K$  is an empirical coefficient representative of secondary effects.

The eddy viscosity can be determined by using different models, from simple ones such as a constant value to more complicated ones, e.g. zero-equation, one-equation, two-equation, and the Smagorinsky turbulence models (Pham Van et al., 2014a; 2014b). In the present study, the zero-equation turbulence model is chosen to compute the eddy viscosity because in the framework of depth-averaged model of uniform open channel flow no significant advantage of simulation results is brought by using the more complicated model such as one- or two-equation turbulence model (Pham Van et al., 2014a; 2014b). This turbulence model is given as:

$$\nu_t = \lambda U_* H, \quad (2)$$

where  $U_*$  is the shear velocity and  $\lambda$  is the non-dimensional eddy viscosity coefficient. The latter is set equal to 0.067, which is obtained by averaging the logarithmic velocity profile over the depth.

The bed shear stress is computed as

$$\tau_b = \rho \frac{g n^2}{H^{1/3}} U^2 \quad (3)$$

where  $n$  is the Manning coefficient.

Equation (1) is discretized by using the finite difference scheme, resulting in an algebraic equation system. The latter is then solved by using Newton-Raphson iteration method. In addition, the free slip condition is applied at river banks.

#### 3.2. Sediment module

The bedload sediment discharge through the river section  $Q_s$  is obtained by integrating the bedload sediment rate  $q_{bs}$  (that is defined as the volume rate of bedload sediment transport per unit the river width) across the section.

$$Q_s = \int_0^B q_{bs} dy \quad (4)$$

where  $B$  is the river width.

The bedload sediment rate  $q_{bs}$  is computed as

$$q_{bs} = \phi_b \sqrt{g(\rho_s / \rho - 1)} d_{50}^3 \quad (5)$$

where  $\rho_s$  is the sediment density ( $\text{kg/m}^3$ ),  $d_{50}$  is the median grain-size of bedload sediment particles, and  $\phi_b$  is the dimensionless sediment transport rate that is often determined empirically based on the Shields parameter  $\theta$  and the critical Shields parameter for initial motion of sediment  $\theta_{cr}$ . Nine bedload relations (listed in **Table 1**) are applied to compute  $\phi_b$ . The purposes of using different bedload relations are to (i) investigate the sensitivity of the bedload sediment transport rate and bedload sediment discharge when using different relations, (ii) compare the accuracy of different bedload relations and their suitability for real applications by validating them against measurements, and (iii) identify approximate relations.

**Table 1. Bedload relations for computing the non-dimensional sediment transport rate**

Bedload relations	$\theta_{cr}$	Reference	Abbreviated
$\phi_b = 8(\theta - \theta_{cr})^{1.5}$	0.047	Meyer-Peter and Müller (1948)	MM
$\phi_b = 12(\theta - \theta_{cr})^{1.5}$	0.047	Wilson (1966)	Wi
$\phi_b = 17(\sqrt{\theta} - \sqrt{\theta_{cr}})(\theta - \theta_{cr})$	0.05	Ashida and Michiue (1972)	AM
$\phi_b = 18.74(\theta - \theta_{cr})(\sqrt{\theta} - 0.7\sqrt{\theta_{cr}})$	0.058	Engelund and Fredsoe (1976)	EF
$\phi_b = 5.7(\theta - \theta_{cr})^{1.5}$	0.05	Fernandez Luque and van Beek (1976)	FV
$\phi_b = 11.2 \frac{(\theta - \theta_{cr})^{4.5}}{\theta^3}$	0.03	Parker (1978)	Pa
$\phi_b = 0.0053 \left( \frac{\theta}{\theta_{cr}} - 1 \right)^{2.2}$	0.03	Wu et al. (2000)	Wu
$\phi_b = 12\theta^{1.5} \exp\left(-4.5 \frac{\theta_{cr}}{\theta}\right)$	0.055	Camenen and Larson (2005)	CL
$\phi_b = 3.97(\theta - \theta_{cr})^{1.5}$	0.0495	Wong and Parker (2006)	WP

#### 4. RESULTS AND DISCUSSION

##### 4.1. Hydrodynamic results

###### 4.1.1. Calibration results

To calibrate the Manning coefficient and  $K$  coefficient, different constant values of these parameters were tested in order to obtain the best fit between the simulated results and observed data of the flow at the studied river section. The value of each parameter was varied separately while keeping the other ones constant. In particular, the value of  $n$  was varied between 0.015 and 0.035 while the value of  $K$  was changed from 0.0001 to 0.0006.

Fig. 2 shows impacts of bottom friction coefficient on the stage-discharge and velocity-discharge while Fig. 3 illustrates impacts of  $K$  coefficient. It is not surprised that both cross-section averaged velocity and water discharge vary significantly if the variable values of parameters are employed. This result suggests that the calculated results of flow are very sensitive to changes in the bed shear stress or

secondary force.

The approximate values of parameters are found to be  $n = 0.0265$  and  $K = 0.0003$ . The root mean square error ( $RMSE$ ) and mean absolute error ( $MAE$ ) of the water discharge corresponding to these parameters values are 250 and 170  $m^3/s$  (<5.4% of observed magnitude measured of water discharge), respectively. The Nash-Sutcliffe efficiency ( $NSE$ ) of water discharge is 0.92 while the correlation coefficient between computed results and observed data is 0.97. The  $RMSE$  and  $MAE$  of section-averaged velocity are 0.127 and 0.093  $m/s$  (<7.2% of observed velocity magnitude measured at the river section), respectively. The correlation coefficient between computed and observed section-averaged velocity equals to 0.80. These results reveal that the model represents well the observed flow data.

A value  $n = 0.022$  is also obtained by using the median grain-size. This value is slightly

smaller than the calibrated value ( $n = 0.0265$ ). The discrepancy can be explained by taking into account the secondary flow in simulations,

which is resulted from the channel curvature, the complex topography in the cross-section, and the presence of a bedform on the river bed.

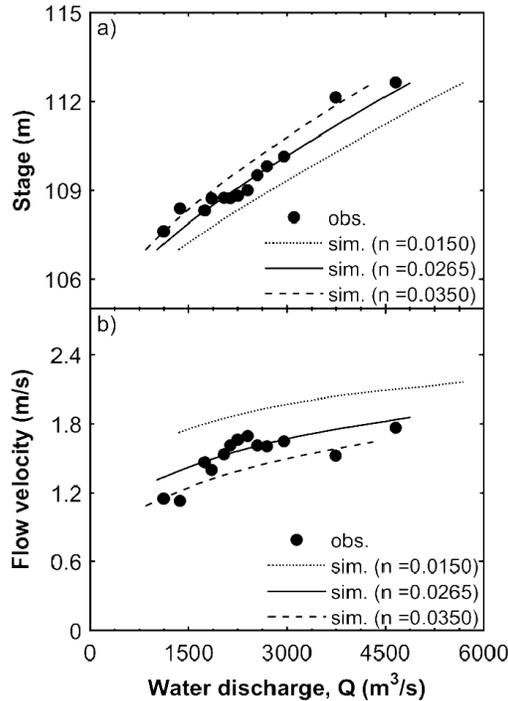


Fig. 2. Impacts of Manning coefficient on: a) stage and b) flow velocity ( $K=0.0003$ )

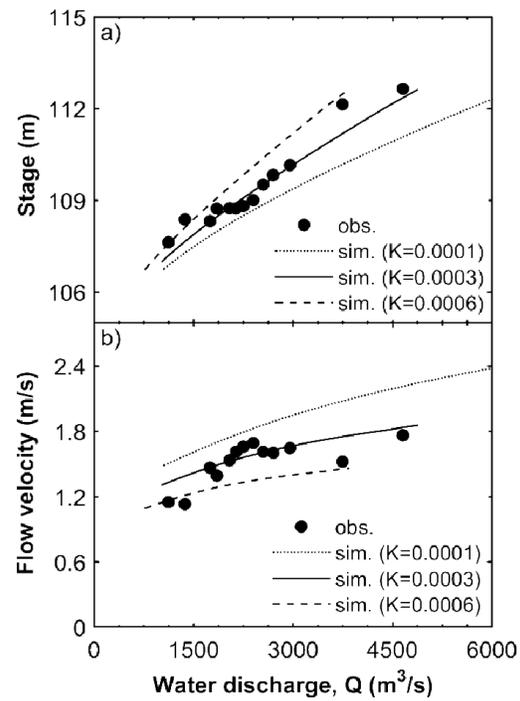


Fig. 3. Impacts of  $K$  coefficient on: a) stage and b) flow velocity ( $n=0.0265$ )

#### 4.1.2. Validation results

Four simulations using constant water discharges, i.e.  $Q = 1250, 1750, 2250, 2750$  ( $m^3/s$ ) are performed to validate the hydrodynamic module. The computed bed shear stress from these simulations is compared to values estimated from experimental data. The latter was obtained from water discharge corresponding to four ranges, i.e. 1000-1500, 1500-2000, 2000-2500, 2500-3000 ( $m^3/s$ ).

Fig. 4 shows the comparison between model predictions and observations of the lateral distribution of the bed shear stress, illustrating that a good agreement is obtained. Model-predicted bed shear stress typically lies in the mid-range of the experimental data, except for low flow conditions (Fig. 4a) in which the model-predicted bed shear stress overestimates

the observed data. Previous studies (Pham Van et al., 2014b; Pham Van, 2016) showed that the bed shear stress can increase sufficiently due to the effects of bed-generated turbulence and lateral shear turbulence when the water depth in the river section is low, which may be used to account for overestimation of the model-predicted bed shear stress in low flow conditions.

The model also reproduces the bed shear stress in the shallow water region (from  $y = 200$  m to  $y = 250$  m) in the river cross-section. Model-predicted bed shear stress is often small in this region while larger values are observed in the adjacent deeper regions of the section. A similar trend is revealed in experimental data. These results suggest that the values  $n = 0.0265$  and  $K_1 = 0.0003$  can be used to reproduce the flow when studying bedload sediment.

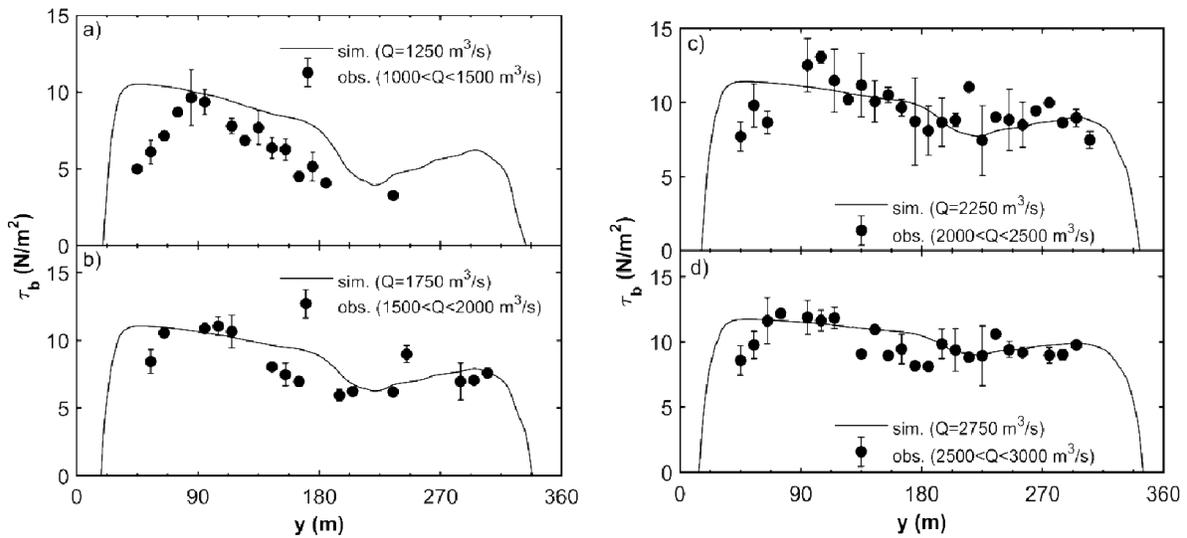


Fig. 4. Lateral distribution of bed shear stress with the water discharge of:  
 a)  $1250 \text{ m}^3/\text{s}$ , b)  $1750 \text{ m}^3/\text{s}$ , c)  $2250 \text{ m}^3/\text{s}$  and d)  $2750 \text{ m}^3/\text{s}$

#### 4.2. Sediment results

Fig. 5 shows the lateral distribution of model-predicted and observed bedload transport rate for four selected water discharges, i.e.  $Q=1250$ ,  $1750$ ,  $2250$ ,  $2750$  ( $\text{m}^3/\text{s}$ ). A good agreement is obtained for model-predicted and observed data for the bedload transport rate. In the low flow conditions ( $Q < 1500 \text{ m}^3/\text{s}$ ), the bedload sediment is concentrated mainly in the section from  $y = 20$  m to  $200$  m, as this is the deepest region in the river section. On the other hand, the bedload sediment is progressively transported in the whole section in the high flow situations, indicating a consistency with the variable trend in bed shear stress.

Nine bedload relations are applied to compute  $\phi_b$ . The MM, Pa, and Wi relations provide the largest bedload transport rate among nine relations while the CL and Wu relations lead to the smallest transport rates. The CL and Wu formulas demonstrate the transport of bedload sediment even in low flow conditions.

This is because the CL formula employs a statistical approach for the initial motion of sediment, and is less sensitive to the uncertainty in estimating the critical Shields parameter (Camenen et al., 2005). In the case of the Wu formula, the small bedload transport rate is due to the small critical Shields parameter. AM, Wu, and WP yield an acceptable computed sediment rate in comparison with the observations while CL relation provides the best fits of observed sediment rate for all four selected water discharges.

Fig. 6 shows the bedload sediment discharge associated with water discharge in the river section. The CL relation provides the best fit between model predictions and observations. However, in the low flow conditions, an overestimation for sediment discharge is obtained, revealing similar trend as those achieved from the other relations. The reason for that is due to an overestimation of the bedload sediment transport rate.

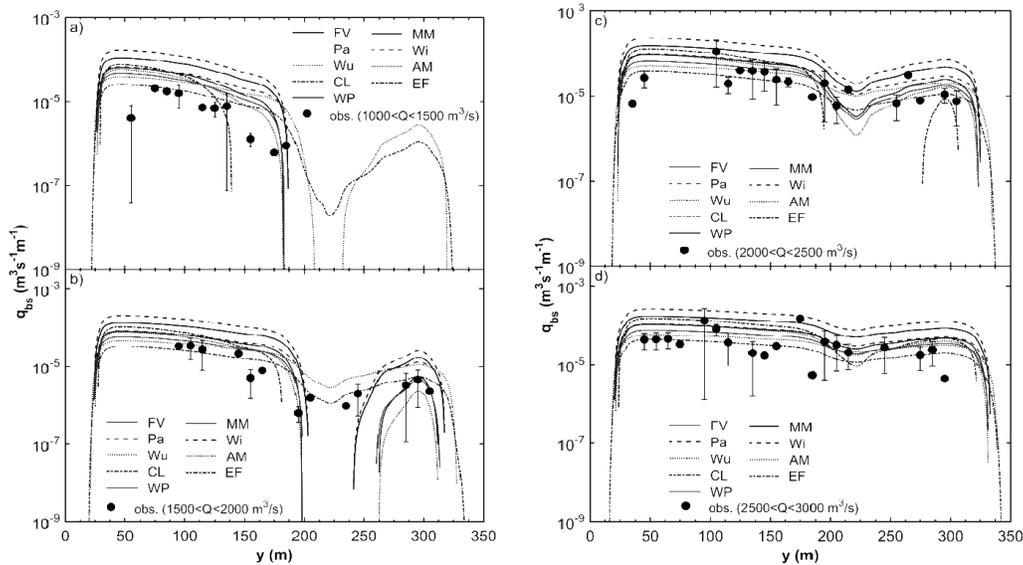


Fig. 5. Lateral distribution of bedload sediment rate, with water discharge of: a)  $Q=1250 \text{ m}^3/\text{s}$ , b)  $Q=1750 \text{ m}^3/\text{s}$ , c)  $Q=2250 \text{ m}^3/\text{s}$ , and d)  $Q=2750 \text{ m}^3/\text{s}$

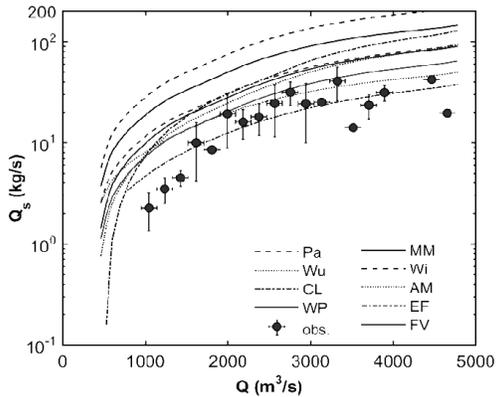


Fig. 6. Sediment discharge versus water discharge

Camenen et al. (2011) reported that bedload samples at the studied river section consist of both coarse and fine gravel particles. Fine gravel particles can be hidden and protected by coarse gravel particles, leading to coarse gravel particles that are more exposed and may be transported easily. In other words, the mobility of fine gravel particles may be reduced as they are hidden, and the mobility of coarse gravel particles may be increased due to their exposition in non-uniform mixture sediments. However, for the sake of simplicity, only one sediment class using the median grain-size was used when calculating bedload transport rate and sediment discharge. The use of median grain-size may not be suitable when considering the bedload sediment in reality. More than one sediment class may be needed in order to take

into account the effects of non-uniform mixture sediments. This will be done in the future modeling efforts of the research.

## 5. CONCLUSION

The use of numerical models for solving sediment transport problems is relatively compared with the use of field measurements and the physical models. The aims of the study were (i) to propose a simple model that can be used to simulate bedload sediment through river sections, (ii) to accurately reproduce the measurement data by using the model, and (iii) to identify suitable relations to compute the bedload sediment transport rate and bedload sediment discharge. The results showed that the proposed model was successfully applied for simulating both flow and bedload sediment transport through the studied river section of the Danube River. An acceptable agreement between simulations and measurements was obtained for cross-section averaged velocity, water discharge, bed shear stress, bedload sediment rate, and bedload sediment discharge. The CL relation, which employs a statistical approach for the initial motion of sediment and it is less sensitive to the uncertainty in estimating the critical Shields parameter, allowed a better representation of both bedload sediment transport rate and bedload sediment discharge than the eight others in the studied river section.

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### Tóm tắt:

## MÔ HÌNH THUYẾT ĐỘNG LỰC/BÙN CÁT CHO MÔ PHỎNG BÙN CÁT ĐÁY TRONG SÔNG

Bài báo này đề xuất một mô hình thủy động lực/bùn cát cho mô phỏng bùn cát đáy chuyển qua mặt cắt sông. Trước tiên, các đặc trưng dòng chảy như vận tốc trung bình theo độ sâu được tính toán thông qua giải phương trình toán học đặc trưng (i) được biến đổi từ các phương trình Reynolds và (ii) cho phép xem xét cả trọng lực, lực ma sát đáy, lực khuếch tán do dòng chảy rối, và dòng chảy thứ cấp trong tính toán. Bốn tiêu chí sai số khác nhau: sai số quân phương, sai số trung bình, hệ số Nash-Sutcliffe, và hệ số tương quan đã được sử dụng để đánh giá chất lượng của kết quả tính toán, thể hiện sự phù hợp tốt giữa kết quả tính toán và số liệu đo đạc tại mặt cắt sông nghiên cứu. Tiếp theo, chín quan hệ khác nhau dùng cho xác định tỷ lệ chuyển động của bùn cát đáy và lưu lượng bùn cát đáy được xem xét và so sánh để xác định các quan hệ phù hợp nhất. Kết quả tính toán thể hiện rằng quan hệ đề xuất bởi Camenen and Larson (2005) cho kết quả tính toán tốt hơn các quan hệ khác. Cuối cùng, hướng phát triển và các áp dụng mở rộng của mô hình đề xuất cũng được thảo luận.

**Từ khóa:** Bùn cát đáy, tỷ lệ bùn cát, mô hình thủy động lực/bùn cát, sông Danube.

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