

# An integrated method for multi-objective optimal design of a piped irrigation network

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**Abstract:** Recently, piped irrigation systems have been getting more and more widely utilized. This paper aims to propose an integrated Non-dominated Sorting Genetic Algorithm II (NSGA II) and Multiply Criteria Decision Making (MCDM) method for finding the ultimately optimal design of piped irrigation networks when simultaneously considering minimum cost of pipes and maximum life span of pipes. The coupled method was applied on a real piped irrigation system consisting of 30 pipe segments. First, 11 Pareto optimal solutions were found by using NSGA II. Then, the optimal solution with the pipe cost of  $11.5 \times 10^9$  VND and the life span of 43.6 years was ultimately selected based on Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) selection methods. The proposed coupled method could be applied to find optimal design for other piped irrigation systems.

**Keywords:** Pipe networks, optimal design, NSGA II, selection methods.

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## 1. Introduction

In an irrigation system, a conveyance network plays an important role in transport of water from the water sources to the fields. The investment cost for the conveyance networks could account for 30-40% of the total project cost. The conveyance networks can use canals or pipes. Compared with the canal networks, the pipe networks have more advantages such as saving water, taking advantage of high water head to increase the irrigation area, ensuring water quality during transportation, occupying small land area, saving management and operation costs, facilitating the installation of water measurement devices, being highly adaptive to complex topographical and geological conditions, natural disasters... Because of such dominations, piped irrigation systems are getting more and more widely utilized (Zhao and Li 2020). To keep a piped irrigation system to be more effective and sustainable, its optimal design should be carefully considered.

Recently, optimization approach has been widely applied to design of water supply networks. Several researches focused on using single objective optimization to find optimal solutions of loop and/or tree networks. As compared with single objective optimization, multi-objective optimization (MOO) approach provides much comprehensive information for decision makers to choose an optimal design for water supply networks. Namely, Artina et al. (2012) improved NSGA II algorithm ( Deb et al. 2002) to optimally design a water distribution network when simultaneously considering both minimum cost and controlling pressure at nodes. Multi-Criteria Decision-Making (MCDM) approaches support decision makers to find the best solution from a set of candidate alternatives against relevant multiple criteria (Hafezalkotob et al. 2019). MOO problems generate many optimal solutions known as Pareto-optimal solutions. MCDM has been integrated with MOO to select one solution of the set of Pareto-optimal solutions for implementation in several fields (Parhi et al. 2020). Among previous MOO researches of water supply networks, the ultimately optimal solution for implementation

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has not yet clarified. Therefore, an integrated MOO and MCDM model to select the optimal design of water supply networks for implementation deserves to develop further.

This paper aims to propose an integrated NSGA II and MCDM method for finding the ultimately optimal design of piped irrigation networks. First, MOO problems of design of piped irrigation networks were established when simultaneously considering minimum cost of pipes and maximum life span of pipes. Second, a set of Pareto optimal design alternatives was found using NSGA II. Finally, one optimal design alternative of Pareto optimal solution set for implementation was selected by using TOPSIS methods (Hwang and Yoon 1981).

## 2. Methods

First, the optimization model of piped irrigation network design was developed to simultaneously achieve the minimum cost of construction and the maximum service life based on selecting different available diameters and material for pipe segments. Secondly, NSGA ii was used to find sets of non-dominated optimal solutions and subsequently define the set of Pareto – optimal solutions. Finally, one of Pareto – optimal solutions was recommended to decision makers for implementation by using several selection methods.

### 2.1. Optimal Model of Piped Irrigation Network Design

#### 2.1.1. Objective functions

The first objective is to minimize the cost of pipe. Here, the pipe cost ( $C_p$ ) includes the pipe construction cost ( $C_c$ ) and the operational cost ( $C_o$ ). The pipe cost depends on diameters, material, construction method, working hours of pipes, discharges.

$$\text{Min } C_p = C_c + C_o \quad (2.1)$$

The construction cost ( $C_c$ ) includes the cost of pipe material ( $C_m$ ) at site and the cost of laying ( $C_L$ ).

$$C_c = C_m + C_L \quad (2.2)$$

According to Hai (2018) and Lin et al. (2016),  $C_m$  and  $C_L$  were defined as follows.

$$C_m = \sum_{i=1}^n C_{m0} D_i^\alpha L_i \quad (2.3)$$

$$C_L = \sum_{i=1}^n (550.9 D_i^2 - 396 D_i + 3066) \quad (2.4)$$

Where:

$D_i$  is the pipe diameter of the  $i^{th}$  pipe segment;

$L_i$  is the length of the pipe segment;

$C_{m0}$  and  $\alpha$  are empirical coefficients, depending on specific material.

$n$  is the total number of pipe segments of piped irrigation networks

The operational cost ( $C_o$ ) was determined in the following equation.

$$C_o = \left( \sum_{i=1}^n 9.81 \cdot k_i \cdot Q_i \cdot H_i \cdot T_i \right) \cdot \frac{((1+i)^m - 1)}{(1+i)^m} \quad (2.5)$$

Where:

$Q_i$  is the calculated discharge of the  $i^{th}$  pipe segment;

$T_i$  is the working duration corresponding  $Q_i$  of the  $i^{th}$  pipe segment;

$m$  is the project life,  $m=30$  years;

$k_i=1$  if the  $i^{th}$  pipe segments locate along the disadvantage route and  $k_i = 0$  if not.

$H_i$  is the hydraulic loss of the  $i^{th}$  pipe segment which is calculated by Hazen Williams formula as follows:

$$H_i = 10.67 \cdot L_i \cdot Q_i^1 \cdot \frac{85}{C_i^1 \cdot 85 \cdot D_i^4 \cdot 87} \quad (2.6)$$

where:

$C_i$  is the roughness coefficient determined by pipe material of the  $i^{th}$  pipe segment and the others are explained above.

The second objective is to maximize the service life of piped irrigation network. Here, the service life of piped irrigation network is defined as the average service life of all pipe segments.

$$\text{Maximize SL} = \frac{1}{n} \sum_{i=1}^n SL_i \quad (2.7)$$

Where:  $SL_i$  is the service life of the  $i^{th}$  pipe segment

### 2.1.2. Constraints

Velocity of the pipe  $i^{th}$  segment ( $V_i$ ) is in the allowable range:  $0.3 \text{ m/s} \leq V_i \leq 3 \text{ m/s}$  (2.8)

Along the main pipeline, diameters of the upstream pipe segments ( $D_{up}$ ) are not smaller than that of the downstream pipe segments ( $D_{down}$ ):  $D_{up} \geq D_{down}$  (2.8)

### 2.1.3. Decision variables

The decision variables are the pipe diameter ( $D_i$ ), the pipe material ( $M_i$ ) and the length of pipe segment ( $L_i$ ).

## 2.2. NSGA-II

The NSGA-II algorithm (Deb et al. 2002) is used to find the set of Pareto optimal solutions for multi-objective optimization problems. The three main features of the NSGA-II algorithm are: developing elites, using a mechanism to preserve the diversity of solutions, and focusing on non-domination solutions. In this paper, the MOO problem was formulated in MS Excel, subsequently solved by MOO program developed by Sharma et al. (2012).

**Table1.** Encoding rule for available diameters and material

No.	Encode	Diameter (mm)			
		1	2	3	4
		PVC	HDPE	MSP	DIP
1	1	63	32	50	50
2	2	75	63	65	60
3	3	90	90	80	65
4	4	110	110	100	80
5	5	125	125	125	125
6	6	140	160	150	150
7	7	160	180	200	200
8	8	180	225	250	250
9	9	200	250	300	300
10	10	225	280	350	350
11	11	250	315	400	400
12	12	280	355	450	450
13	13	315	500	500	500

No.	Encode	Diameter (mm)			
		1	2	3	4
		PVC	HDPE	MSP	DIP
14	14	355	520	520	600
15	15	400	600	600	700
16	16	450	700	700	800
17	17	500	800	800	900

Note: PVC is Poly-Vinyl Chloride; HDPE is High Density Polyethylene Pipe; MSP is Mild Steel Pipe; DIP is Ductile Iron Pipe.

### 2.2.1. Selection Methods for Pareto-Optimal Solutions

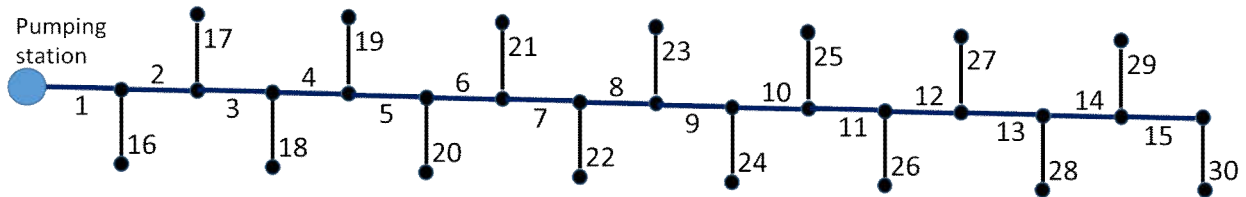
First, all Pareto-optimal solutions of MOO problem which were generated by using NSGA II several times were combined. Subsequently, the set of Pareto-optimal solutions were sorted in non-domination principle, consequently finding the true Pareto-optimal front. Finally, combinations of the entropy weighting method against TOPSIS selection methods were utilized to recommend one optimal solution for the decision makers. The entropy method is based on a measure of uncertainty in information, formulated in terms of probability theory (Li; et al. 2014). The TOPSIS selected optimal solution has the smallest Euclidean distance from the positive ideal solution (PIS) and the largest Euclidean distance from the negative ideal solution (NIS). The PIS is comprised of the best value of each objective in the given optimal solutions, while the NIS is a combination of the worst value of each objective in the given optimal solutions (Hwang and Yoon 1981). Here, TOPSIS selection method were solved by using MS Excel program developed by Wang et al. (2020).

### 2.2.2. Study Case of Water Supply Pipe Network

The study area is located at Tho Xuan District, Thanh Hoa province, Vietnam. The study area is 162 hectares which apply modern agriculture practices to various crops including

tea, dragon fruit, sugarcane, oranges and pomelo. The average annual rainfall of the area is 1911 m. The climate is divided into two distinct seasons: rainfall season from May to October and dry season from November to May. The average humidity of the area is 86%. The land in the study area is mainly low hills. The piped irrigation system is designed to pump

water from Chu river to the modern agricultural area and Lam Son Sugar Factory (Figure 1). The piped irrigation system consists of 30 pipe segments including 15 main pipe segments and 15 branched pipe segments. Material of each pipe segment could be one of PVC, HDPE, MSP and DIP. The parameters of each material are shown in Table 2.



**Figure 1.** Calculation diagram of the piped irrigation system

**Table 2.** Parameters of pipe materials

No.	Parameters	Material			
		PVC	HDPE	MSP	DIP
1	$C_{mo}$	285.3	233	287	476
2	$\alpha$	0.0034	0.0059	0.0076	0.0044
3	$C$	140	150	130	120

Note:  $C_{mo}$  and  $\alpha$  are empirical coefficients which are extracted by a regressive analysis for each material;  $C$  is pipe roughness coefficient.

The calculated flow of each pipe segment is calculated based on the service area and the irrigation coefficient of each crop. Irrigation coefficient, the amount of irrigation water and irrigation duration of

each crop are referred to in the Irrigation Manual for Dry Crops (MARD 2013). Calculation results of flow rate and irrigation duration for pipe segments are shown in Tables 3 and 4.

**Table 3.** Flow rate and irrigation duration of branch pipes

No.	Branch pipe	Areas (ha)	Irrigation rates (l/s.ha)	Discharges ( $m^3/s$ )	Crops	Lengths(m)	Irrigation duration (h/year)
1	16	19.3	2.47	0.048	Dragon fruit	211.3	324
2	17	25.7	2.55	0.065	Orange	483.9	216
3	18	11.1	4.17	0.046	Tea	209.5	192
4	19	16.0	3.01	0.048	Sugar cane	731	398
5	20	13.5	2.55	0.034	Orange	247.7	216
6	21	13.1	3.01	0.039	Sugar cane	332.2	398
7	22	13.0	2.47	0.032	Dragon fruit	296.1	324
8	23	15.8	3.01	0.048	Sugar cane	334.9	398
9	24	5.6	2.55	0.014	Orange	72.4	216

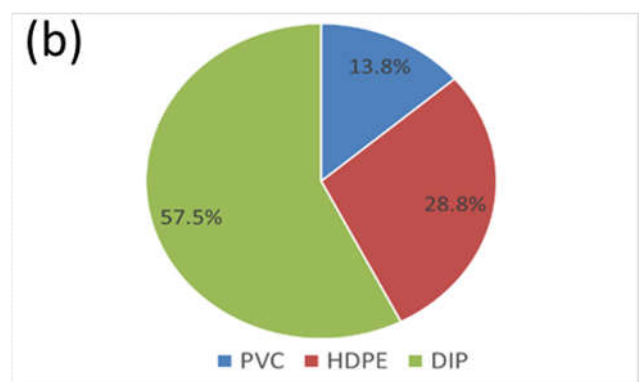
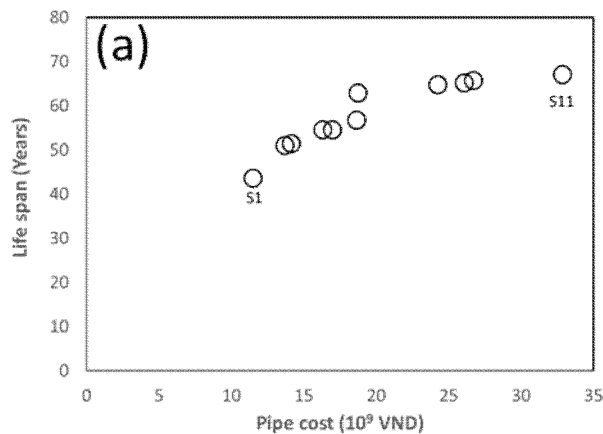
No.	Branch pipe	Areas (ha)	Irrigation rates (l/s.ha)	Discharges (m <sup>3</sup> /s)	Crops	Lengths(m)	Irrigation duration (h/year)
10	25	4.3	1.30	0.006	Factory	289.8	1056
11	26	6.6	1.91	0.013	Grapefruit	320.4	216
12	27	5.1	1.30	0.007	Factory	181.5	1056
13	28	3.6	2.55	0.009	Orange	91.9	216
14	29	5.3	2.55	0.013	Orange	81	216
15	30	3.5	2.55	0.009	Orange	158.5	216

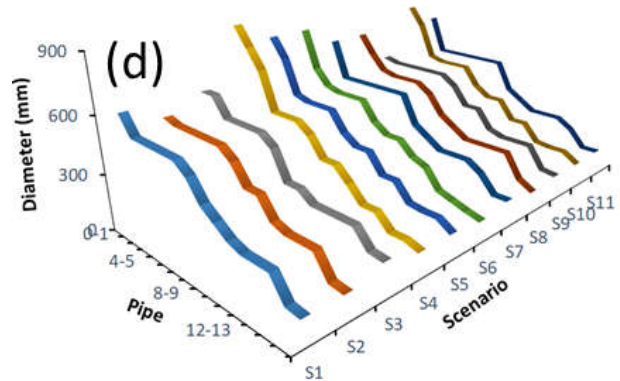
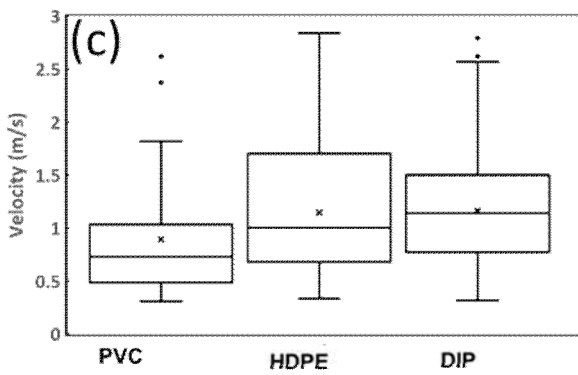
**Table 4.** Discharges of main pipes

No.	Main pipe	Area (ha)	Discharge (m <sup>3</sup> /s)	Length (m)
1	1	162	0.43	316.7
2	2	142	0.38	125.3
3	3	117	0.32	108.2
4	4	105	0.27	35.9
5	5	89	0.22	110.7
6	6	76	0.19	156
7	7	63	0.15	99.7
8	8	50	0.12	66.7
9	9	34	0.07	179.5
10	10	28	0.06	65.4
11	11	24	0.05	53.1
12	12	17	0.04	56
13	13	12	0.03	34.2
14	14	9	0.02	138.3
15	15	4	0.01	25

### 3. Results and discussion

#### 3.1. Minimization of cost together with maximization of life span





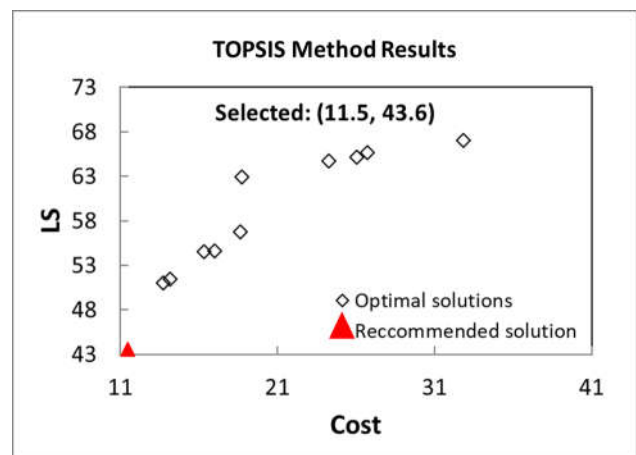
**Figure 2.** Results of minimization of cost coincident with maximization of life span:

- (a) Pareto optimal solutions, (b) Pipe materials of Pareto solutions,
- (c) Pipe diameters of Pareto solutions, and (d) Velocities of Pareto solutions

Figure 2a describes the optimal trade-off solution for two objectives, which are minimizing the implementation cost and maximizing the life span of pipes. For each pipe segment, 17 possible diameter alternatives and four possible material alternatives were evaluated. For total 30 pipe segments, the solution space consists of  $4^{30} \times 17^{30}$  solutions. The parameters were set for NSGA II including 300 generations, a crossover rate of 0.9, a mutation rate of 0.65, and a population size of 600. NSGA II generated from 3 to 8 optimal solutions for each run. A total of 102 optimal solutions were combined through 22 run times. Each optimal solution is a combination of 30 pipe segments with defined diameters and materials. By using non dominated sorting in MS Excel for 102 optimal solutions, 11 non dominated optimal solutions were defined and plotted in Figure 2a. Figure 2a indicate that the pipe cost is in the range of from 11.5 to  $32.8 \times 10^9$  VND and the corresponding life span is from 43.6 to 67 years. This means that an increase by 23 years in the life span required an additional investment of  $30 \times 10^9$  VND. No pipe segments utilized MSP material due to its high price unit. DIP utilization (Fig. 2b) is the most popular in all optimal solutions. The second and third popularities in material utilization are HDPE

and PVC, respectively. Figure 2c indicates that velocities of all pipe segments of all solutions strictly followed the constraint, in the range of from 0.3 m/s to 3 m/s. The mean velocities of the pipe segments made of PVC (0.89 m/s) is lower than those made of HDPE (1.05 m/s) or DIP (1.37 m/s). Figure 2d shows that the constraint of diameters along the main pipe routine is completely satisfied because there is not an increase in diameters from the upstream to downstream pipes. These prove that all constraints were strictly followed when finding the optimal solutions.

### 3.2. Selection one of Pareto optimal solutions



**Figure 3.** Recommended optimal solutions selected by TOPSIS selection methods

To recommend the decision makers to choose one Pareto-optimal solution for implementation, TOPSIS selection methods were utilized. The entropy weighting method objectively generated two weights of 0.856 and 0.144 which were respectively assigned for cost and life span objectives of TOPSIS selection methods. The optimal solution having cost of  $11.5 \times 10^9$  VND and life span of 43.6 years was chosen for implementation

because of its most popular recommendation by 11 selection methods (Figure 3). The decision variables of the final chosen optimal solution was shown in Table 5. The results indicate that the percentage of pipe material were 27%, 33% and 40% for PVC, HDPE and DIP, respectively. The velocities of the main pipes are in the range of 0.8-2.3 m/s which is narrower than the range of 0.3-2.8 m/s in the branch pipes.

**Table 5.** Decision variables of the chosen optimal solution

No.	Pipe	Material	Velocity (m/s)	Diameter (mm)	No.	Pipe	Material	Velocity (m/s)	Diameter (mm)
1	16	HDPE	0.8	280	16	1	DIP	1.5	600
2	17	PVC	0.4	450	17	2	HDPE	2.0	500
3	18	HDPE	2.3	160	18	3	DIP	1.6	500
4	19	DIP	1.5	200	19	4	DIP	1.4	500
5	20	DIP	2.8	125	20	5	HDPE	1.1	500
6	21	PVC	0.5	315	21	6	DIP	1.0	500
7	22	PVC	2.1	140	22	7	DIP	0.9	450
8	23	PVC	2.4	160	23	8	DIP	1.2	350
9	24	PVC	1.5	110	24	9	DIP	1.0	300
10	25	PVC	0.5	125	25	10	DIP	1.1	250
11	26	PVC	0.8	140	26	11	HDPE	1.3	225
12	27	HDPE	0.3	160	27	12	HDPE	1.0	225
13	28	PVC	0.4	180	28	13	HDPE	0.8	225
14	29	DIP	1.7	100	29	14	HDPE	2.3	110
15	30	HDPE	0.9	110	30	15	DIP	1.1	100

In Vietnam, dendritic pipe networks such as piped irrigation networks and rural water supply networks have been designed through selecting velocity for each pipe segment based on the range of economical velocities ruled in the codes without additional specific guidance (MOC 2006). In fact, economical velocities depend on material, price of pipes and consequently the economical velocities change with various pipe material as well as places to install pipes. Therefore, referring only the range of economic velocity in the codes for design of different dendritic pipe networks could consist

of high uncertainty. The integrated NSGA II and MCDM method to optimally design dendritic pipe networks was considered as novel contribution to fill the gap. In future, various construction methods of pipe segments should be included in the proposed method to adapt the complicated characteristics of topography and geology.

#### 4. Conclusions

This paper proposed the coupled method of NSGA II and Multiply Criteria Decision Making to find one optimal design alternative of piped irrigation systems when simultaneously

considering two objectives including the minimum pipe cost and the maximum life span. Subsequently, the coupled method was applied on the real piped irrigation system consisting of 30 pipe segments. Each design solution of a pipe segment included one of 17 available diameter sizes and one of 4 material types. From  $4^{30} \times 17^{30}$  possible design solutions of the piped irrigation system, NSGA II finds 11 Pareto-optimal solutions. Accordingly, the pipe cost is in the range of from 11.5 to  $32.8 \times 10^9$  VND corresponding to the life span range varying from 43.6 to 67 years. By using TOPSIS selection methods, the optimal solution with the pipe cost of  $11.5 \times 10^9$  VND and the life span of 43.6 years was selected based on its most popular recommendation from 14 selection methods.

The proposed coupled method could be applied to find optimal design of other piped irrigation systems. In future, the life span should be considered as a function of material, diameter and buried depth.

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