

# CALCULATION OF THE DRAFT OF THE FLOATING BRIDGE BUILT FROM PLASTIC BUOYS WHEN THE LOAD IS A TRUCK

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## Abstract

In some emergency traffic situations such as rescue, natural disasters,... it is necessary to build a temporary bridge to cross the river. Using a quick-assembled floating bridge made of plastic buoys is the appropriate solution. This article presents the study of calculating the draft of continuous floating bridge assembled from plastic buoys. The author calculates the draft by two different methods, which are theoretical calculation and finite element modeling using SAP2000 software, to verify the discrepancy of the results of these two methods. The results obtained show that the discrepancy between these two methods is insignificant, and continuous floating bridge assembled with a layer of plastic buoys can withstand a small truck load of up to 12.3 tons.

*Keywords: Floating bridge; plastic buoy; truck; draft; elastic background.*

## 1. Introduction

In our country, in some emergency traffic situations such as rescue, natural disasters, repairing old bridges, etc., it is necessary to build a temporary bridge to cross the river [1-3]. When the load requirement is not large, a floating bridge assembled from plastic buoys is one of the feasible and suitable options. Pedestrian floating bridges assembled from these plastic buoys are being used today. However, there are no documents or actual projects using this type of bridge for vehicles with gross vehicle weight up to 12.3 tons. Therefore, in-depth research and practical tests are needed to evaluate the demand before applying it in practice. In this article, the authors focus on calculating the draft of continuous floating bridge when withstanding to the load of small trucks up to 12.3 tons. It is the scientific basis for evaluating the applicability of this type of bridge in practice.

## 2. Scientific basis for calculating the draft of floating bridges assembled from plastic buoys

### 2.1. Introduction to plastic buoys of floating bridge

In our country today, plastic buoys are increasingly widely used in many fields.

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Plastic buoys are floating blocks made from HDPE plastic. Floating plastic buoys are designed in modular form, and can be certainly connected to each other continuously in 3 dimensions. Dimensions of 1 float are 500 mm × 500 mm × 430 mm (length × width × height), and float weight is about 6.3 kg [4]. Physical properties of HDPE plastic materials include: Density (0.95 g/cm<sup>3</sup>), and elastic modulus ( $E = 995$  MPa) [5, 6]. Floating plastic buoys have many advantages such as: easy to transport, relocate, install, dismantle, and change structure shape as desired; short installation time; low installation cost (because of modular installation details); high buoyancy (because of lightweight), and highly durability. Floating plastic buoys can form a flat surface on the water so it is very convenient to assemble into a floating bridge.



*Fig. 1. Plastic buoy.*



*Fig. 2. The pedestrian floating bridge is assembled from floating plastic buoys.*

## **2.2. Scientific basis for calculating the draft of continuous floating bridges**

The floating bridges that are assembled from plastic buoys are continuous floating bridge. Calculating the draft of this type of floating bridge is a complicated problem. In this article, the authors use a beam model on an elastic foundation to calculate the draft of a continuous floating bridge, and presents two methods for determining the draft of this type of floating bridge. The first method is an analytical method, and the second method is a finite element method using SAP2000 software.

According to the analytical method [7-9]: Continuous floating bridges are considered beams on an elastic foundation that is water. The calculation diagram is shown in Fig. 2. When a concentrated load  $P$  (kN) is placed on a beam with stiffness  $EI_z$  that is placed on a uniform and isotropic elastic foundation. The beam will bend and sink into the foundation. The foundation reaction force  $q$  (kN/m) has intensity distributed along the length of the beam and is balanced with the load  $P$ . The intensity of the reaction force of the ground  $q$  (kN/m) is proportional to the deformation of the ground  $y$  (m) at a certain section, so

$$q = K \cdot y \tag{1}$$

The proportionality coefficient  $K$  (kN/m<sup>2</sup>) (the model under consideration is a 2-dimensional plane problem, so the width of the beam is taken as 1m,  $K$  has dimensions of kN/m<sup>2</sup> because  $q$  has dimensions of kN/m;  $y$  has dimensions of m) is called the foundation coefficient and it characterizes the elastic properties of the foundation, then:

$$EI_z \frac{d^4 y}{dx^4} = -q \tag{2}$$

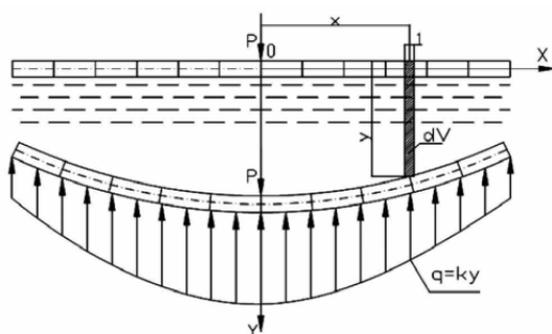


Fig. 3. Calculation diagram of continuous floating bridge.

From formulas (1) and (2), the following is obtained:

$$EI_z \frac{d^4 y}{dx^4} + K \cdot y = 0 \tag{3}$$

Set  $\beta = \sqrt[4]{\frac{K}{4EI_z}}$  (1/m), and solving Eq. (3) will obtain the equation representing the elastic curve of the beam, consider the beam as infinitely long and the concentrated force  $P = 1$ , then:

$$y = \frac{\beta}{2K} (e^{-\beta x} \cdot \cos \beta x + e^{-\beta x} \cdot \sin \beta x) \tag{4}$$

For the problem of the floating bridge on water with an elastic foundation in water, load per 1 m length  $q = \gamma \cdot B_0 \cdot y$ ;  $q(\frac{kN}{m})$ ; or foundation coefficient

$K = \gamma \cdot B_0$ ;  $K(\frac{kN}{m^2})$  then:

$$\beta = \sqrt[4]{\frac{K}{4EI_z}} = \sqrt[4]{\frac{\gamma \cdot B_0}{4EI_z}} \tag{5}$$

where  $\gamma$  is specific gravity of water,  $B_0$  (m) is width of the part in contact with the water of the floating.

On the basis of the above formulas, the displacement influence line, also known as the draft influence line, of the continuous floating bridge can be constructed as shown in Fig. 4. Loading on the displacement (draft) influence line will result in the displacement (draft) of the floating bridge when withstanding to load.

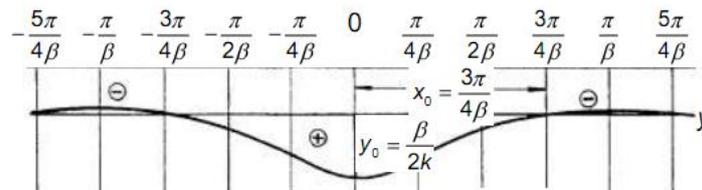


Fig. 4. Displacement influence line (water draft) of continuous floating bridge.

According to the finite element method using SAP2000 software [10, 11] is summarized as follows:

In the finite element method, the continuous object is replaced by a finite number of discrete elements. The basic steps of the finite element method include:

- Discretize the real structure into a pre-selected mesh of molecules suitable to the geometric shape of the structure and the accuracy requirements of the problem. These elements are connected by nodes.

- Build element properties.

- Build the element equilibrium equation  $[K] \cdot \{d\} = \{Q\}$ .

in which  $[K]$  is element stiffness matrix,  $\{d\}$  is element's nodal displacement vector,  $\{Q\}$  is nodal load vector. Representing the displacement at any point of the element through nodal displacements  $\{U\} = [N] \cdot \{d\}$  where  $[N]$  is coordinate function matrix (shape function).

- Calculate the element mass matrix.

- Calculate the resistance matrix.

- Build a general equilibrium equation from the element equations  $[K] \cdot \{r\} = \{P\}$ .
- Solve the system of balanced equations to determine the nodal displacement of the entire structure.
- From the found node displacement, determine each element.

While using finite elements, the floating bridge is placed directly on the water surface so it can be considered an ideal one-coefficient beam model on an elastic foundation. The water surface is replaced by a system of springs which is distributed at the bottom of the buoys. The spring stiffness is determined from the background coefficient. In SAP2000 software, elastic connections are described by spring bearings located at the nodes.

When calculating floating bridges, the stiffness of the spring depends on the area of the elements in its zone of influence. The stiffness coefficient of a spring can be determined by the product between the area of its influence zone and the background coefficient. When calculating the middle section of the river, using the finite element method with a finite bridge length, the spring stiffness coefficient will include two coefficients:  $K_1$  - for the bridge head and bridge edge  $K_1 = 0.5 \cdot \gamma \cdot a \cdot b \left(\frac{\text{kN}}{\text{m}}\right)$ , and  $K_2$  - for the inner sections  $K_2 = \gamma \cdot a \cdot b \left(\frac{\text{kN}}{\text{m}}\right)$  where  $\gamma$  is specific gravity of water,  $\left(\frac{\text{kN}}{\text{m}^3}\right)$ ,  $a$  is the distance between buoys vertically in the structure, (m),  $b$  is distance between buoys horizontally across the structure, (m).

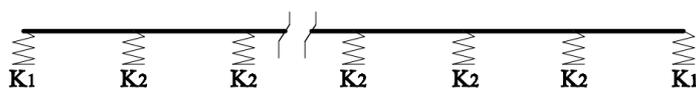


Fig. 5. Elastic bearing diagram when calculating the floating bridge.

### 2.3. Example of calculating the draft of a floating bridge assembled from plastic buoys

To better illustrate the method of determining the draft of an continuous floating bridge assembled from plastic buoys, the authors chose a floating bridge with the following dimensions: length  $L = 72(\text{m})$  consists of a layer with 144 buoys that ensure the length in the middle of the river, and width  $B = 3.5(\text{m})$  that ensure one-way traffic, the cross-section has a layer including 7 buoys as Fig. 6.

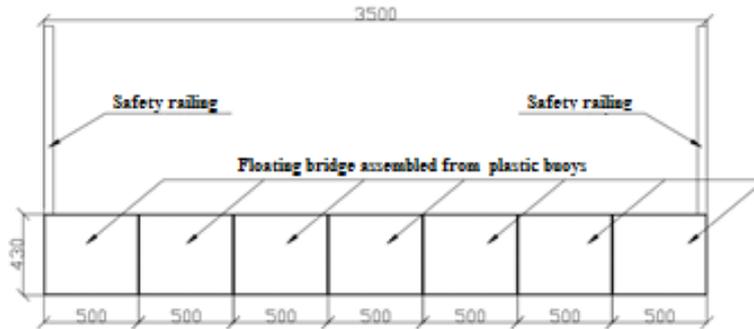


Fig. 6. Cross section of floating bridge used for survey.

Calculation assumptions included: the vehicle crosses the bridge at a slow speed, ignoring the dynamic impact of the vehicle on the bridge, and the impact of the flow on the floating bridge is not considered. The connection between the pontoons is not considered. In the article, only the draft of the mid-river section of the pontoon bridge is calculated, the pontoon bridge has a special connection with the near-shore section, so it is not considered in this article.

The load selected for the survey is the Kamaz 4326 truck. This is a fairly common truck in our country. The total weight of the vehicle and goods is 12.3 tons [12]. The vehicle has 4 wheel axles, of which 2 wheel axles in the front and 2 wheel axles in the rear. Load capacity of 2 rear axles  $P_1 = 7.1(T)$  or  $P_1 = 71(kN)$ ; Load capacity of the two front axles  $P_2 = 5.2(T)$  or  $P_2 = 52(kN)$ . The distance between the two vehicle axles is  $l_{xe} = 4.2(m)$ .

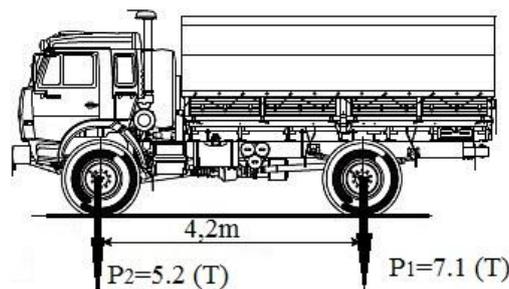


Fig. 7. Truck with a load capacity of 12.3 tons.

a) Using the analytical method

The moment of inertia is calculated as:  $I_x = 0.002013m^4$ , and the density of water is  $\gamma = 10(\frac{kN}{m^3})$  width of the surface in contact with the water of the floating  $B_0 = 3.5(m)$ ,

According to formula (5), the coefficient value can be calculated:

$$\beta = \sqrt[4]{\frac{K}{4EI_z}} = \sqrt[4]{\frac{10 \cdot 3.5}{4 \cdot 995 \cdot 10^3 \cdot 0.002013}} = 0.257 \left(\frac{1}{m}\right)$$

According to formula (4), calculate the ordinate of the water draft influence line at positions  $x_i = i \frac{\pi}{4 \cdot \beta}$ , including  $x_0$  at the middle position of the bridge. From there, build a influence line of the draft. Place the truck on this as shown below (Fig. 8), get the results: Draft value due to self-load of floating bridge:  $Y_{GT}^{TT} = q \cdot \omega = 0.885 \cdot 0.0268 = 0.025$  (m) in which  $q = 0.885 \left(\frac{kN}{m}\right)$  is the load of the floating bridge itself distributed over 1 meter of bridge length,  $\omega = 0.0286$  is the area of the draft influence line.

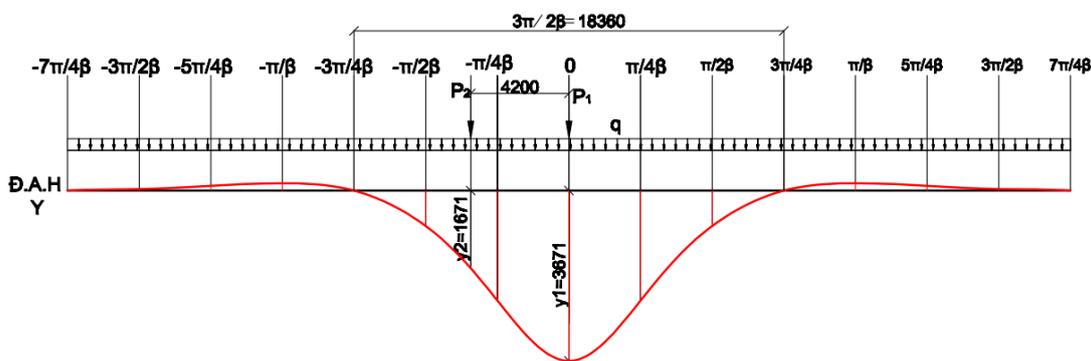


Fig. 8. Road loading affects the draft of the floating bridge.

Maximum draft value due to vehicle load:

$$Y_{GT}^{HT} = P_1 \cdot y_1 + P_2 \cdot y_2 = 71 \cdot 3.671 + 52 \cdot 1.671 = 347 \text{ (mm)} = 0.347 \text{ (m)}$$

in which  $P_1, P_2$  are the vehicle load transmitted to the front and rear axles of the vehicle,  $y_1$  and  $y_2$  are ordinate of draft's influence line. The total draft value caused by both self-load and vehicle load is:

$$Y_{GT} = Y_{GT}^{TT} + Y_{GT}^{HT} = 0.025 + 0.347 = 0.372 \text{ (m)}$$

b) Using the finite element method on SAP2000 software

Modeling plastic buoys from shell elements, each buoy consists of 6 panels that are rigidly linked together. When modeling structures on software (Fig. 9 and Fig. 10), the authors declare the elastic coefficient at the nodes outside the edge (boundary) of the floating bridge with the elastic coefficient:

$$K_1 = 0.5 \cdot \gamma \cdot a \cdot b = 0.5 \cdot 10 \cdot 0.5 \cdot 0.5 = 1.25 \left( \frac{\text{kN}}{\text{m}} \right)$$

and the inner (middle) nodes have coefficients:

$$K_2 = \gamma \cdot a \cdot b = 10 \cdot 0.5 \cdot 0.5 = 2.5 \left( \frac{\text{kN}}{\text{m}} \right)$$

The truck is modeled as a 4 load axles. The vehicle's load acting on the floating bridge is transmitted through the wheels at 4 axles. Thus the load passes through each wheel of the rear axle  $\frac{P_1}{2} = 35.5 \text{ (kN)}$ , and each wheel of the front axle  $\frac{P_2}{2} = 26 \text{ (kN)}$ . These loads are distributed over the tire track area pressing on the bridge deck with dimensions of  $0.5 \text{ m} \times 0.5 \text{ m}$ .

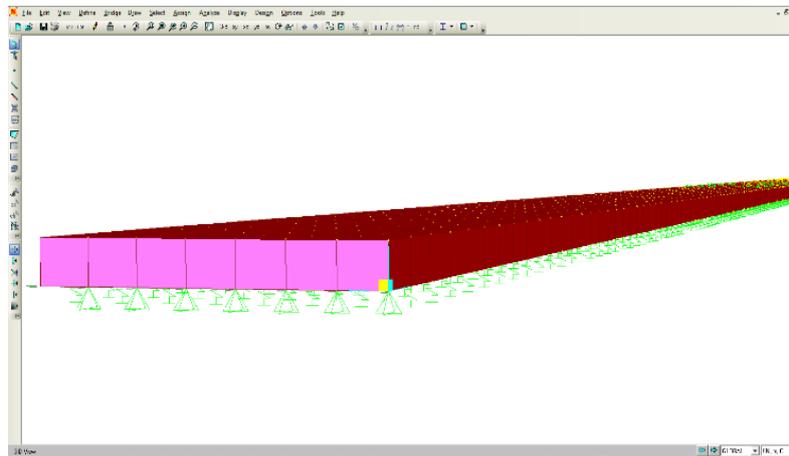


Fig. 9. Model of the entire floating bridge assembled from plastic buoys.

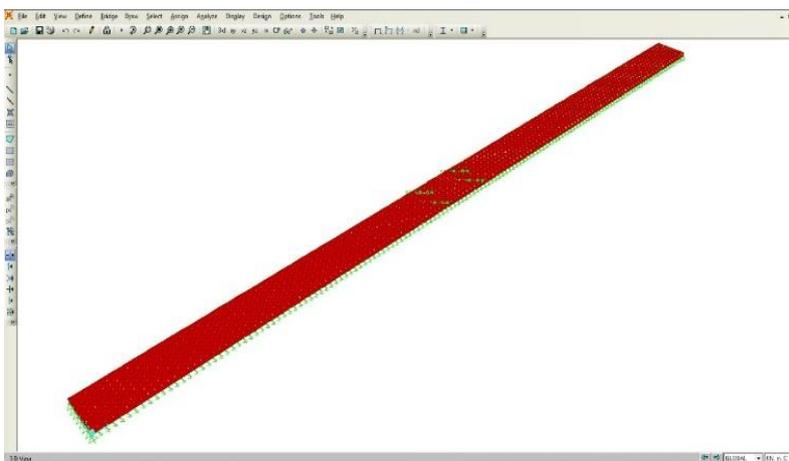


Fig. 10. Load declaration on the floating bridge model.

The results obtained are presented in the figures and tables below:

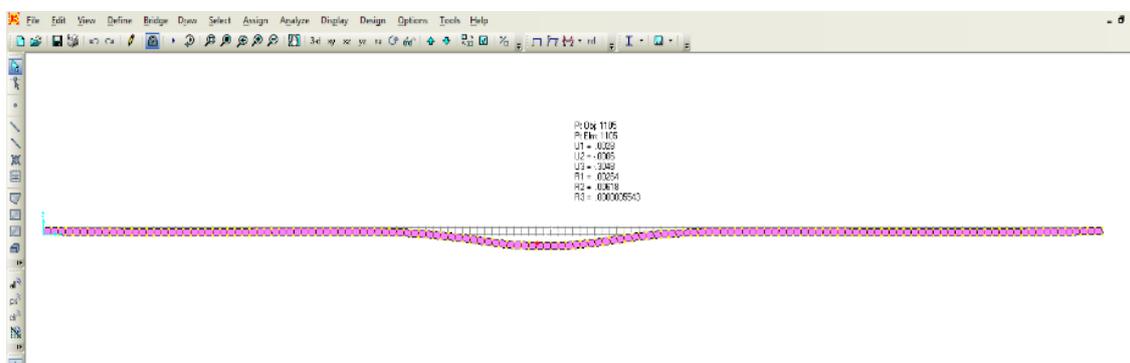


Fig. 11. Displacement of floating bridge under the effect of vehicle load.

Table 1. Maximum vertical displacement value (draft) received from SAP2000 software

TABLE: Joint Displacements			
Joint	Output Case	Case Type	U3
Text	Text	Text	m
1143	DEAD	LinStatic	-0.025846
1143	Kamaz 4326 Truck	LinStatic	-0.353515
1143	DEAD+Truck	Combination	-0.379362

The maximum vertical displacement (draft) due to dead load:

$$Y_{PTHH}^{TT} = U_{3max}^{DEAD} = 0.026 \text{ (m)}$$

The maximum vertical displacement (draft) due to live load:

$$y_{PTHH}^{HT} = U_{3max}^{Truck} = 0.354 \text{ (m)}$$

Thus, the maximum vertical displacement (draft):

$$Y_{PTHH}^{max} = Y_{PTHH}^{TT} + Y_{PTHH}^{HT} = 0.026 + 0.354 = 0.380 \text{ (m)}$$

Table 2. Results draft of floating bridge obtained from 2 different calculation methods

No.	Parameters	Analytic method	Finite element method	Difference (%)
1	Draft due to static load, (m)	0.025	0.026	
2	Draft due to live load, (m)	0.347	0.354	
3	Total draft, (m)	0.372	0.380	2.2

The difference in water draft results between the analytical method and the finite element method is 2.2%. This is an acceptable error.

It shows that the calculated water draft value is reliable and two methods of calculating the water draft of a floating bridge can be applied. The larger draft value is selected to compare with the height of the plastic buoy:

$$Y^{\max} = \max(Y_{GT}^{\max}; Y_{PTHH}^{\max}) = \max(0.372; 0.380) = 0.380(\text{m})$$

The height of the plastic buoy is:  $H_{phao} = 0.43(\text{m})$ .

Thus, the condition  $Y_{\max} \leq H_{phao}$  is satisfied.

$$\text{The ratio: } \frac{Y^{\max}}{H_{phao}} = \frac{0.380}{0.430} = 88(\%).$$

This demonstrates that buoyancy reserves remain. This reserve capacity can be considered a safety factor.

Calculation results show that the floating bridge is assembled from plastic buoys and a layer of buoys with buoyancy that can withstand a truck load of 12.3 tons. Thus, plastic buoys assembled into continuous floating bridge that can withstand truck loads have a scientific basis for practical application.

### 3. Conclusion

Research results show that calculating the water draft of a continuous floating bridge can use the following methods: analytical method or finite element method. The draft value obtained by the two methods have negligible errors. This shows that the results are reliable.

Floating bridge assembled from HDPE plastic buoys with dimensions of 500 mm  $\times$  500 mm  $\times$  430 mm (length  $\times$  width  $\times$  height), with 1 layer of buoys, 3.5 m width, 1 lane with enough buoyancy to carry a 12.3 tons truck load across the bridge, and vehicles crossing the bridge at low speed do not cause significant dynamic effects on the bridge.

Due to the limited scope of the article, the authors were only able to calculate the water draft to evaluate the buoyancy of floating bridges using theoretical methods. Calculation results show the high applicability of plastic buoys to assemble into continuous floating bridges that can withstand motor vehicle loads. It shows the scientific basis and practical application prospects of this type of floating bridge. To develop this research direction, the authors will continue to study other calculation contents of this floating bridge such as calculating internal forces in the elements, connections between floatings, experimental evaluation... The authors hope to be able to discuss this issue further with colleagues.

### Acknowledgement

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## TÍNH TOÁN MÓN NƯỚC CỦA CẦU PHAO ĐƯỢC LẮP GHÉP TỪ PHAO NHỰA CHỊU TẢI TRỌNG CỦA XE TẢI

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**Tóm tắt:** Trong một số tình huống giao thông khẩn cấp như cứu hộ cứu nạn, thiên tai..., cần bắc cầu tạm để vượt qua sông thì việc sử dụng cầu phao lắp ghép nhanh từ các phao nhựa là một giải pháp phù hợp. Bài báo trình bày việc nghiên cứu tính toán món nước của cầu phao dạng băng được lắp ghép từ các phao nhựa. Các tác giả tính toán món nước bằng hai phương pháp khác nhau là tính toán theo phương pháp lý thuyết và phương pháp mô hình phần tử hữu hạn bằng phần mềm SAP2000, để kiểm chứng sự sai lệch kết quả của hai phương pháp này. Kết quả nhận được cho thấy sự sai lệch giữa hai phương pháp này là không đáng kể và cầu phao dạng băng lắp ghép bằng một lớp phao nhựa có thể chịu được tải trọng xe tải loại nhỏ đến 12,3 tấn.

**Từ khóa:** Cầu phao; phao nhựa; xe tải; món nước; nền đàn hồi.

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