

STABILITY MECHANISM OF MASONRY SEMI-CIRCULAR ARCHES

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Abstract

Masonry arches are ancient structures, applied from tunnel works to complex constructions such as churches, temples, and castles, bearing the aesthetic and unique style of each civilization across various periods. A masonry arch consists of a series of blocks of a specific size stacked atop one another to create an upward curve that can maintain its shape and stability through the interaction of forces among the blocks within the structure. This research elucidates the principles and stability mechanisms of masonry arches in the case of support displacements. Focus on investigating the influence of structural thickness on the stability of masonry semi-circular arches. Simultaneously, it assesses the influence of the surrounding soil on the stability of the structural system. From there, determine the minimum structural thickness to ensure stability under different working conditions of masonry semi-circular arches.

Keywords: *Masonry arches; masonry semi-circular arches; thrust lines.*

1. Introduction

Robert Hooke described the relationship between a hanging chain and an arch: a hanging chain forms a catenary in tension under its own weight, while an arch stands in compression [1]. According to Hooke's hypothesis, if an "inverted hanging chain" shaped line of thrust can be found that lies entirely within the masonry, then the structures are safe, with a minimum dimension of $t/R = 0.11166$ [2], [3] where the variables t and R represent the radial thickness and radius measured to the center-line of a circular arch.

Based on Hooke's hypothesis, Couplet defined the compression force transmission path in masonry arches to determine the stable mechanism and minimum thickness under its own weight. According to Couplet, masonry arches become unstable, collapsing into four parts due to the formation of five hinges in the structure at A, T, K, R, and F (as shown in Fig. 1). From static equilibrium analysis of the structure, the relationship between the thickness t of the masonry arches and the radius R can be determined. Couplet determined the minimum thickness of masonry arches under their own weight as $t/R = 0.101$ [4]. However, the author did not determine the exact location of hinges T and K, which were assumed to be at an angle of 45 degrees. Analyzing the static equilibrium of block AK, the force acting at A is horizontal and represented by AG. The weight of

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the block passes through the center of gravity H and is represented by GH. Then, the force transmitted at K must have the same direction as GK, which is not tangential to the intrados of the masonry arches at K.

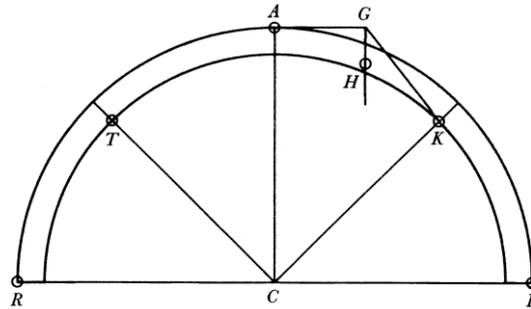


Fig. 1. Couplet's analytical model.

Later, Heyman used Couplet's collapse mechanism to analyze masonry semi-circular arches, ensuring that the thrust at intrados hinges was tangential to the intrados. The positions of hinges T and K were then determined at $\beta = 58.8$ degrees, and the minimum thickness value was $t/R = 0.106$ [5].

Ochsendorf, after demonstrating that this solution must be incorrect, provided results with a minimum thickness for semi-circular arches of $t/R = 0.10748$ and the position of the intrados hinge at $\beta = 54.484$ degrees (Fig. 2) [7].

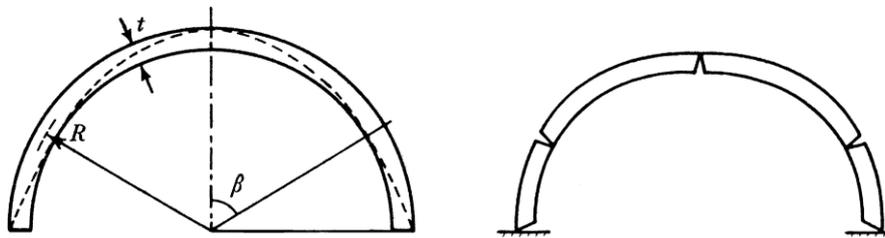


Fig. 2. Collapse of minimum thickness arch.

Mars, in his study, employed the limit analysis method to assess the static stability of masonry arches. Mars pointed out that in a structure consisting of blocks, the contact surface of two adjacent blocks is likely to be compressed, broken, or slide if it exceeds the allowable strength limit state. With a structure of 16 blocks, up to 6018 cases of instability can occur [8].

Thus, the research on masonry arches mentioned above is mainly based on analyzing a compressive force transmission line within the structure and designing the structure to ensure that its thickness completely covers those lines of thrust (Fig. 3). The analysis of the stability mechanism of masonry arches is still limited to specific cases and is not general.

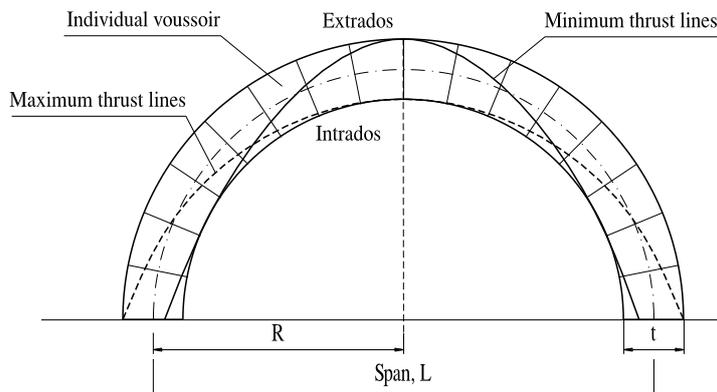


Fig. 3. Definition of geometry and schematic illustrations showing minimum and maximum thrust lines in a semi-circular arch.

2. Stability mechanism of masonry semi-circular arches under its own weight

To analyze the stability mechanism of masonry semi-circular arches, the author conducted research on experimental models. From practical observations, construction works using masonry arches are destroyed mainly due to the displacement of the arch foot support. This gives an idea: move the support of the structural model until it collapses, observing the process to derive a rule about the instability mechanism of the structure.

2.1. Instability mechanism of semi-circular arches undergoing support displacements

Two experimental models had 20 blocks (voussoir angle = 9 degrees), with structural thicknesses ratio (t/R) of 0.15 and 0.24, respectively. Experiments were conducted on the models in three cases: span expansion, span reduction, and abutment subsidence (results shown in Figs. 4-6).

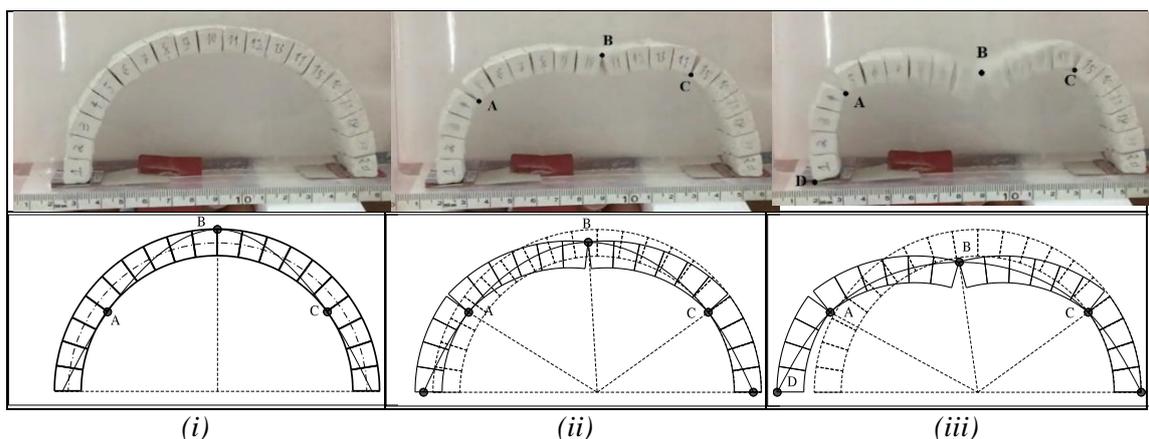


Fig. 4. Schematic illustrations of the procedure for the determination of maximum span increase for a semi-circular arch.

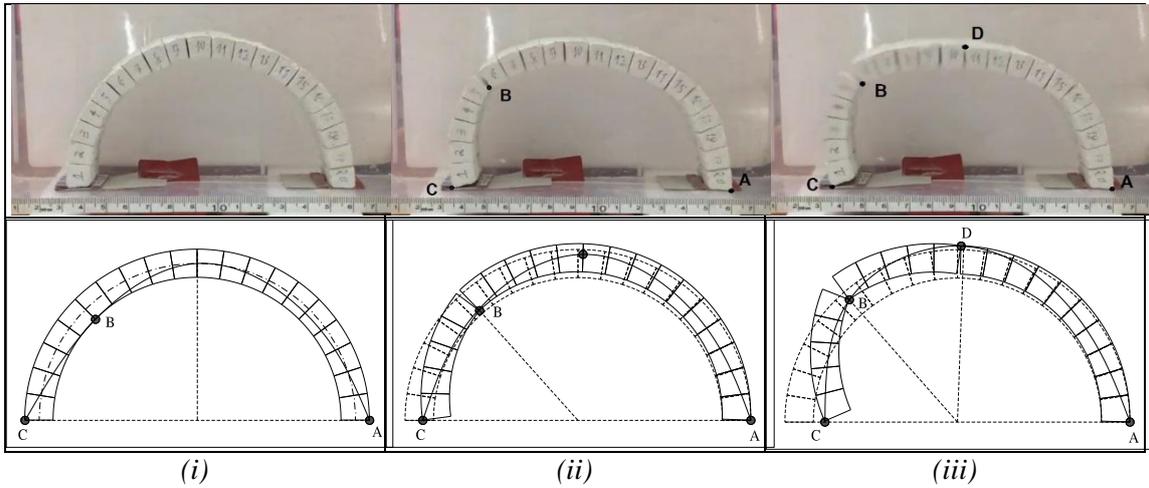


Fig. 5. Schematic illustrations of the procedure for the determination of maximum span decrease for a semi-circular arch.

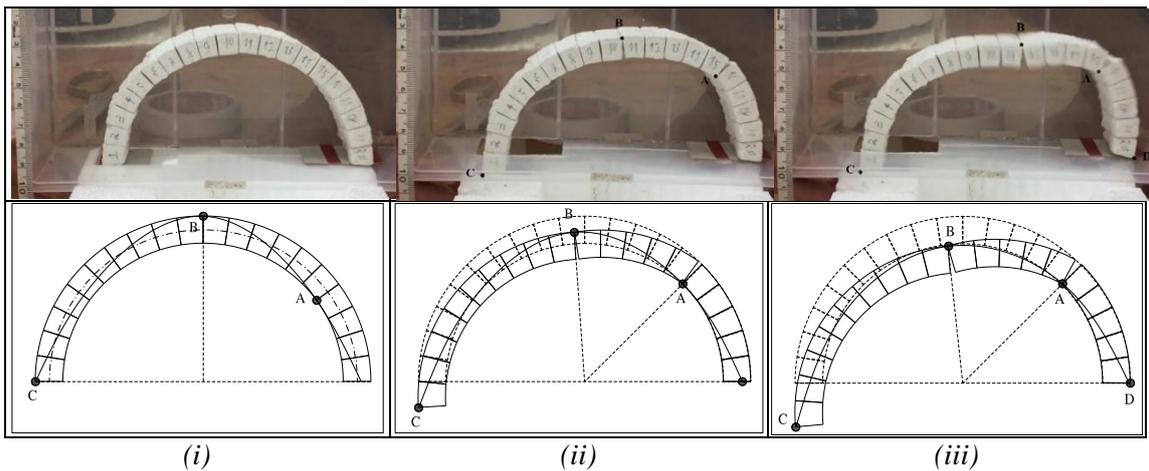


Fig. 6. Schematic illustrations of the procedure for the determination of maximum abutment subsidence for a semi-circular arch.

Thus, the instability mechanism of a semi-circular arch due to support displacement goes through three stages:

(i) Stage 1: When displacement begins, the force interaction of the blocks in the structure changes, the thrust lines in the structure change, and hinges A, B, and C begin to appear.

(ii) Stage 2: As displacement increases, the rotation angle at hinges A, B, and C increases, but the structural system still ensures overall stability.

(iii) Stage 3: Instability occurs when one of the following two conditions is satisfied:

- The thrust lines in the structure move to the extrados of the structure (at the arch foot in the case of span expansion and abutment subsidence; at the crown of the arch in the case of span reduction). Then hinge D (the 4th hinge) appears in the structure.

- In the case of span expansion and abutment subsidence, block AB rotates horizontally. In the case of span reduction, block CB rotates vertically.

After this, we have the statically determinate system to determine the stable equilibrium of the structure in cases of support displacement, as shown in Figs. 7-9.

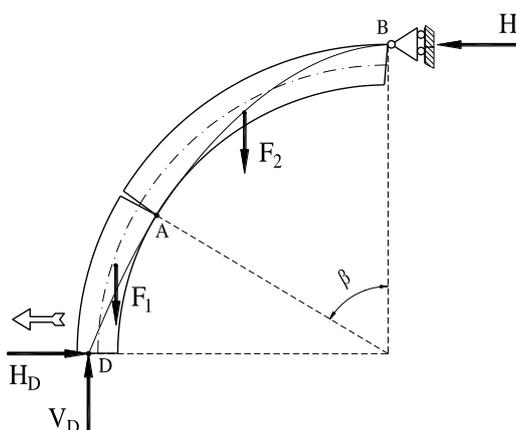


Fig. 7. Schematic illustration for determining the equilibrium conditions of the internal forces in a semi-circular arch in the case of supports moving apart (F_1 and F_2 are weights of the arch segments between two centers; H is the thrust).

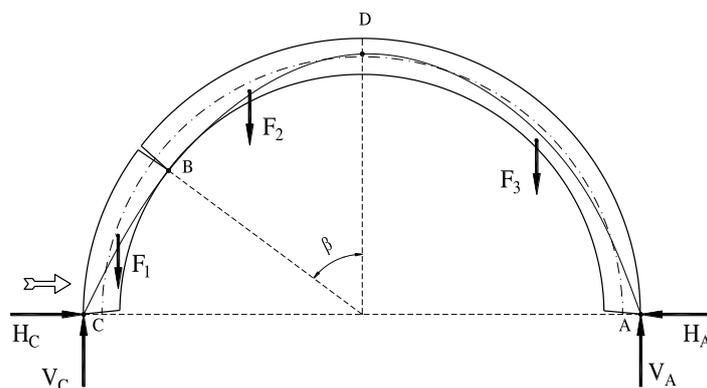


Fig. 8. Schematic illustration for determining the equilibrium conditions of the internal forces in a semi-circular arch in the case of supports moving inwards.

Another challenge lies in identifying the precise location of the hinge on the intrados of the structure (hinge A or B), which corresponds to determining the exact angle at which it forms. According to Heyman, the intrados hinge tends to appear at the position associated with the maximum horizontal thrust when the supports move apart.

Conversely, when supports move inwards, the hinge appears at the position with the smallest horizontal thrust. Figs. 10, 11 show the results of calculating the horizontal thrust value at each position between the blocks in a semi-circular arch of 20 blocks. The results indicate that when the span is increased, the hinge forms at an angle of 54 degrees. When the supports move inward, the hinge position shifts to an angle of 45 degrees. These findings are in complete agreement with the results observed in the experimental model.

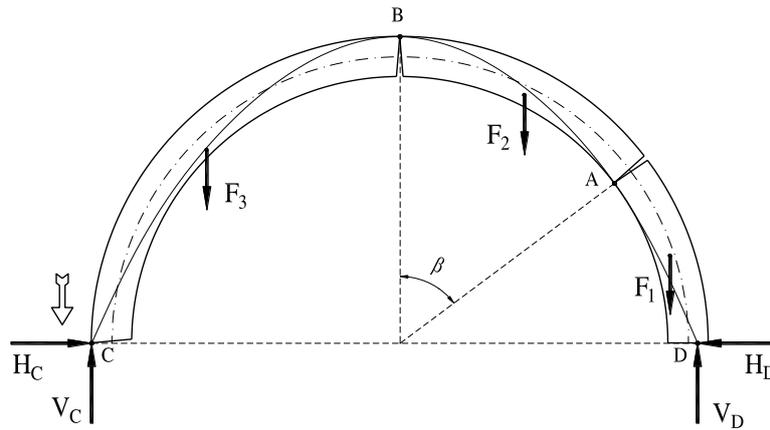


Fig. 9. Schematic illustration for determining the equilibrium conditions of the internal forces in a semi-circular arch in the case of differential vertical support movement.

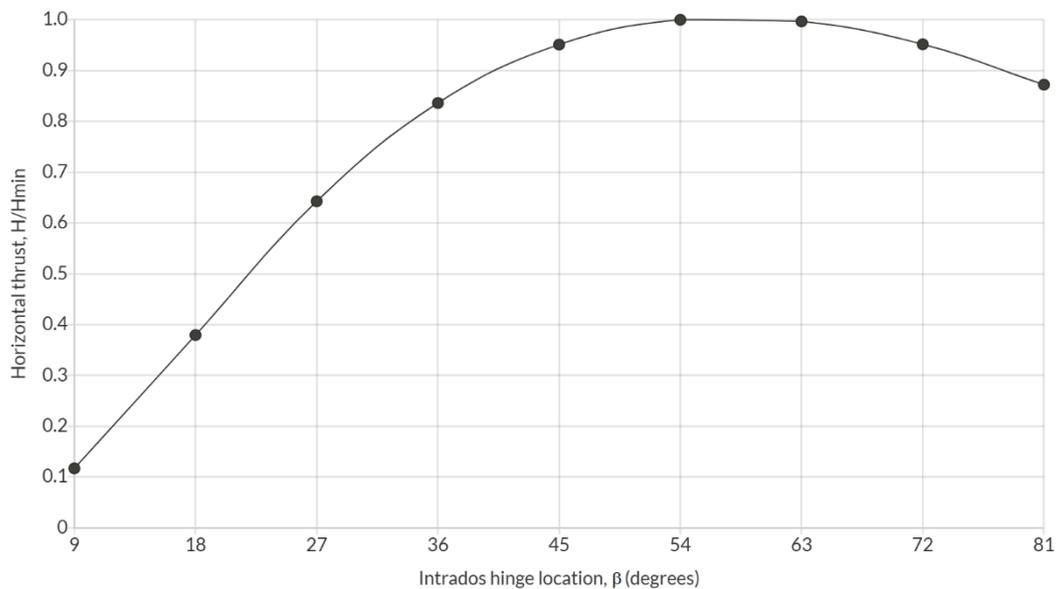


Fig. 10. Graphs of horizontal thrust of a semi-circular arch to find the location of the intrados hinge β in the case of spreading supports.

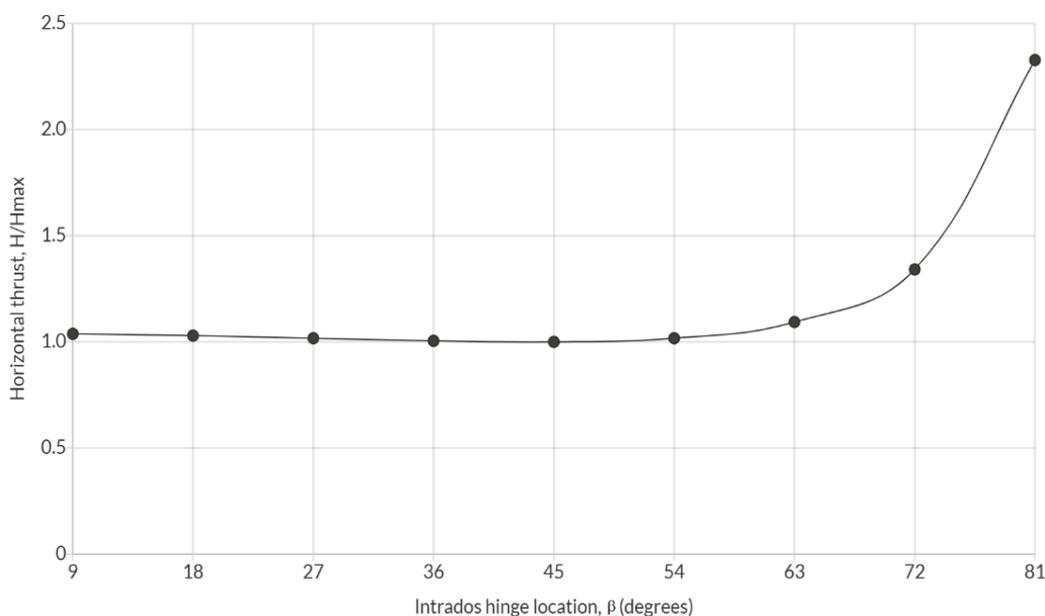


Fig. 11. Graphs of horizontal thrust of a semi-circular arch to find the location of the intrados hinge β in the case of inward-moving supports.

2.2. Stability mechanism of masonry semi-circular arches under its own weight

This section investigates the stability of semi-circular arches undergoing support displacements, based on the statically determinate system given above, and analyzes the structure when the arch foot is displaced until it collapses.

Determining the exact location of the intrados hinge helps to determine the exact geometric dimensions and loads acting on the components in the structural system. From there, an initial equilibrium structural system has been determined, and continuous calculation of the equilibrium state of a newly formed system due to the displacement of the support will also be performed until the structural system loses balance and collapses. The calculation volume is quite substantial because, at each stage of support displacement, both the shape of the structural system and the applied loads change.

To address this issue, the author has created a calculation program on MATLAB software, the program's algorithm is shown in Fig. 12. The calculation result for the largest displacement of the arch foot with changing structural thickness is shown in Fig. 13.

It can be seen that the results of the support displacement analysis in the three cases converge as the structural thickness decreases. From this, it can be determined that the minimum thickness for a semi-circular arch structure under its own weight is $t/R = 0.108$.

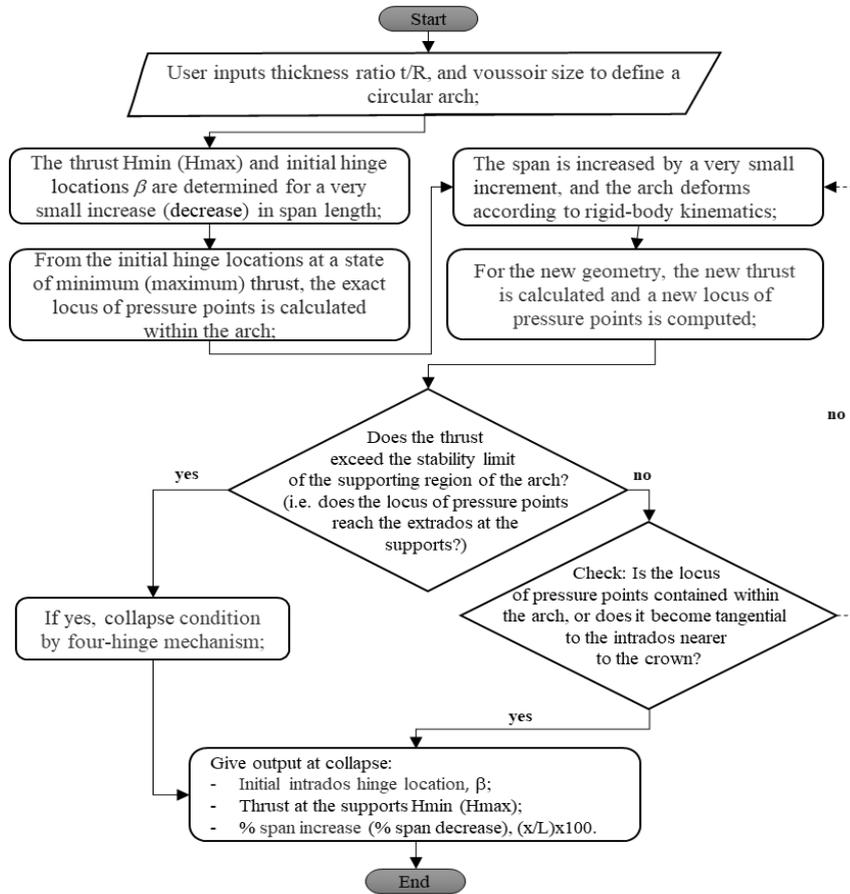


Fig. 12. Algorithm to determine the collapse state of an arch on displacement of the arch foot.

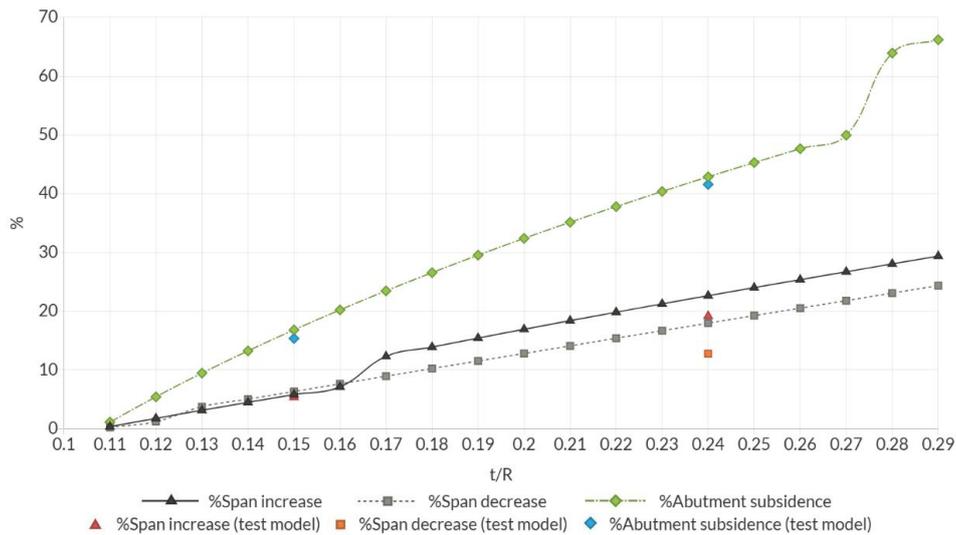


Fig. 13. Graphs of the support displacements required for arch collapse with varying thickness ratio t/R .

The voussoir sizes in a semi-circular arch will affect the position of the hinge on the intrados, thereby affecting the result of the support displacements required for arch collapse. To evaluate the influence of voussoir sizes on the stability of the structural system, the support displacements required for arch collapse were calculated when the voussoir sizes changed. For a selected thickness structure of $t/R = 0.15$, the calculation result is shown in Fig. 14.

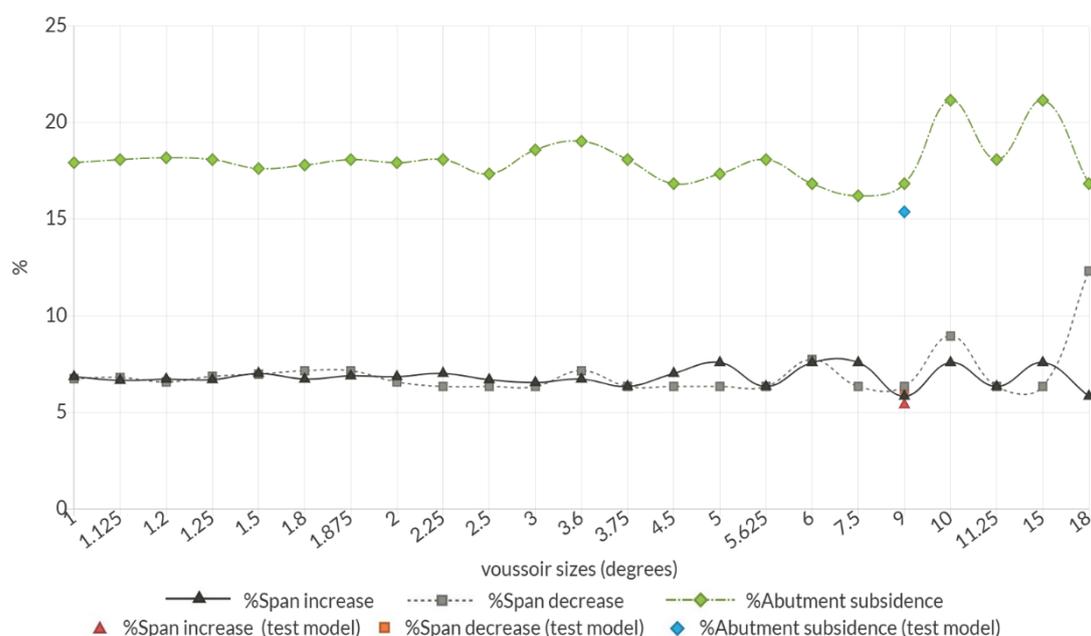


Fig. 14. Graphs of the support displacements required for a semi-circular arch collapse with varying voussoir sizes.

The above results show that the voussoir sizes in the structure do not significantly affect the maximum allowable displacement of the arch foot. When the voussoir sizes are less than 9 degrees, the results show little variation. This provides flexibility in choosing voussoir sizes when applying in practice.

2.3. Stability mechanism of masonry semi-circular arches in soil

In soil, the structural system is subject to soil pressure and limited in movement due to the elastic resistance of the soil. In Fig. 15, it can be seen that support displacements, in the case of span expansion and span reduction, are limited because the structure must move inwards into the soil. The case of abutment subsidence can occur due to the structural system moving towards the inside of a void, so we will only focus on studying this case.

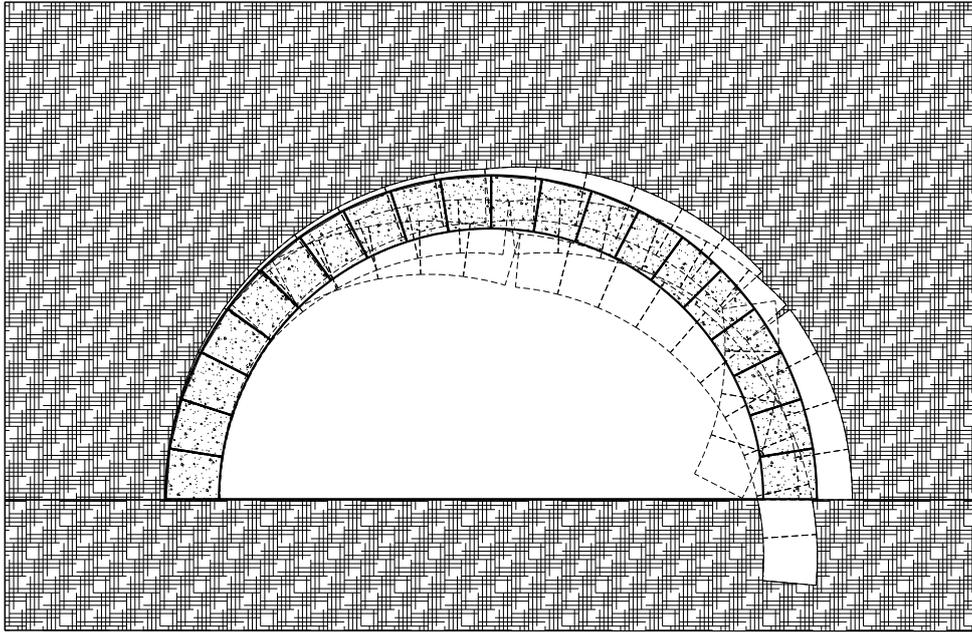


Fig. 15. The deformation of semi-circular archs in soil undergoing support displacements.

Observing the process, it can be seen that the collapse mechanism of the structural system also occurs in three stages, similar to when only bearing its own weight. The results from the experimental model are shown in Fig. 16.

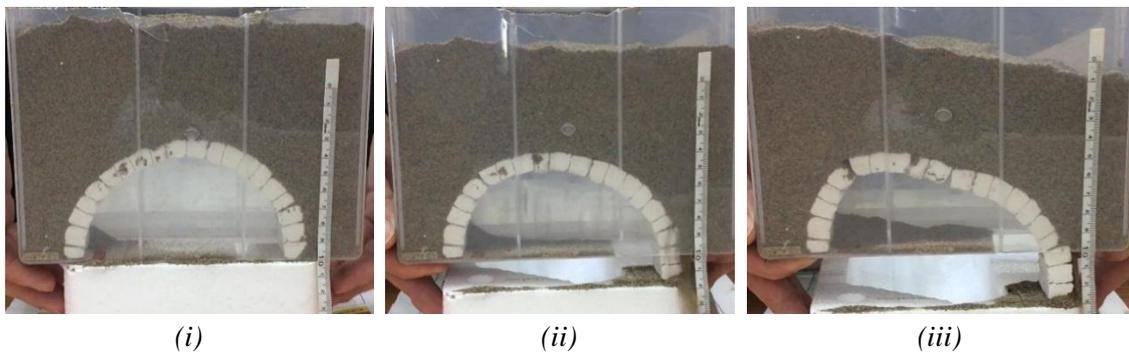


Fig. 16. Experimental results of the procedure for the determination of maximum abutment subsidence for a semi-circular arch in soil.

To simplify the calculation process, the soil pressure acting on the structural system in the vertical and horizontal directions is considered to be evenly distributed, as shown in Fig. 17a. Due to the similarity in the instability mechanism of semi-circular arches when bearing only their own load and when in soil, the statically determinate arch for differential vertical support movement is built as shown in Fig. 17b.

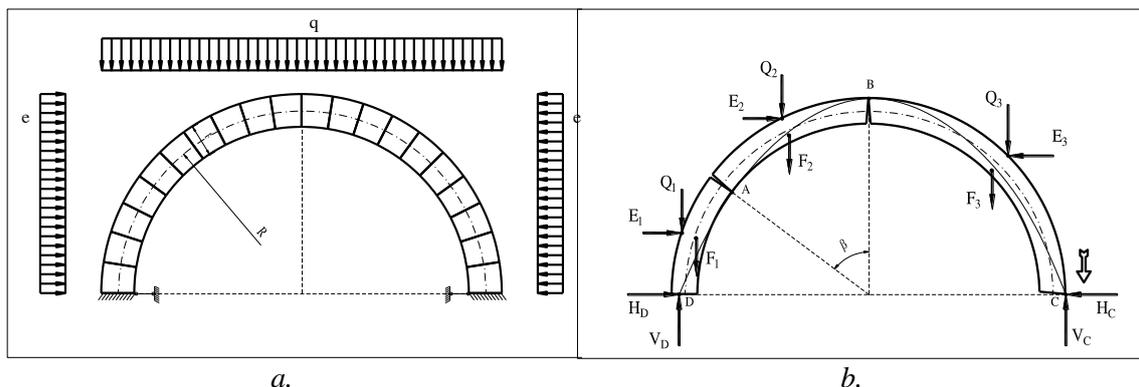


Fig. 17. The semi-circular arches in soil as differential vertical support movement
(a) Calculation diagram; (b) Statically determinate arch.

To evaluate the influence of soil on the stability of semi-circular arches, the maximum displacement of the arch foot in different soils was calculated. Using the soil and rock classification system according to M. M. Protodiakonov's stiffness coefficient f_{kp} , with weak soil having a stability coefficient in the range $f_{kp} = 0.3 \div 3.0$. The calculation results are shown in Fig. 18.

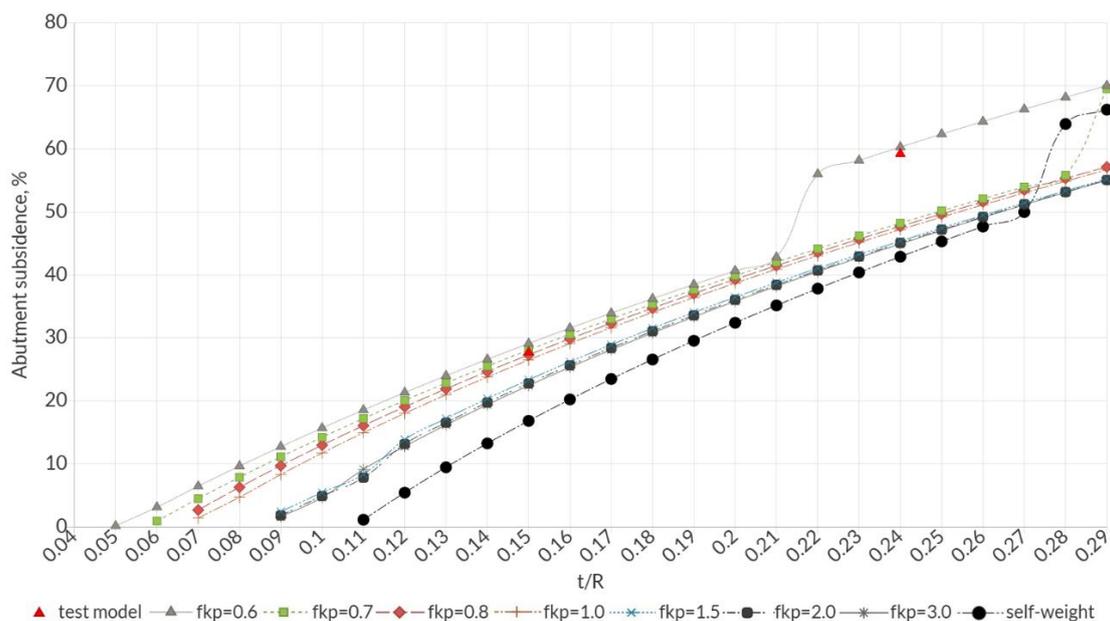


Fig. 18. Graphs of the support displacements required for arch collapse with varying thickness ratio t/R in different soils.

Note: The experimental model was carried out in moist sand, corresponding to a consolidation coefficient of $f_{kp} = 0.6$. The value of soil pressure was determined according to M. M. Protodiakonov's theory of landslide arch formation.

From the above results, it is quite interesting that soil is beneficial to the stability of semi-circular arches. In weaker soil, the stability of the structural system increases even more. Thus, it can be seen that horizontal pressure is an important factor affecting the stability of the structural system. The greater the horizontal pressure is, the more stable the structural system becomes.

From the diagram in Fig. 17, the minimum thickness of the semi-circular arch structure can also be determined to ensure stability in different geological conditions.

3. Conclusion

In this research, the author conducted tests on experimental models to determine the instability mechanism of semi-circular arches when support displacements occur. By determining the support displacements required for a semi-circular arch to collapse in different cases of voussoir sizes, it is possible to evaluate the influence of varying voussoir sizes on the stability of the structural system. The results show that voussoir sizes do not significantly affect the stable operation of the structure. Under the effect of self-weight, for the structural system to be stable, the thickness of the structure t/R has a minimum value of 0.108.

In soil, under the influence of soil pressure and elastic resistance, the semi-circular arches are more geometrically stable than when bearing only their own weight. An increase in the soil's horizontal pressure coefficient contributes positively to structural stability. However, the weaker the soil (the smaller the f_{kp} coefficient), the greater the value of soil pressure acting on the structure, and it is necessary to check the durability of the structural material.

Acknowledgement

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CƠ CHẾ ỔN ĐỊNH CỦA HỆ KHỐI XẾP DẠNG VÒM TRÒN

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Tóm tắt: Kết cấu khối xếp dạng vòm là dạng kết cấu cổ xưa, được áp dụng từ các công trình đường hầm đến các công trình phức tạp như các nhà thờ, đền, lâu đài... mang đậm tính thẩm mỹ và phong cách riêng của từng nền văn minh qua từng thời kỳ phát triển khác nhau. Vòm dạng khối xếp là tập hợp các khối có kích thước xác định được đặt chồng lên nhau tạo thành một đường cong hướng lên có khả năng tự duy trì hình dạng và sự ổn định thông qua tương tác lực giữa các khối trong cấu trúc. Bài báo làm rõ quy luật và cơ chế làm việc ổn định của hệ kết cấu khối xếp dạng vòm trong trường hợp gối tựa có sự dịch chuyển. Tập trung khảo sát ảnh hưởng của chiều dày kết cấu tới độ ổn định của hệ kết cấu khối xếp dạng vòm tròn. Đồng thời, đánh giá ảnh hưởng của môi trường đất đá xung quanh đến sự làm việc ổn định của hệ kết cấu. Từ đó, xác định được chiều dày kết cấu tối thiểu đảm bảo ổn định trong các điều kiện làm việc khác nhau của hệ kết cấu khối xếp dạng vòm tròn.

Từ khóa: *Kết cấu khối xếp; hệ khối xếp dạng vòm tròn; đường truyền lực nén.*

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