

STATISTICAL MODELING OF UNIT SKIN FRICTION BETWEEN PILES AND CORAL GRAVELLY SAND USING IN-SITU SPT DATA

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Abstract

This paper presents a statistical analysis of unit skin friction between piles and coral sand layers, based on 245 in-situ Standard Penetration Test (SPT) records collected from a coral reef area. The unit skin friction was estimated using Meyerhof's empirical correlation. Field data were used to assess the statistical characteristics of the pile-soil interface in coral sand. The results reveal that the distribution of unit skin friction does not follow a single normal (Gaussian) distribution, as is typical for terrestrial granular soils, but instead indicating the presence of multiple distinct geotechnical regimes. This distinctive feature reflects the unique interaction mechanism between piles and calcareous soil, which should be carefully considered in the foundation design of coastal and island structures.

Keywords: *Coral gravelly sand; standard penetration test; pile; unit skin friction; borehole; mean; standard deviation; weight.*

1. Introduction

Calcareous soil is a type of marine sand originating from biogenic sediments, widely distributed across islands and coral reefs. Its primary components are fragmented coral and broken shells of marine organisms, with calcium carbonate (CaCO₃) content accounting for more than 90% of the total mass [1], derived from the exoskeletons of these organisms. This type of sand possesses distinctive characteristics such as a porous structure, irregularly shaped particles, high crushability, and high compressibility. These properties result in mechanical behaviors that differ significantly from those of typical continental sand [2], [3]. In practical foundation design and construction, applying standards developed for continental sand to calcareous soil can lead to serious errors, and thus, a clear distinction must be made. Numerous studies have shown that under low-stress conditions, coral sand particles are highly prone to crushing, which alters the interparticle bonding state. This phenomenon greatly affects the bearing capacity of piles in coral sand foundations, making it distinctly different from that in ordinary sandy soils [4], [5].

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Particle breakage significantly influences the strength and stress–strain behavior of granular soils [6]. This, in turn, affects the bearing capacity and stability of structures built on these soils [7]. Hagerty *et al.* conducted triaxial compression tests on sand under high confining pressures, and the results indicated that relative density, particle shape, and particle size have a pronounced influence on the degree of particle breakage. Furthermore, denser sands require a higher initial stress to induce particle breakage, and vice versa [8]. Einav [9], [10] developed a new theoretical framework for particle breakage based on four parameters: shear modulus, bulk modulus, friction angle, and critical breakage energy, along with the initial grain-size distribution and void ratio, to predict the ultimate grain-size distribution of sand. Daouadji and Hicher [11] analyzed and classified particle breakage into three types:

(1) Fracture – the particle breaks into several fragments of similar size;

(2) Abrasion – the particle breaks into slightly smaller fragments, often with fine particles appearing along the edges;

(3) Scratching – the particle remains mostly intact, but fine particles appear along the edges due to friction [12]-[15].

The bearing capacity of piles is primarily influenced by the mechanical properties of the surrounding soil and the geometry of the pile. In coral reef environments, particularly in coral gravelly sand layers, characteristics such as distinctive particle structures, a pronounced tendency for particle breakage [16]-[20], and high variability in particle size and frictional properties lead to mechanical behavior that differs markedly from that of continental sand. These factors introduce significant uncertainties in the input parameters for geotechnical analysis and must be carefully considered in foundation design. To account for the effects of random variations in the geotechnical parameters of coral foundations, reliability-based design methods offer an appropriate approach. However, effectively applying this method requires a clear understanding of the probability distributions of the relevant parameters. In his study, N. T. Sang [21] assumed that certain parameters such as the internal friction angle and unit weight of coral gravelly sand follow a Gaussian distribution, and based on this, applied reliability theory to analyze the bearing capacity of helical steel pipe piles installed in coral foundations. The aim was to evaluate the impact of random variability on the reliability and load-bearing capacity of single piles.

However, due to the unique nature of coral gravelly sand, which differs significantly from conventional sand, specific experimental studies are necessary to accurately determine the probability distributions of input parameters characteristic of

this material. In this study, the authors evaluate the probability distribution of the unit skin friction between coral gravelly sand and piles, determined based on SPT (Standard Penetration Test) data and Meyerhof's empirical formula. Statistics were collected from boreholes at various locations within the milky white coral gravelly sand layer in an offshore island area of Vietnam. The analysis results serve as a basis for reliability assessments, tailored to the unique geological conditions of coral-based foundations.

2. Field data collection of SPT values for the coral gravelly sand layer in the offshore island area

The Standard Penetration Test (SPT) is a widely used in-situ geotechnical investigation method for assessing the mechanical properties of granular soils. The test is conducted in a borehole by driving a standard split spoon sampler into the soil using a 63.5 kg hammer dropped from a height of 76 cm. The SPT value (N-value) is defined as the total number of hammer blows required to drive the sampler through the final 30 cm of penetration, after disregarding the initial 15 cm (considered as the seating or "initial driving" stage). The N-value reflects the relative density of the soil layer and is commonly used in empirical formulas to estimate parameters such as unit skin friction between pile and soil, internal friction angle, soil deformation modulus, or to evaluate liquefaction potential.

The drilling and SPT testing method on coral terraces using a pontoon platform system combined with a chain hoist offers a flexible and efficient solution in island marine environments, where conventional land-based drilling equipment cannot access. The pontoon system consists of interconnected floating pontoons that form a stable working platform capable of supporting drilling equipment and conducting in-situ tests. A lightweight steel drill frame is installed on the platform and equipped with a manual chain hoist, which facilitates the lifting and lowering of drilling tools, drill bits, and, in particular, the SPT apparatus. The chain hoist, typically operated manually, enables precise control of the standard 63.5 kg SPT hammer drop from the specified height, ensuring test accuracy under overwater conditions. During operation, the pontoon platform is anchored in place using a mooring system to maintain stability and minimize the effects of waves and wind. This method is especially suitable for shallow water areas around coral islands, where access for large survey vessels is limited, offering a cost-effective and rapidly deployable solution. The testing procedure is illustrated in Fig. 1, and the sample retrieved from the SPT split spoon sampler is shown in Fig. 2.

The borehole data from the study area can be categorized into the following geological layers:

- Layer 1: Coral boulders mixed with coral branches, milky white in color.
- Layer 2: Milky white coral gravelly sand with some porous coral branches.
- Layer 3: Milky white coral gravelly sand with few coral boulders, ranging from moderately dense to dense.
- Layer 4: Coral boulders mixed with coral gravel, milky white in color.

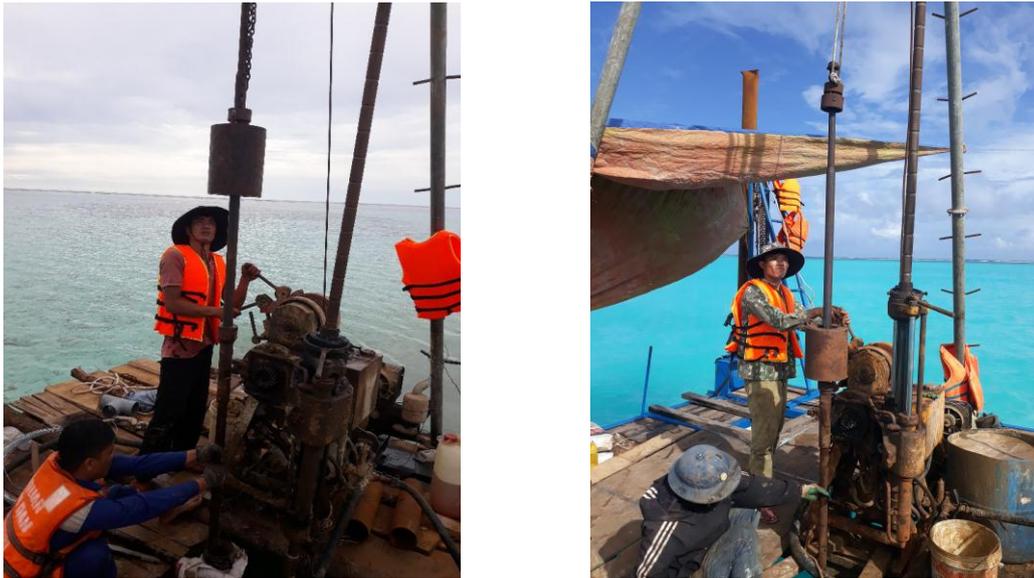


Fig. 1. Drilling and Standard Penetration Test (SPT) procedure.

The distribution of these geological layers in each borehole depends on its location on the island, with the following general characteristics: For boreholes located near the sea – areas frequently exposed to strong wave actions – the surface layer is typically Layer 1, which is a high load-bearing material. Below it is Layer 3, consisting of milky white coral gravelly sand with few coral boulders, ranging from moderately dense to dense (Fig. 2). In contrast, boreholes located within the lagoon – where wave energy is dissipated due to coral reefs – primarily have Layer 2 or Layer 3 as the surface layer, rather than Layer 1 as found near the sea.

In this study, the authors focus on analyzing Layer 3 – the milky white coral gravelly sand with few coral boulders, with a structure ranging from moderately dense to dense. From the collected borehole data, the research team obtained 245 SPT samples. The results show that the SPT values for Layer 3 range from 10 to 38, reflecting a high level of variability in density and a distinct heterogeneity of this soil layer.

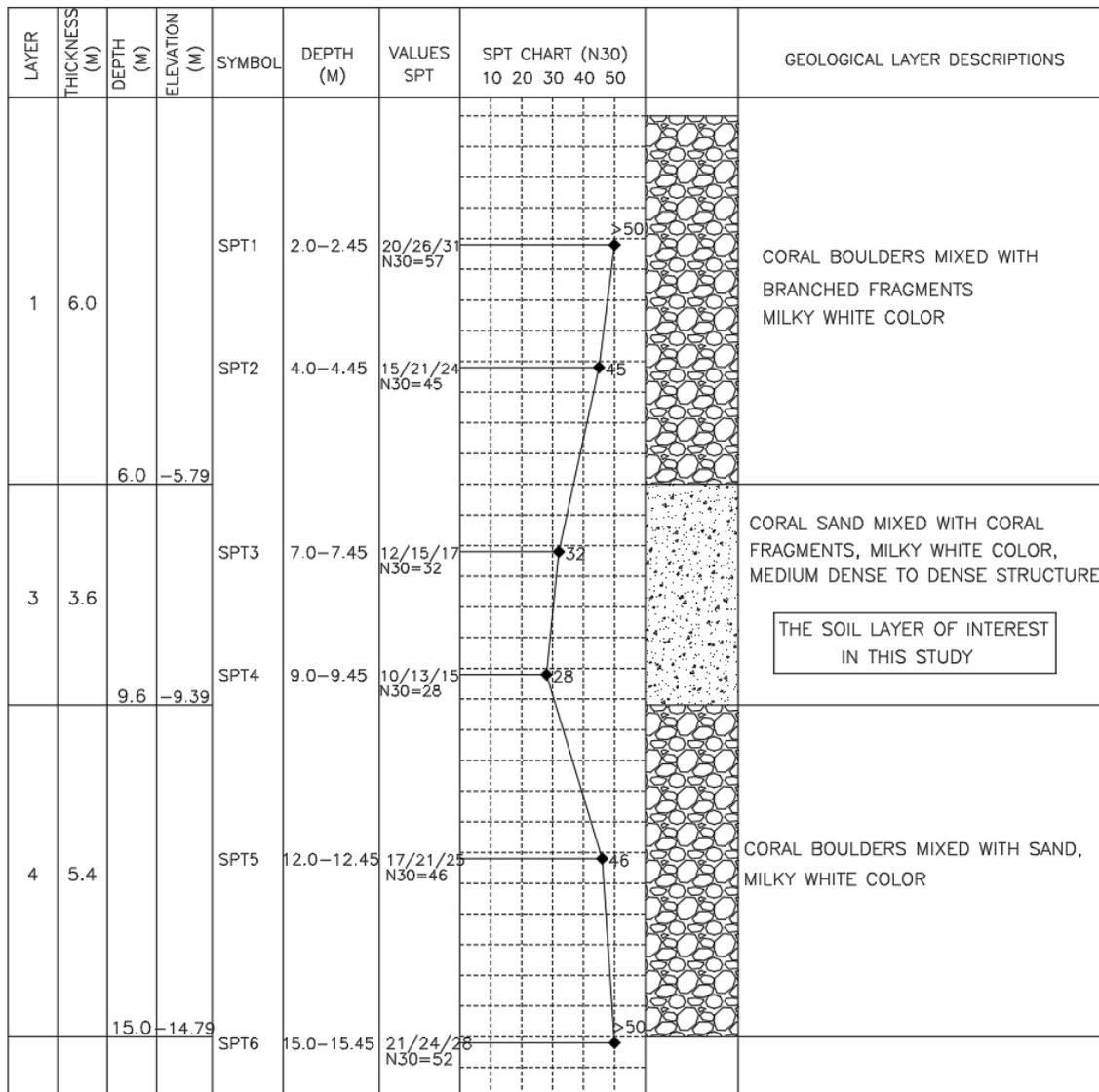


Fig. 2. A typical geological column in the research area on the offshore coral platform.

3. Statistical methods for determining the distribution of unit skin friction between piles and coral sand substrate

3.1. Determination of the distribution of unit skin friction between piles and coral gravelly sand

The unit friction between piles and coral sand substrate can be indirectly determined through the Standard Penetration Test (SPT - N) index. According to the TCVN 10304:2014 standard [22] applicable to loose soils, two commonly used formulas to determine the unit friction of the pile shaft are as follows:

- Meyerhof's formula:

$$f = 2 \times N \text{ (kPa)} \quad (1)$$

- Formula of the Japan Architectural Institute:

$$f = 3.3333 \times N \text{ (kPa)} \quad (2)$$

Angelmeer *et al.* [23] conducted static compression tests on driven steel pipe piles in coral sand. The results showed that although calcareous soil has a higher internal friction angle compared to conventional sand, the skin friction and end bearing resistance tend to be lower.

When comparing the two formulas mentioned above, it is evident that Meyerhof's formula yields lower values of unit friction, thus offering a more conservative approach for design purposes. Based on the experimental results of Angelmeer *et al.* and the geological characteristics of the surveyed area (milky white coral gravel sand with minor coral fragments, ranging from medium dense to dense), the authors propose using Meyerhof's formula to calculate the unit friction between piles and the coral foundation.

With a dataset of 245 collected SPT samples, this method enables the construction of a reliable dataset to support statistical analysis of the unit friction characteristics.

The statistical processing of data collection ($n = 245$) for the unit friction values between piles and coral gravel sand (Layer 3) is carried out according to the following steps:

- Step 1: Determine the minimum and maximum values of data collection

$$f^{min} = 20 \text{ kPa}; f^{max} = 76 \text{ kPa}.$$

- Step 2: Divide the range into 20 intervals, each with a width of $d = 2.8$ kPa. Determine the starting and ending value of each interval.

- Step 3: Count the number of unit friction values falling within each interval.

- Step 4: Calculate the frequency density for each interval using the following formula:

$$p_i = \frac{n_i}{n \times d} \quad (3)$$

The probability distribution histogram of unit friction is presented in Fig. 3. Observation of the bar chart indicates a left-skewed distribution with three distinct peaks. This type of distribution does not conform to common distribution models such as the Gaussian, log-normal, or Weibull distributions. Instead, the shape of the histogram suggests that the unit friction characteristics may be described by a mixture distribution consisting of three Gaussian components. The probability density function takes the following form:

$$f(x) = w_1N(x | \mu_1, \sigma_1^2) + w_2N(x | \mu_2, \sigma_2^2) + w_3N(x | \mu_3, \sigma_3^2) \quad (4)$$

where w_j is the weight of the j -th Gaussian component, with the condition that $\sum_{j=1}^3 w_j = 1$, x is the mean value of each interval, for $i = 1$ to 20, μ_j and σ_j are the mean and standard deviation of the three Gaussian components, respectively.

To determine these 9 parameters, the authors applied the nonlinear least squares method with the following objective function:

$$g(w_1, \mu_1, \sigma_1, w_2, \mu_2, \sigma_2, w_3, \mu_3, \sigma_3) = \sum_{i=1}^{20} S_i^2 \Rightarrow \min \quad (5)$$

where S_i is the total squared deviation between the empirical frequency (p_i) and the theoretical probability density value (f_i) at interval i .

$$f_i = f(x_i) = w_1N(x_i | \mu_1, \sigma_1^2) + w_2N(x_i | \mu_2, \sigma_2^2) + w_3N(x_i | \mu_3, \sigma_3^2) \quad (6)$$

x_i is the mean value of each interval ($i = 1$ to 20)

$$S_i = |P_i - f_i| \quad (7)$$

To estimate the parameters of the three-component Gaussian mixture model (GMM), the authors used Microsoft Excel's Solver tool to solve the optimization problem based on the least squares method. The results, including the 9 estimated parameters, are presented in Tab. 1.

Tab. 1. Components of the Gaussian mixture distribution

w_1	0.643	w_2	0.221	w_3	0.136
μ_1 - kPa	26.278	μ_2 - kPa	35.357	μ_3 - kPa	48.020
σ_1 - kPa	3.094	σ_2 - kPa	1.625	σ_3 - kPa	5.000
Coefficient of determination, $R^2 = 0.94$					

3.2. Discussion

The analysis results indicate that the three-component GMM provides an excellent fit for describing the characteristics of unit friction, with a coefficient of determination of $R^2 = 0.94$. The probability distribution chart (Fig. 3) clearly shows three local peaks, corresponding to the three components of the mixture, reflecting the existence of three

distinct groups of unit friction values within the dataset. Each component of the mixture distribution carries a clear physical meaning:

- The first component, with the highest weight $w_1 = 0.643$, a mean value $\mu_1 = 26.278$ kPa, and a standard deviation $\sigma_1 = 3.094$, represents the dominant group accounting for approximately 64% of the samples. It characterizes the typical subsoil conditions in the survey area, where piles frequently penetrate coral sand layers with stable and relatively uniform mechanical properties.

- The second component, with a moderate weight $w_2 = 0.221$, a higher mean $\mu_2 = 35.357$ kPa, and a smaller standard deviation $\sigma_2 = 1.625$, indicates low variability. This group reflects areas with more specialized geological conditions, possibly denser sand zones or regions containing fragmented coral materials, resulting in higher but stable unit friction.

- The third component, with the smallest weight $w_3 = 0.136$, has a very high mean $\mu_3 = 48.020$ kPa and a large standard deviation $\sigma_3 = 5.00$, indicating strong dispersion. This component likely represents abnormally high unit friction values, which may occur where piles encounter weathered coral rock layers or interbedded hard materials, causing significant localized friction.

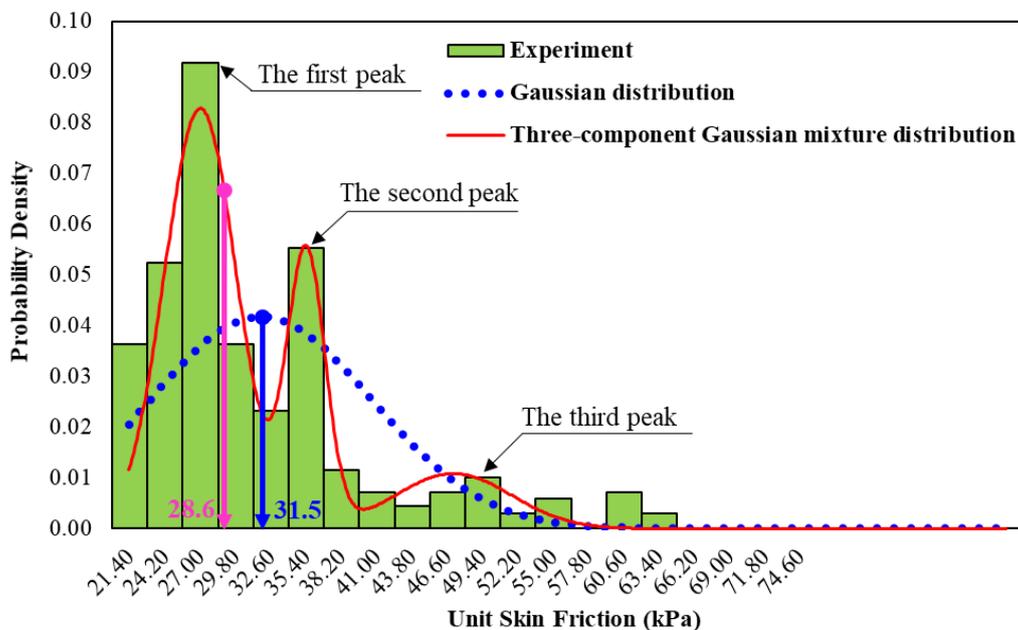


Fig. 3. Probability distribution of unit skin friction

Modeling with a three-component mixture distribution allows for a clearer identification of different geological condition groups, which cannot be distinguished using a single normal distribution. This approach provides deeper insight into the variation of unit friction in specific geological settings like calcareous soil, and offers valuable support for the design and analysis of pile bearing capacity.

In deterministic design, when the input variable is assumed to follow a normal distribution, the mean value is commonly used for calculation due to its favorable statistical properties. Specifically, the normal distribution is symmetric, so the mean coincides with the median - the value that divides the distribution into two equal halves (50% probability on each side). Moreover, the probability density is highest around the mean, making it a clear and representative indicator of the central tendency of statistics. Therefore, using the mean in design provides a typical and reasonable reflection of ground conditions.

Applied to the problem of pile design in coral sand foundations, if it is assumed that the unit skin friction between the pile and the coral sand follows a normal distribution, the representative value used would be 31.5 kPa which is both the mean and the median of the distribution. However, statistical analysis shows that this variable does not follow a normal distribution but instead fits better with GMM. In this case, using the overall mean may lead to misleading results, as it is influenced by extreme data clusters (e.g., a group with high friction values but a small proportion of occurrence).

Therefore, using the median of the mixture distribution the value corresponding to a cumulative probability of 50% is a more reasonable choice in deterministic design. Based on the parameters given in Tab. 1, the median of the mixture distribution is 28.6 kPa, which is significantly lower than the mean value of 31.5 kPa. Choosing 28.6 kPa offers several clear advantages:

- It helps reduce the risk of overestimation, especially when rare high values distort the mean.
- It better reflects the most common ground condition, as 50% of the actual data is below or equal to this value.
- It represents a more conservative and safer choice in deterministic design, especially when full probabilistic analysis is not performed.
- It aligns with the true nature of the data distribution, thereby improving the reliability and effectiveness of the design.

In conclusion, when the input variable follows a multi-component mixture distribution, using the median instead of the mean in design calculations is statistically sound, better reflects observed field conditions, and contributes to the overall safety and reliability of the structure. The authors recommend that if the calculation is based on the average value, a reduction factor of $k = 28.6/31.5 = 0.91$ should be applied.

4. Conclusion

The unit skin friction between piles and coral sand substrate can be effectively estimated through the Standard Penetration Test (SPT) index. Among available formulas, Meyerhof's equation was selected due to its compatibility with the geological conditions of the study area and its inherent design conservatism. Based on 245 collected SPT samples, the authors developed a comprehensive dataset of unit skin friction values and conducted a detailed statistical analysis.

The analysis revealed that the distribution of unit friction does not follow simple models such as Gaussian or Weibull, but is better represented by a three-component Gaussian mixture distribution. This model demonstrated a high goodness of fit ($R^2 = 0.94$) and allowed for the identification of three distinct geological condition groups, corresponding to the three peaks observed in the probability distribution. These components reflect varying subsurface characteristics, from homogeneous calcareous soil, to denser sand with coral fragments, and even cases involving interbedded hard materials.

Importantly, rather than using the mean value of 31.5 kPa, as would be the case under the assumption of a normal distribution, the study recommends adopting the median value of the mixture distribution, identified as 28.6 kPa, as the representative parameter in deterministic design. The median, representing the 50% cumulative probability point, better reflects the most frequently encountered soil conditions, minimizes the influence of rare extreme values, and offers a more conservative and safer basis for design, particularly in cases where full probabilistic analysis is not performed. The authors recommend that if the calculation is based on the average value, a reduction factor of $k = 0.91$ should be applied.

The conclusions of this study are primarily drawn from the analysis of data representing a typical coral gravelly sand layer in an offshore island area. To obtain a more comprehensive and reliable assessment of this type of coral foundation, it is necessary to continue collecting and analyzing additional datasets from different coral sand layers across various geological settings. This will support the development of appropriate correction factors for the design of foundations on coral soils in offshore island environments.

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MÔ HÌNH THỐNG KÊ MA SÁT ĐƠN VỊ GIỮA CỌC VÀ LỚP CÁT SAN SAN HỒ DỰA TRÊN SỐ LIỆU SPT TẠI HIỆN TRƯỜNG

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Tóm tắt: Bài báo trình bày phân tích thống kê về ma sát đơn vị giữa cọc và lớp cát san hồ, dựa trên 245 kết quả thí nghiệm xuyên tiêu chuẩn (SPT) được thu thập tại một khu vực rạn san hô. Ma sát đơn vị được ước tính thông qua công thức thực nghiệm của Meyerhof. Dữ liệu thực địa được sử dụng để đánh giá các đặc trưng thống kê của ma sát đơn vị giữa cọc và lớp cát san hồ. Kết quả cho thấy phân bố của ma sát đơn vị không tuân theo phân bố chuẩn (Gauss) như thường thấy ở đất hạt rời trên đất liền, mà thay vào đó biểu hiện sự tồn tại của nhiều chế độ địa kỹ thuật khác nhau. Đặc điểm này phản ánh cơ chế tương tác đặc thù giữa cọc và cát san hồ, cần được xem xét cẩn trọng trong thiết kế nền móng cho các công trình ven biển và trên đảo.

Từ khóa: Cát san hồ; xuyên tiêu chuẩn; cọc; ma sát đơn vị; lỗ khoan; kỳ vọng; độ lệch chuẩn; trọng lượng.

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