IDENTIFICATION OF DAMAGE LOCATIONS IN STEEL BEAMS BASED ON CHANGES IN NATURAL FREQUENCIES

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Abstract

When damage occurs in a structure, it results in changes to its dynamic characteristics including natural frequencies, mode shapes, and damping ratios, compared to an undamaged structure. Based on this principle, several methods have been researched and applied to identify damaged regions in structures. In this article, a cantilever steel beam with constant cross-section is studied by dividing it into regions, where each region is classified based on the first four normalized natural frequencies. The damaged region is identified by classifying the normalized frequencies of the structure through the ratio of the natural frequency of the damaged beam to that of the undamaged beam. A simulation is conducted using SAP2000 for both the intact and damaged cantilever beam to determine its natural frequencies, serving as a basis for identifying the damage location and severity. The identified damage region matches the assumed location and is independent of damage severity. The research results demonstrate that changes in natural frequencies can be utilized to identify damage on cantilever beam structures, providing a basis for the application in detecting damage locations in more complex structures and real-world constructions.

Keywords: *Natural frequency; mode shape; structural damage.*

1. Introduction

Many mechanical structures, such as wind turbine blades, aircraft wings, and other civil engineering structures, can be considered as cantilever beams. Any changes in material properties or geometry that negatively impact their performance are considered damage. Early detection of damage helps prevent structural failures and extends the service life of structures.

The presence of damage in a structure alters its dynamic characteristics, including natural frequencies, mode shapes, and damping ratios. Therefore, many engineers and researchers have developed various damage detection techniques based on these dynamic properties.

In recent years, damage can be directly diagnosed using various methods such as changes in the frequency response function (FRF), variations in natural frequencies, alterations in mode shapes, curvature-based methods, and strain energy-based approaches.

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Since the dynamic parameters of a structure reflect its actual working conditions, internal damage within the structure inevitably leads to changes in frequency response and dynamic characteristics. By comparing the experimentally determined dynamic parameters with the calculated parameters of the intact structure, the damage state and severity can be identified based on variations in these dynamic properties. Depending on the type of structure and damage characteristics, the changes in dynamic properties follow different patterns. Therefore, accurate damage detection methods require evaluating these variations under different conditions.

Several authors have identified damaged structural elements by comparing mode shapes of damaged and undamaged structures. West [1] was one of the first to introduce the idea of using mode shapes to locate structural damage. He employed the mode assurance criterion (MAC) to determine the correlation between the mode shapes of a damaged and an undamaged structure. Lieven and Erwins [2] proposed a damage indicator based on mode shapes, known as the coordinate modal assurance criterion (COMAC).

Many researchers have used mode shape curvature (the second derivative of the mode shape) to detect damage, as this method is more sensitive to local changes in mode shape by amplifying the effects of damage. Pandey *et al.* [3] were the first to suggest using changes in mode shape curvature to detect and locate damage in beams.

However, mode shapes require measurements at multiple structural points and take considerable time to estimate. In contrast, natural frequencies are easier and more cost-effective to determine. For this reason, many researchers have focused on detecting damage using changes in natural frequencies. According to Doebling *et al.* [4], Lifshitz and Rotem [5] were the first to use vibration measurements for damage detection. They analyzed shifts in natural frequencies due to changes in dynamic modulus. Messina *et al.* [6] introduced the Damage Location Assurance Criterion (DLAC), which is based on frequency shifts. Subsequent studies [7], [8] extended this approach to multiple damage cases, introducing the Multiple Damage Location Assurance Criterion (MDLAC), which uses statistical correlation between frequency shifts from analytical predictions and experimental measurements.

Gillich and Praisach [9] demonstrated that the curves of natural frequency shifts have the same shape for different damage levels at a specific location. Therefore, to identify the damage location, they compared measured frequency shift curves with those obtained through analytical methods or FEM simulations. The position where the calculated curve best matches the measured curve indicates the damage location. Later studies by Gillich and Praisach [10] and Gillich *et al.* [11] mathematically showed that normalized frequency shifts correspond to squared normalized mode shape curvatures at the damage location, multiplied by a coefficient dependent on the damage severity. By normalizing the frequency shift values, they eliminated the influence of damage severity, obtaining a set of values independent of damage extent.

The natural frequencies of a damaged structure depend on both the damage location and crack depth. Many researchers have used contour plot methods based on frequency shifts. Nahvi and Jabbari [12] plotted contour lines of normalized frequency shifts based on crack location and depth using finite element methods. By discretizing the beam into elements, with cracks located at different depths in each element, they determined damage location and depth at the intersection of contour lines corresponding to constant natural frequencies. Barada *et al.* [13] used analytical methods with crack modeling based on a spring approach to generate contour plots.

Currently, domestic research on structural damage localization remains limited. This study builds upon international research, combining theoretical analysis and numerical simulations to identify damaged regions in cantilever steel beams, with the aim of applying these techniques to more complex structures and real-world engineering applications.

A cantilever steel beam with a constant cross-section divided into distinct zones. Each zone is uniquely characterized by a specific classification of the first four normalized natural frequencies. The research methodology involves simulating and calculating the natural frequencies for both the undamaged and damaged beam states. The damaged zone is identified solely by the classification of these normalized natural frequencies, which are defined as the ratio of the natural frequency of the damaged beam to that of the undamaged beam. By arranging these normalized frequencies in increasing order, a unique classification system is established for each zone, allowing for the effective determination of the damage location.

2. Theoretical basis for identifying characteristics of steel beams

2.1. Cantilever steel beam model

Consider a beam structure have any distribution mass $m(x)$, with distributed load $q(x,t)$ (Fig. 1).

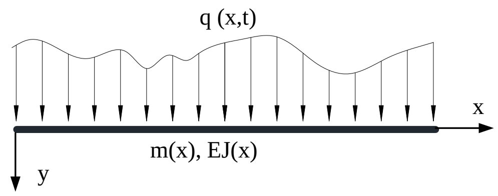

Fig. 1. Analytical diagram.

Differential equation for free vibration without considering the effect of resistance is written in the form.

$$\frac{d^2}{dx^2}\left[EJ(x)\frac{d^2X}{dx^2}\right]=\omega^2 m(x)X \tag{1}$$

in which E is the elastic modulus of the beam material, $J(x)$ is the moment of inertia of the beam cross-section, X is the bending form of beam structure (mode shape) only depends on x, ω is natural frequency, $m(x)$ is the mass per unit length, x is the distance from the fixed end.

If beams have constant stiffness and mass evenly distributed, we have:

$$\frac{d^4X}{dx^4}-\omega^2\frac{m}{EJ}X=0 \tag{2}$$

With the above equation and the boundary conditions corresponding to the cantilever beam, we can write the formula to calculate the specific vibration frequency as follows:

$$\omega_i=\alpha_i^2\sqrt{\frac{EJ}{ml^4}} \tag{3}$$

in which E is the elastic modulus of the beam material, J is the moment of inertia of the beam cross-section, m is the mass per unit length, l is the length of the cantilever beam. α_i is the coefficient, get the values $\alpha_i=1.875;\ 4.694;\ 7.885\ ;...;\pi(2i+1)/2$. Corresponding to the natural frequency ω_i, we have the i^{th} mode shape.

2.2. The undamaged beam

The cantilever beam is shown in Fig. 2:

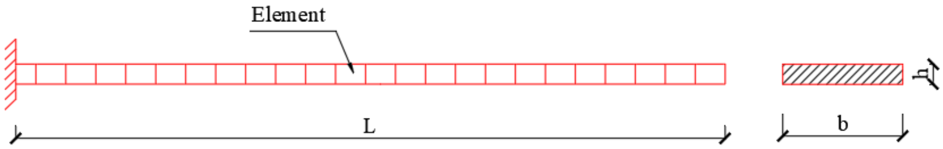

Fig. 2. The undamaged beam model.

The undamaged elemental stiffness $[k_e]$ and the mass matrix $[m_e]$ are expressed as follows:

$$[K]_e=\frac{EI}{L_e^3}\begin{bmatrix}12 & 6L_e & -12 & 6L_e\\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2\\ -12 & -6L_e & 12 & -6L_e\\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2\end{bmatrix} \tag{4}$$

$$[m]_e = \frac{\rho A L_e}{420}\begin{bmatrix} 156 & 22L_e & 54 & -13L_e \\ 22L_e & 4L_e^{\ 2} & 13L_e & -3L_e^{\ 2} \\ 54 & 13L_e & 12 & -22L_e \\ -13L_e & -3L_e^{\ 2} & -22L_e & 4L_e^{\ 2} \end{bmatrix} \tag{5}$$

2.3. The damaged beam

The cantilever damaged beam is shown in Fig. 3:

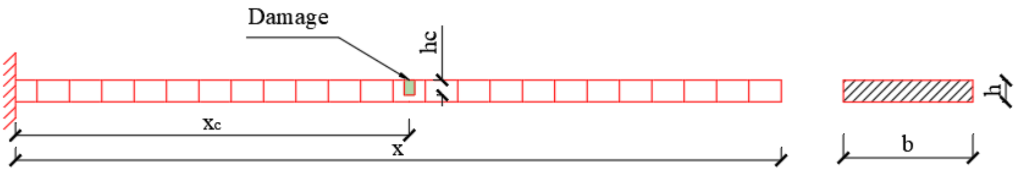

Fig. 3. The damaged beam model.

The local stiffness is reduced whenever a damage occurs in the beam. The elemental stiffness matrix of a cracked element $[K_{ec}]$ is given from the inverse of the flexibility matrix:

$$K_{ec} = [T]^t[C]^{-1}[T] \tag{6}$$

where $[T]$ is the transformation matrix and $[C]$ is the flexibility matrix. The flexibility matrix $[C]$ is the addition of the flexibility matrix of the intact beam $[C_n]$ and the additional flexibility matrix due to the presence of the crack $[C_c]$ calculated from the Fracture Mechanics Approach:

$$[C] = [C_n] + [C_c] \tag{7}$$

The transformation matrix $[T]$:

$$[T] = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{8}$$

The flexibility matrix of the non-cracked element $[C_n]$:

$$[C_n] = \begin{bmatrix} \dfrac{l_e^3}{3EI} & \dfrac{l_e^2}{2EI} \\ \dfrac{l_e^2}{2EI} & \dfrac{l_e}{EI} \end{bmatrix} \tag{9}$$

The terms of the additional flexibility matrix of the element due to the crack $[C_c]$

are calculated in Nahvi and Jabbari [12].

$$\left[C_c\right] = [C_{(i,j)}] \tag{10}$$

$$C_{11} = \frac{2\pi(1-\nu)}{Eb}\left[\frac{9l_e^2}{h^2}\int_0^{\alpha}\alpha F_1^2(\alpha)d\alpha + \int_0^{\alpha}\alpha F_2^2(\alpha)d\alpha\right] \tag{11}$$

$$C_{12} = C_{21} = \frac{36\pi l_e(1-\nu)}{Ebh^2}\int_0^{\alpha}\alpha F_1^2(\alpha)d\alpha \tag{12}$$

$$C_{22} = \frac{72\pi(1-\nu)}{Ebh^2}\int_0^{\alpha}\alpha F_1^2(\alpha)d\alpha \tag{13}$$

where ν is the Poisson's ratio and α is the crack depth ratio, F_1 and F_2 are the correction functions for a rectangular cross-section. They are expressed as follows:

$$\alpha = \frac{h_c}{h} \tag{14}$$

$$F_1\ (\alpha) = \sqrt{\frac{\tan(\frac{\alpha\pi}{2})}{\frac{\alpha\pi}{2}}}.\frac{[0.923 + 0.199(1-\sin(\frac{\alpha\pi}{2}))^4]}{\cos(\frac{\alpha\pi}{2})} \tag{15}$$

$$F_2\ (\alpha) = [1.122 - 0.561(\alpha) + 0.85(\alpha)^2 + 0.18(\alpha)^3]/\sqrt{1-\alpha} \tag{16}$$

2.4. Discretize the cantilever beam in zones

Consider the damaged cantilever steel beam as shown in Fig. 3. The damaged steel beam is divided into different regions based on the relationship between the ratio of the normalized natural frequency shift $\Delta F_{(i,j)}$ (calculated as the ratio of normalized mode frequencies df_i, df_j) and the ratio of the damage location to the total length of the steel beam, according to the following formula:

$$\Delta F_{(i,j)} = \frac{df_i}{df_j} \tag{17}$$

$$df_i = \frac{f_i^{damaged}}{f_i^{undamaged}} \tag{18}$$

$$df_j = \frac{f_j^{damaged}}{f_j^{undamaged}} \tag{19}$$

$$x_n = \frac{x_c}{x} \cdot 100 \tag{20}$$

Consider a steel beam with a length of x = 0.71 m; with damage at position x_c, the relationship diagram between $\Delta F_{(i,j)}$ and x_c is shown in Fig. 4.

The damaged region will be identified through the natural frequencies of the damaged steel beam and the initial steel beam.

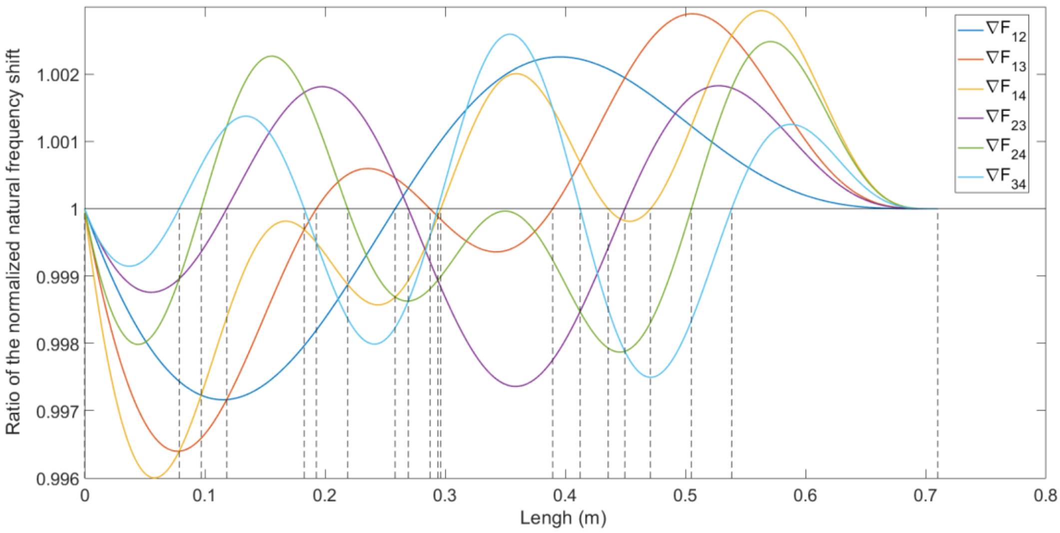

Fig. 4. The relationship diagram between $\Delta F_{(i,j)}$ and x_c.

3. Numerical simulation of structural damage based on natural frequency changes

3.1. Simulation model

The SAP2000 software is used to simulate the vibration of the structure. The simulated structure is a cantilever steel beam with a rectangular cross-section, fixed at one end and free at the other, as shown in Fig. 5. The physical parameters of the structure are presented in Tab. 1.

Tab. 1. The physical parameters of the structure

No.	Parameter	Symbol	Unit	Value
1	Length	L	mm	710
2	Density	ρ	Kg/m^3	7850
3	Young's modulus	E	MPa	2.03×10^5
4	Width	B	mm	60
5	Height	H	mm	8

The initial structural diagram before any damage is presented as follows:

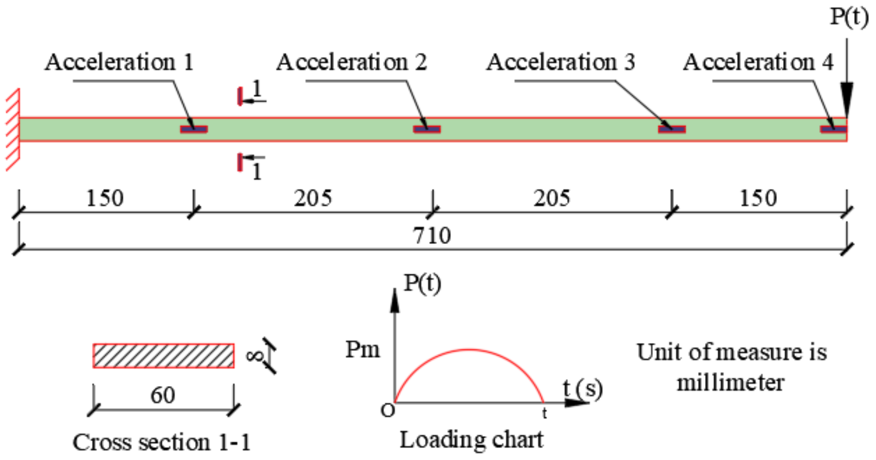

Fig. 5. The initial structural diagram.

Conduct structural vibration simulation for both the initial undamaged state and the state with cross-section reduction. Utilize the Frequency Domain Decomposition (FDD) method [14] to identify natural frequencies from the acceleration values at two arbitrary points extracted from the SAP2000 software.

Simulations consider cases where the cross-section reduction occurs at a position 1/2 of the span length as shown in Fig. 6 and at a position 1/4 of the span length from the fixed end as shown in Fig. 7.

For both cases, assume the cross-section depth reduction ratios are 25%, 35%, 45%, and 55%, with the reduction applied across the entire width B = 60 mm.

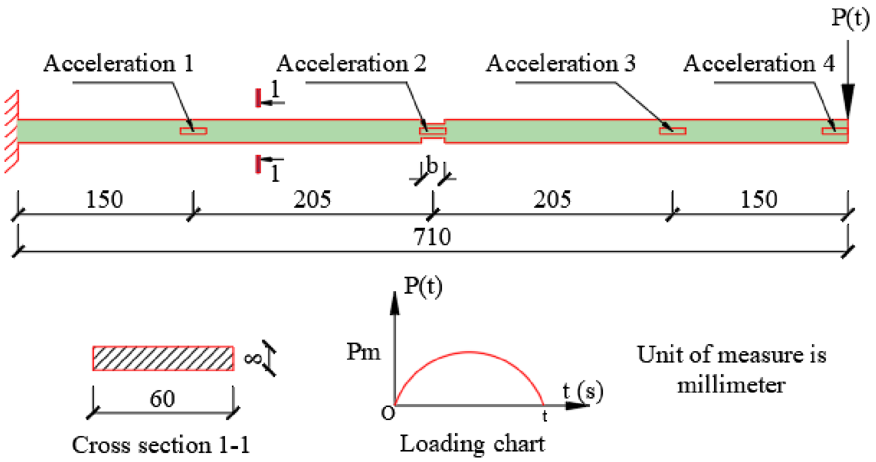

Fig. 6. Structural diagram of damage located at the mid-span of the beam.

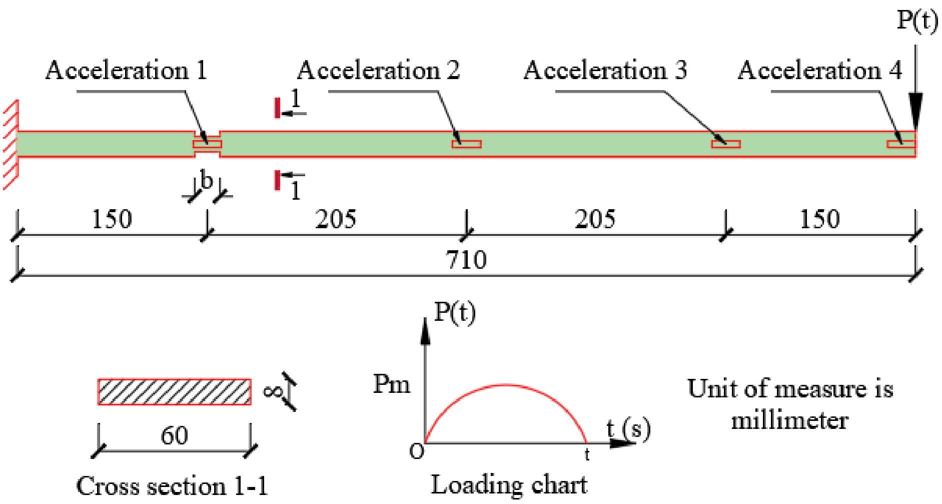

Fig. 7. Structural diagram of damage located at 1/4 of the span length.

3.2. Results of damage identification in the steel beam region

3.2.1. Results when the damage is located at a position 1/2 L

Simulating and calculating the natural frequencies when the damage is located at a position 1/2L with the cross-section depth reduction ratios are 25%, 35%, 45%, and 55%. Results of natural frequencies as shown in Tab. 2.

The damaged zone is identified by the classification of normalized natural frequencies, through the ratio of the natural frequency of the damaged beam to that of the undamaged beam. Results of damage detection when the damage is located at a position 1/2L as shown in Fig. 8.

Tab. 2. Results of natural frequencies identification

Mode	Natural frequencies				
	Undamage	Damage with various damage depth ratios α			
		25%	35%	45%	55%
1	13.25	12.88	12.75	12.75	12.63
2	80.75	79.5	77.5	75.5	74.38
3	226.8	226.5	226.5	226.5	225.5
4	435.5	429.5	420.5	413.5	410.5

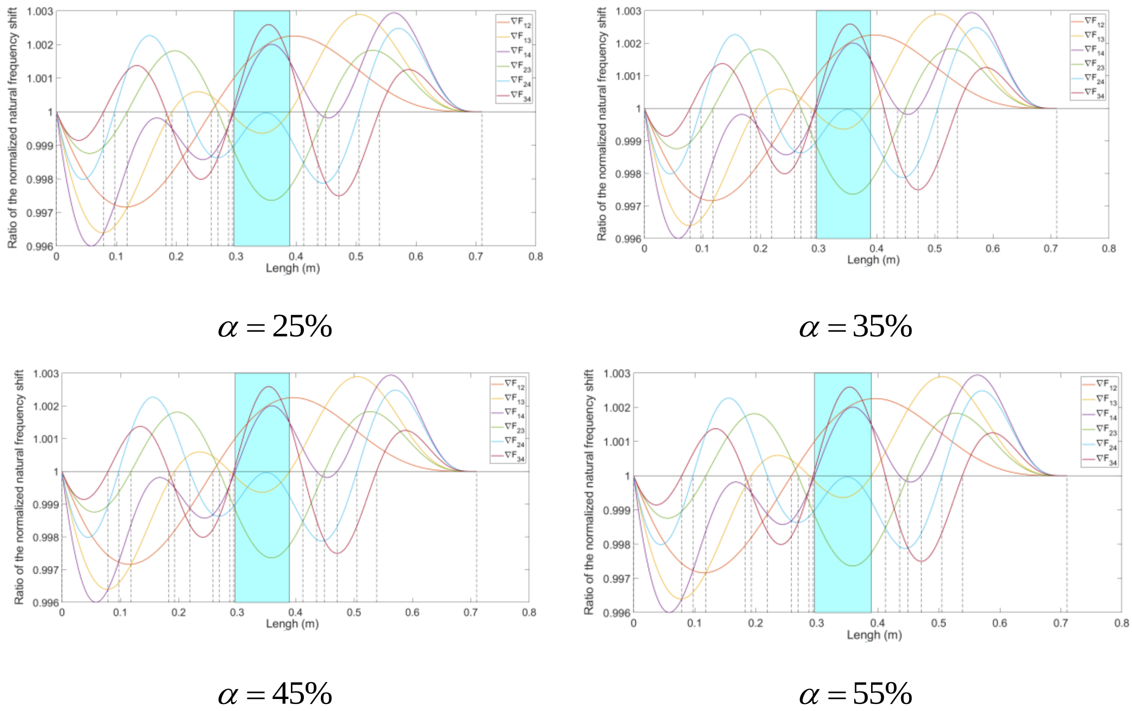

Fig. 8. Results of damage detection when the damage is located at 1/2 L.

3.2.2. Results when the damage is located at a position 1/4 L

Simulating and calculating the natural frequencies when the damage is located at a position 1/4L with the cross-section depth reduction ratios are 25%, 35%, 45%, and 55%. Natural frequencies as shown in Tab. 3.

Tab. 3. Results of natural frequencies identification

Mode	Natural frequencies				
	Undamage	Damage with various damage depth ratios α			
		25%	35%	45%	55%
1	13.25	12.75	12.25	12	11.5
2	80.75	80.75	81.25	81.25	81.25
3	226.8	225.25	223.5	222	221.25
4	435.5	428.25	419.5	415.5	413.25

The damaged zone is identified by the classification of normalized natural frequencies, through the ratio of the natural frequency of the damaged beam to that of the undamaged beam. Results of damage detection when the damage is located at a position 1/4L as shown in Fig. 9.

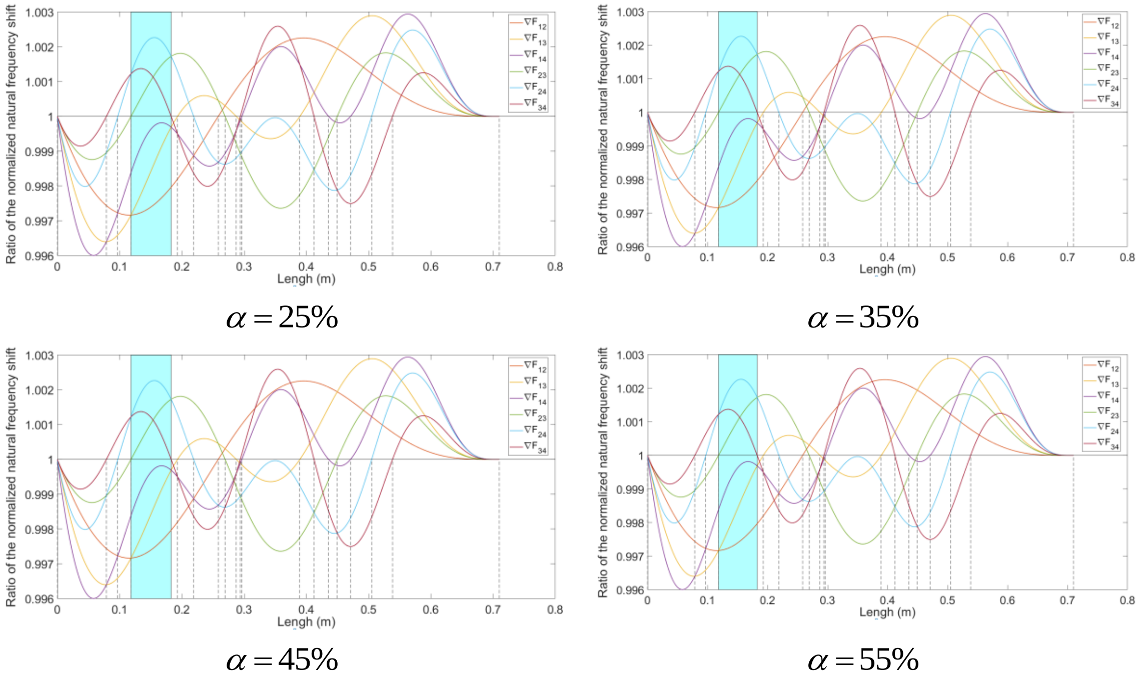

Fig. 9. The results of damage detection when the damage is located at 1/4 L.

The results in Fig. 8 and Fig. 9 show that the damage region identified using the method based on changes in the natural frequency matches the initially simulated damage locations. The cases of damage region identification are independent of the damage severity at the damaged location.

4. Conclusion

The proposed method, based on normalized frequency changes, accurately identifies damage regions in cantilever steel beams, independent of damage severity. The identified damage region matches the initially simulated damage location.

The damage region identification is independent of the severity of the structural damage, indicating the feasibility of the method for identifying the damage location in steel beams based on changes in natural frequency. Further research is needed to identify damage locations in structures with different types of damage.

The research results can be used to conduct experiments to determine the damage location in real steel structures, complex structures, and real-world projects under environmental excitation, based on changes in natural frequency.

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XÁC ĐỊNH VỊ TRÍ HƯ HỎNG CỦA KẾT CẤU DẦM THÉP THÔNG QUA SỰ THAY ĐỔI TẦN SỐ DAO ĐỘNG RIÊNG

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Tóm tắt: Khi hư hỏng xuất hiện trong kết cấu sẽ dẫn đến thay đổi những đặc trưng động lực học như tần số dao động riêng, dạng dao động riêng, tỉ số cản... so với kết cấu không hư hỏng. Dựa trên điều này, một số phương pháp được nghiên cứu và sử dụng để xác định vị trí vùng hư hỏng của kết cấu. Trong bài báo này, tiến hành nghiên cứu trên dầm thép công xôn có tiết diện không thay đổi, dầm công xôn được phân tách thành các vùng, trong đó mỗi vùng có một phân loại cụ thể của bốn tần số dao động riêng đầu tiên được chuẩn hóa. Vùng hư hỏng được phân biệt bằng cách phân loại các tần số chuẩn hóa của kết cấu thông qua tỉ lệ giữa tần số dao động riêng của dầm hư hỏng và tần số dao động riêng của dầm ban đầu. Tiến hành mô phỏng với kết cấu dầm công xôn ban đầu và kết cấu dầm công xôn hư hỏng bằng phần mềm SAP2000 và xác định tần số dao động riêng của dầm công xôn, làm cơ sở cho việc nhận dạng vị trí và mức độ hư hỏng của kết cấu. Kết quả nhận dạng cho thấy vùng hư hỏng nhận dạng được phù hợp với vị trí hư hỏng giả định ban đầu và không phụ thuộc vào mức độ hư hỏng của kết cấu. Kết quả nghiên cứu cho thấy có thể dựa vào sự thay đổi tần số dao động riêng để xác định được vị trí hư hỏng trên kết cấu dầm thép công xôn, làm cơ sở ứng dụng cho việc xác định vị trí hư hỏng của kết cấu phức tạp hơn và các công trình thực tế.

Từ khóa: *Tần số dao động riêng; dạng dao động riêng; hư hỏng kết cấu.*

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