APPLICATION OF NONLINEAR REGRESSION METHOD TO CALCULATE APPARENT COHESION OF GEOGRID- REINFORCED SOILS

Duc Tiep Pham[1,*], Tien Nam Nguyen[1], Nam Hung Tran[1], Thanh Trung Nguyen[1]

[1]*Le Quy Don Technical University, Hanoi, Vietnam*

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Abstract

In this article, the authors use Plaxis 2D software to simulate triaxial testing that shows the behavior of the geogrid reinforced soil structures under the effect of loads. Based on the Mohr - Rankine limit equilibrium conditions, the apparent cohesion generated by the geogrid layers is determined. The article has established 62 different numerical simulation problems on input parameters such as the internal friction angle of soil, the distance of layers, and the strength of geogrid. From numerical analysis results, combined with the nonlinear regression method, the authors have built a formula to determine the apparent cohesion of the reinforced soil structures.

Keywords: *Angle of internal friction; cohesion; apparent cohesion; finite element method; reinforced embankments; geogrid.*

1. Introduction

The reinforced soil calculation method was first implemented by Henri Vidal with the following idea: if the stress-deflection angle θ acting at the contact position between the soil and the geogrid is smaller than the friction angle φ* at this position, then the apparent cohesion between the soil particles and the geogrid is present. Complying with this condition ($\theta < \varphi^*$), there will be a mutual relationship between the soil particles and the geogrid. When the geogrid is placed in the direction of the main tensile stress and the condition $\theta > \varphi^*$ is satisfied, the tensile strain is limited by the dependence on the relative elongation of the geogrid. This characteristic can be further explained through the following example: An axial load acting on a loose soil sample will lead to a deformation of the side expansion, however, if the geogrid layers are placed in the horizontal direction, they will resist this deformation thanks to the appearance of friction between the geogrid elements and the soil. Then the state of the soil sample is similar to that of the case when a side load is added to it.

Destruction can occur solely as a result of the absence of friction between the soil and the geogrid or from the rupture of the geogrid element. The details of these

[*] Email: phamductiep@lqdtu.edu.vn

phenomena are studied and proven by the authors Schlosser F, Long NT (1973) [1], and Haussman (1976) [2]. These authors have established a theoretical correlation between the distance of the longitudinal geogrid elements, the tensile strength of the geogrid, and the apparent anisotropic cohesion occurring in the reinforced soil.

Recently, thanks to the development of experimental models and numerical simulation tools, many authors have studied and clarified the interaction between geosynthetics and soil [3-5]. However, there are not many studies evaluating the apparent cohesion of reinforced soils.

The accurate determination of the apparent cohesion value of the reinforced soil is very meaningful in predicting the bearing capacity of the ground reinforced with several layers of geogrids so that a suitable choice can be made type as well as the distance between layers. In this study, the authors will use Plaxis 2D software to simulate a triaxial test to model the behavior of geogrid-reinforced soil structures and based on the Mohr-Rankine limit equilibrium condition to determine the apparent cohesion produced by the geogrid. The results calculated by this method will be compared with some analytical methods to evaluate the reliability of the modeling method.

2. Analytical methods for determining the apparent cohesion of reinforced soils

The triaxial testing model is the basis for determining the apparent cohesion. Henri Vidal first considers a triaxial testing model of two loose soil samples. The first sample (Figure 1a) is not reinforced, the sample reaches the limit equilibrium when the principal stresses are σ_1, σ_2 and is represented by Mohr's circle (Figure 1c). Then the tangent line is determined by the friction angle in φ and passes through the origin O. In this case, the relationship between the principal stresses is as follows:

$$\sigma_1 = \frac{\sigma_2}{\mathrm{tg}^2(45^\circ - \frac{\varphi'}{2})} \tag{1}$$

In the same test with loose sand samples, the relationship between the principal stresses at the time of failure will change. The limit state is advanced when the vertical stress is σ1 and the lateral stress is reduced by an amount $\Delta\sigma_2$, that is, σ_2-$\Delta\sigma_2$. Then the characteristic of the Mohr’s circle is transformed accordingly (Figure 1c). The shear strength line will tangent to the Mohr circle and intersect the vertical axis with the corresponding coordinate apparent adhesion force $\tau = \Delta C_a$. In addition, the lateral pressure decrease $\Delta\sigma_2$, in this case, may be related only to the role of apparent cohesion:

$$\Delta\sigma_2 = 2 \cdot \Delta C_a \cdot \mathrm{tg}\left(45^\circ - \frac{\varphi'}{2}\right) \tag{2}$$

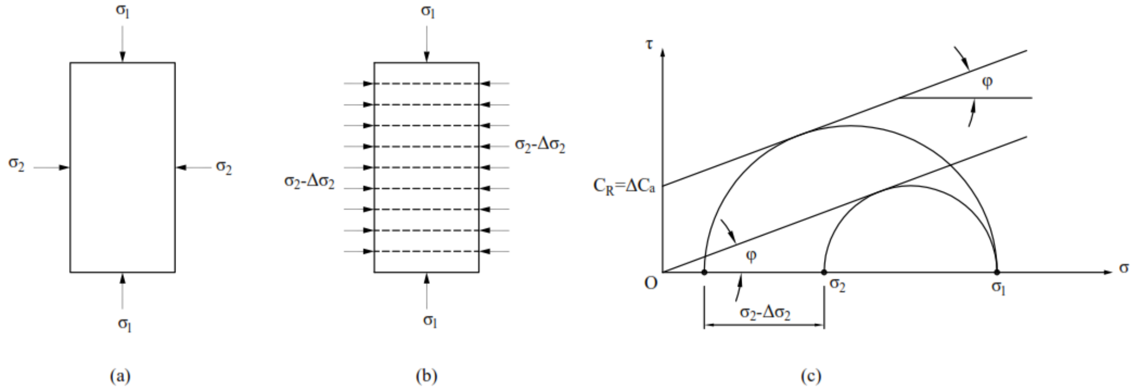


Figure 1. Diagram of triaxial testing results
(a) Sand sample without geogrid; (b) Reinforced sand sample; (c) Mohr's circle.

From formula (2), the apparent cohesion value is determined as follows:

$$\Delta C_a = \frac{\Delta\sigma_2}{2\cdot \mathrm{tg}\left(45^o - \frac{\varphi'}{2}\right)} = \frac{\Delta\sigma_2}{2\cdot\sqrt{\xi}} \tag{3}$$

where ξ is active earth pressure coefficient, $\xi = K_a = \mathrm{tg}^2(45^o - \frac{\varphi'}{2})$;

$\Delta\sigma_2$ is tensile strength of the geogrid per unit height of the triaxial specimen [6].

$$\Delta\sigma_2 = \frac{R_t}{\Delta H} \tag{4}$$

with ΔH is distance between layers of geogrid; R_t is tensile strength of geogrid.

Therefore, the apparent cohesion force is determined by the following formula:

$$\Delta C_a = \frac{R_t}{2\cdot\Delta H\cdot\sqrt{\xi}} \tag{5}$$

Equation (5) implies that geogrid strength and distance between layers of geogrid play the same role in the increase in apparent cohesion. That is to say, if the strength of the geogrid increases n-times, it is the same as the distance between layers of geogrid reduces n-times. On the other hand, Henri Vidal hypothesized that when the geogrid was placed in fill soil, it would exert its full strength. Many experimental results show that the geogrid layer spacing plays a more important role in increase of apparent cohesion than its strength. In the other words, a decrease in the geogrid layer spacing is more efficient than an increase in geogrid strength for increase of apparent cohesion.

According to [7], there is a new method to evaluate the increase in tensile strength of geogrid elements for triaxial samples, and is determined as follows:

$$\Delta\sigma_2 = W \cdot (\frac{R_t}{\Delta H}) \quad (6)$$

$$W = r^{\left(\frac{\Delta H}{\Delta H_{ref}}\right)} \quad (7)$$

where R_t is tensile strength of geogrid; r is dimensionless coefficient; ΔH_{ref} is the conventional distance is determined as follows:

$$\Delta H_{ref} = 6d_{\max}; \quad (8)$$

$d_{\max}$ is the largest diameter of soil particle; ΔH is distance between layers of geogrid.

Ketchart - Wu gives the following formula for determining W:

$$W = 0.7^{\left(\frac{\Delta H}{\Delta H_{ref}}\right)}; \quad (9)$$

$$\Delta\sigma_2 = W \cdot (\frac{R_t}{\Delta H}) = 0.7^{\left(\frac{\Delta H}{\Delta H_{ref}}\right)} \cdot \left(\frac{R_t}{\Delta H}\right) \quad (10)$$

- For cohesionless soil:

$$C_R = \Delta C_a = \frac{\Delta\sigma_2}{2\cdot\sqrt{\xi}} = \frac{0.7^{\left(\frac{\Delta H}{\Delta H_{ref}}\right)} \cdot \left(\frac{R_t}{\Delta H}\right)}{2\cdot\sqrt{\xi}} = 0.7^{\left(\frac{\Delta H}{\Delta H_{ref}}\right)} \cdot \left(\frac{R_t}{2\cdot\Delta H\cdot\sqrt{\xi}}\right) = 0.7^{\left(\frac{\Delta H}{6\cdot d_{\max}}\right)} \cdot \left(\frac{R_t}{2\cdot\Delta H\cdot\sqrt{\xi}}\right) \quad (11)$$

- Cohesive soil:

$$C_R = \Delta C_a + C' = 0.7^{\left(\frac{\Delta H}{\Delta H_{ref}}\right)} \cdot \left(\frac{R_t}{2\cdot\Delta H\cdot\sqrt{\xi}}\right) + C' = 0.7^{\left(\frac{\Delta H}{6\cdot d_{\max}}\right)} . \left(\frac{R_t}{2\cdot\Delta H\cdot\sqrt{\xi}}\right) + C' \quad (12)$$

where ξ is active earth pressure coefficient, $\xi = K_a = \text{tg}^2(45^\circ - \frac{\varphi'}{2})$; C' is soil cohesion.

So, according to the formulas for determining the apparent cohesion of Ketchart - Wu (11) or (12), we see that the strength of the geogrid is not fully promoted, in addition, this formula also considers the influence of the particle size of the surrounding soil.

In fact, when designing the fill embankment with geogrid, they pointed out 3 causes of failure as follows: 1) broken geogrid (insecure strength), 2) too large elongation (each type of geogrid has a limit on elongation at break), and 3) insufficient geogrid length. Both the analytical formulas of Henri Vidal and Ketchart - Wu have not considered the deformation of the geogrid.

3. The finite element method determines the apparent cohesion of the geogrid-reinforced soil

To overcome the disadvantages of the two analytical methods as presented above, the authors consider the finite element method with the 2D Plaxis software application to determine the apparent cohesion of the reinforced soil. Indeed, when using this method, both failure causes (1 and 2) of reinforced soil samples are considered through the value included when calculating the axial stiffness of geogrid EA, because it is determined through the ratio between the ultimate load at geogrid break and the relative elongation.

Based on how to build an experimental model and describe the experimental process through the Plaxis software according to [7], the authors present a model to determine the apparent cohesion of reinforced soil by the finite element method as shown in Figure 2.

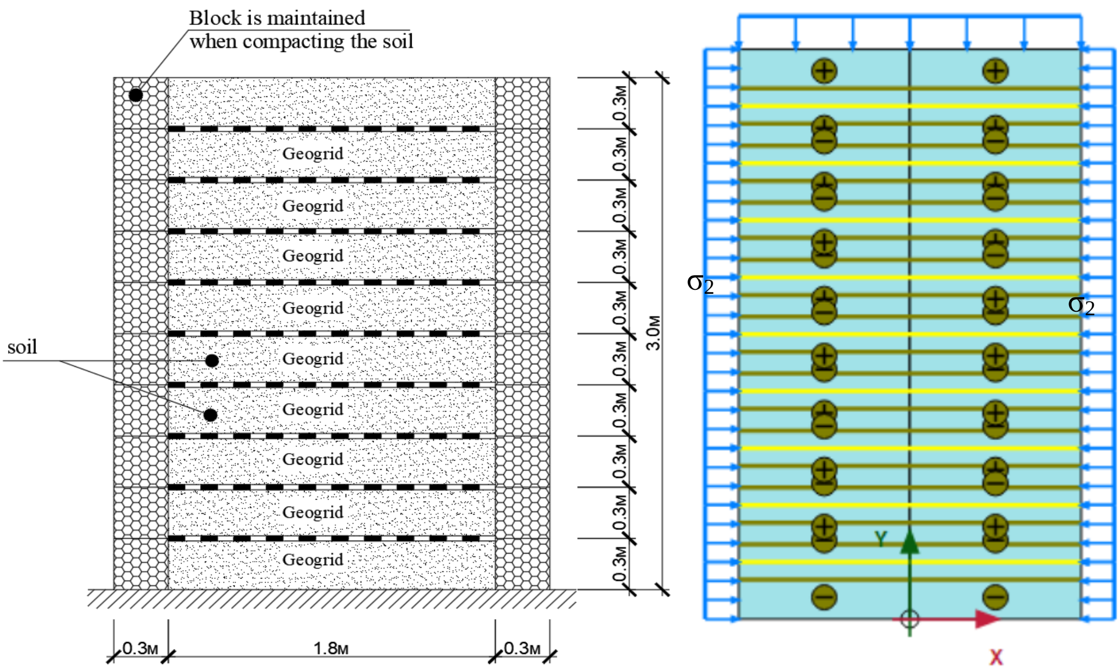
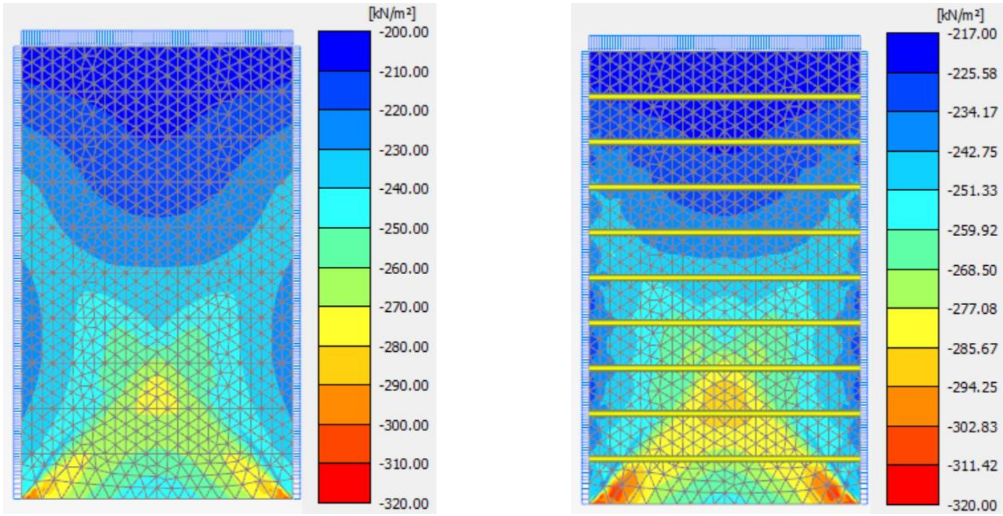


Figure 2. Characteristics of the model when determining the apparent cohesion.

Table 1. Input parameters when building model

Parameters of soil (lean clay mixed gravel) in Pho Yen - Thai Nguyen							Parameters of geogrid		
E_{50}^{ref}	E_{oed}^{ref}	E_{ur}^{ref}	Power (m)	γ	φ'	c'	R	EA	ΔH
kPa	kPa	kPa		kN/m^3	degree	kPa	kN/m	kN/m	m
30000	30000	90000	0.5	20	18	20	15	115	0.30
Hardening soil model							Linear model		



a) Sand sample without geogrid *b) Sand sample with geogrid*

Figure 3. Vertical stress (σ_y) in the soil mass at the time of failure (initial side pressure acting on the soil block σ_2 is 100 kN/m^2).

The formula for determining the ultimate vertical pressure acting on the specimen is as follows [8]:

$$\sigma_1 = \sigma_2 + \Sigma M_{stage} (\sigma_{10} - \sigma_2) \quad (13)$$

where σ_2 is the lateral pressure (σ_2 is 100 kN/m^2); σ_{10} is vertical presure applied to the sample (σ_{10} is 320 kPa); ΣM_{stage} is vertical presure factor.

According to the calculation results of 2 cases of soil block without geogrid and a case of block reinforced with geogrid (Figure 3), when the vertical load coefficient at the time of failure is $\sum M_{\text{Stage}}^{\text{Non-geogrid}} = 0.4675$ and $\sum M_{\text{Stage}}^{\text{Geogrid}} = 0.5320$, respectively. Applying the Mohr - Rankine limit equilibrium condition, we can determine the corresponding apparent cohesion of the soil block reinforced with the geogrid. The results are shown in Table 2.

Table 2. Result comparing the calculation of apparent cohesion by the finite element method (FEM) with the analytical method

Lateral pressure σ_2	Initial establish vertical pressure σ_{10}	ΣM_{Stage} Non-geogrid	ΣM_{Stage} Geogrid	FEM (Plaxis 2D)			Ketchart – Wu
				σ_1 Non-geogrid	σ_1 Geogrid	ΔC	ΔC
kPa	kPa			kPa	kPa	kPa	kPa
100	320	0.4675	0.532	203	217	5.15	4.74

With input data as in Table 1, the authors determine the apparent cohesion of reinforced soil according to the analytical formula (12). The defference between FEM and the analytical formula of Ketchart - Wu is 8%. This confirms that the geogrid-reinforced soil block modeling method as presented is appropriate.

4. Using the regression method to determine the apparent cohesion of the geogrid-reinforced soil

In this section, the authors change parameters: distance of geogrid layers (ΔH); its strength (R), and the internal friction angle of the soil. For each set of input parameters changed, use FEM with the same modeling as above to calculate the value of ultimate vertical pressure, thereby determining the apparent cohesion due to reinforced geogrids.

Here the authors proceed with 62 different input parameters. The results are shown in Table 3.

Table 3. Determination of apparent cohesion according to different input data

No.	**Soil**		**Geogrid**			**Results (Plaxis 2D)**		
	φ'	**c'**	**R**	**EA**	**ΔH**	**$\sigma_{1\ \text{Non-geogrid}}$**	**$\sigma_{1\ \text{Geogrid}}$**	**ΔC**
	degree	kPa	kN/m	kN/m	m	kPa	kPa	kPa
1	18	20	15	115	0.30	**203**	**217**	**5.15**
2	18	20	20	154	0.30	**203**	**222**	**7.11**
3	18	20	30	231	0.30	**203**	**233**	**11.01**
4	18	20	40	308	0.30	**203**	**243**	**14.43**
5	18	20	50	385	0.30	**203**	**242**	**14.36**
6	14	20	30	231	0.30	**174**	**201**	**10.89**
7	16	20	30	231	0.30	**188**	**216**	**10.52**
8	18	20	30	231	0.30	**203**	**233**	**11.01**
9	20	20	30	231	0.30	**219**	**256**	**12.98**
10	22	20	30	231	0.30	**236**	**275**	**13.13**
11	18	20	30	231	0.25	**203**	**242**	**14.15**
12	18	20	30	231	0.30	**203**	**233**	**11.01**
13	18	20	30	231	0.50	**203**	**216**	**4.80**
14	18	20	30	231	0.60	**203**	**213**	**3.78**
15	14	20	30	231	0.25	**174**	**208**	**13.18**
16	14	20	30	231	0.30	**174**	**201**	**10.77**

No.	Soil		Geogrid			Results (Plaxis 2D)		
	φ'	c'	R	EA	ΔH	σ_1 Non-geogrid	σ_1 Geogrid	ΔC
	degree	kPa	kN/m	kN/m	m	kPa	kPa	kPa
17	14	20	30	231	0.50	**174**	**187**	**5.20**
18	14	20	30	231	0.60	**174**	**183**	**3.68**
19	22	20	30	231	0.25	**236**	**278**	**14.10**
20	22	20	30	231	0.30	**236**	**275**	**13.05**
21	22	20	30	231	0.50	**236**	**251**	**5.18**
22	22	20	30	231	0.60	**236**	**249**	**4.21**
23	18	20	15	115	0.25	**203**	**222**	**6.86**
…	…	…	…	…	…	…	…	…
60	18	20	30	231	0.60	**203**	**213**	**3.78**
61	20	20	30	231	0.60	**219**	**230**	**3.91**
62	22	20	30	231	0.60	**236**	**249**	**4.29**

In data analysis, a common task is to study the dependence of a random variable $Y \in R^D$ on many independent variables $X = [X_1, X_2, ..., X_D] \in R^D$. The model given in the form of a mathematical equation $Y = f(X)$ describing the relationship between *Y* and *X* is called a regression model. If the function f has a first-order form, the technique is called multivariable linear regression. If the function f has a higher order form (greater than 1), the technique is called multivariable nonlinear regression [9].

To evaluate the accuracy of the established regression model, the study uses the R^2 index (coefficient of determination). The R^2 index has values between 0 and 1, the closer R^2 is to 1, the higher the accuracy of the regression model is.

Based on the formula of Ketchart - Wu, the authors propose a regression function showing the correlation between the apparent cohension with input parameters such as the internal friction angle of soil, the distance of geogrid layers (Δ*H*), and geogrid strength (*R*).

$$\Delta C = \frac{a^{\Delta H} \cdot (b \cdot R)}{2 \cdot \Delta H \cdot \mathrm{tg}(45^{o} - \frac{\varphi'}{2})} \tag{14}$$

- Regression coefficients: *a*, *b*;

- Observable sample set (*D*) is the numerical experimental data set up as shown in Table 3.

$$D = [\text{Apparent cohesion } \Delta C_i,\ i = 1 \div N] \quad (15)$$

where *N* is the sum of all testing data (*N* is 62).

To determine the above regression coefficients, the author applies Solver tool in Microsoft Excel software for nonlinear regression analysis for the established function *S*(*t*). This analysis is performed on the basis of least squares method, provided that the following function must be minimized:

$$f(\Delta H, R, \varphi') = \sum_{i=1}^{N} \left[\Delta C(\Delta H_i, R_i, \varphi'_i) - \Delta C_i \right]^2 \rightarrow Min \quad (16)$$

The regression coefficient is determined as follows: *a* is 0.5412 and *b* is 0.1713.

The regression function is built with R^2 is 0.86 and RMSE is 1.73.

$$\Delta C = \frac{0.5412^{\Delta H} \cdot (0.1713 \cdot R)}{2 \cdot \Delta H \cdot \text{tg}(45^\circ - \frac{\varphi'}{2})} \quad (17)$$

Figures 4 and 5 show that the deviation between the analytical formula of Ketchart - Wu and the proposed regression formula is not much. The apparent cohesion result according to the analytical method of Ketchart - Wu gives a more secure value.

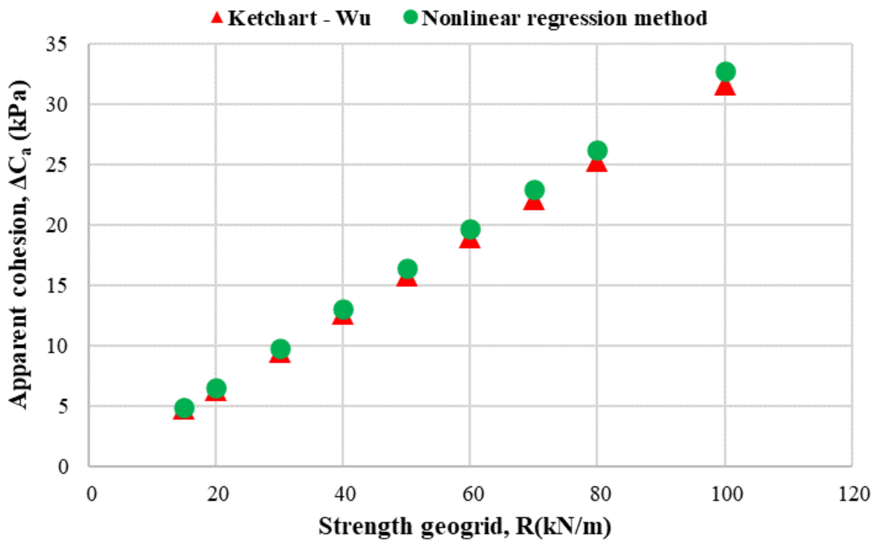


Figure 4. Surveying the dependence of apparent cohesion on geogrid strength.

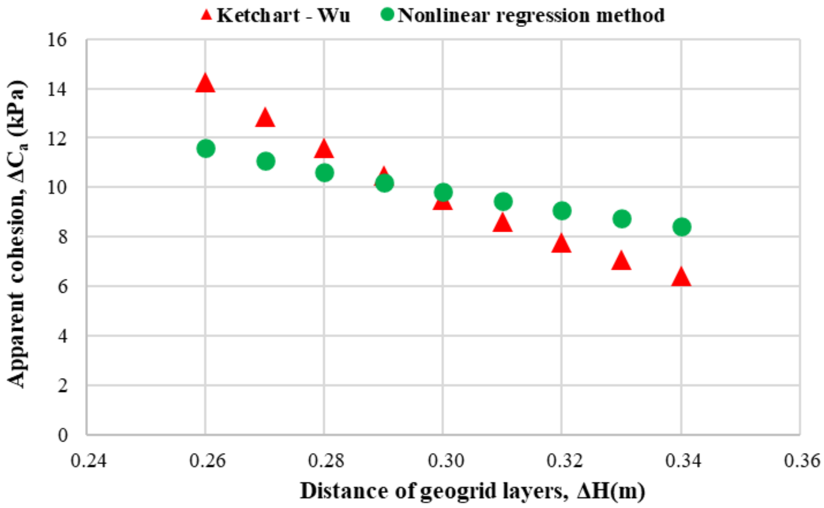


Figure 5. Surveying the dependence of apparent cohesion on distance of geogrid layers.

5. Conclusion

The accurate determination of the apparent cohesion value of the geogrid-reinforced soil is very meaningful in predicting the bearing capacity of the geogrid-reinforced soil structures so that a suitable choice can be made for type of geogrid as well as the distance between layers. In this study, the authors analyze the analytical methods and present the finite element method to determine the apparent cohesion of the geogrid reinforced soil. On the basis of building a numerical test data set of different simulation problems on input parameters (soil friction angle, strength, and distance of geogrid layers), the authors apply the nonlinear regression method to establish the formula for determining the apparent cohesion of the geogrid reinforced soil. The formula is set to have an accuracy level of R^2 equal 0.86 and RMSE equal 1.73.

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ỨNG DỤNG PHƯƠNG PHÁP HỒI QUY PHI TUYẾN TÍNH TOÁN CƯỜNG ĐỘ LỰC DÍNH BIỂU KIẾN CỦA ĐẤT ĐẮP ĐƯỢC GIA CƯỜNG CÁC LỚP LƯỚI ĐỊA KỸ THUẬT

Phạm Đức Tiệp[a], Nguyễn Tiến Nam[a], Trần Nam Hưng[a], Nguyễn Thành Trung[a]

[a]*Trường Đại học Kỹ thuật Lê Quý Đôn*

Tóm tắt: *Bài báo sử dụng phần mềm Plaxis 2D để mô phỏng thí nghiệm 3 trục thể hiện ứng xử của khối đất nền được gia cường cốt lưới địa kỹ thuật dưới tác dụng của tải trọng và trên cơ sở điều kiện cân bằng giới hạn Mohr - Rankine sẽ xác định được lực dính biểu kiến do các lớp cốt tạo ra. Bài báo đã thiết lập được 62 bài toán mô phỏng số khác nhau về các tham số đầu vào như: góc ma sát trong của đất nền, khoảng cách các lớp và cường độ của lưới địa kỹ thuật. Từ các kết quả khảo sát này, kết hợp với phương pháp hồi quy đa biến, các tác giả đã xây dựng được công thức xác định lực dính biểu kiến của khối đất nền có cốt gia cường.*

Từ khóa: Góc ma sát trong; lực dính đơn vị; lực dính biểu kiến đơn vị; phương pháp phần tử hữu hạn; nền đất gia cường cốt; lưới địa kỹ thuật.

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