

# IDENTIFICATION OF THE NATURAL FREQUENCY OF THE OFFSHORE TRUSS STRUCTURE FROM MULTIPLE SIMULATED MEASURED DATA

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## Abstract

This article presents identification methods of the natural frequency of structures from single or multiple measurements in the frequency domain and its application to identify the natural frequencies of the offshore truss structure. The simulated measurements of structural vibrations in the time domain are determined by analyzing the finite element model of the offshore truss structure subjected to random dynamic loads using SAP2000 software. The frequency domain measurements are obtained by the fast Fourier transform of time domain measurements using MATLAB software. The identification method from the multiple combined frequency response function measurements can estimate more natural frequencies than using a single measurement. A study of the considered frequency range was also performed showing the accuracy of the identification methods compared with the finite element model. By subdividing the considered frequency range, it is possible to improve the accuracy of the obtained natural frequencies.

*Keywords:* Identification of natural frequencies; rational fraction polynomial method; offshore truss structure; modal analysis; simulated measurements; frequency domain.

## 1. Introduction

The offshore truss structure plays an important role in the development of the country. Under the impact of extreme offshore environments, some structures at offshore can get damaged or even fall down. Therefore, determining the technical status of structures such as offshore trusses is a regular and urgent requirement.

Monitoring and evaluating the technical status of structures is a complex problem, with the identification of the natural frequency of the structure being one of the fundamental issues. There are many methods to identify the natural frequency of the structure, including the group of identification methods in the frequency domain [1, 2] and the group of identification methods in the time domain [3, 4]. The frequency domain methods use frequency response function (FRF) measurements of the structure to identify modal parameters such as natural frequency, mode shape, and damping ratio. Some popular methods are known as the peak-picking method [5, 6], integral method [7],

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rational fraction polynomial (RFP) method [8], global rational fraction polynomial (GRFP) method [9], and frequency domain decomposition method [10].

Peak-picking method assumes that each significant peak in the frequency-response function corresponds to exactly one natural mode and that any effects due to the other modes can be ignored. This method is only suitable for structures with separated natural frequencies and the damping ratio is not so light but which, on the other hand, is not so heavy [5].

RFP method proposed by Richardson [8], can identify multiple modes simultaneously from a single measured data. This method transforms the FRF from partial fraction form into rational fraction form and uses the least squares procedure of error between the experimental FRF and the theoretical FRF. The RFP method can extract the natural frequencies quite accurately, but sometimes the number of modes in the considered frequency range is not sufficiently estimated because the amplitude of certain modes in the measured data is zero.

GRFP method uses multiple measurement data for the least squares procedure and estimates a larger number of modes. The results of this method depend on many parameters such as considered frequency range, polynomial order, and sometimes fitting the mathematical model but not the physical model [11].

This article presents methods to identify the natural frequency of offshore truss structures from single or multiple simulated measurements in the frequency domain. The natural frequencies are estimated using the identification methods, which are compared with each other and with the finite element (FE) model. A study of the considered frequency range was also performed to improve the accuracy of the identification method based on multiple measurement data.

## **2. The identification methods of natural frequencies**

### ***2.1. The identification method of multiple natural frequencies from a single measurement***

The identification method of multiple natural frequencies from a single measurement data is based on fitting both the experimental FRF curve with the theoretical FRF curve over a specified frequency range. This frequency range can contain many natural frequencies of the structure for which the FRF plot may not show resonant peaks.

The theoretical FRF of the multiple degree of freedom system is as follows:

$$H(\omega) = \sum_{k=1}^N \left[ \frac{A_k}{i\omega - p_k} \exp(i\varphi_k) + \frac{A_k}{i\omega - p_k^*} \exp(i\varphi_k) \right] \quad (1)$$

or can be written as a polynomial fraction form:

$$H(\omega) = \frac{a_0 + a_1(i\omega)^1 + a_2(i\omega)^2 + \dots + a_{2N-1}(i\omega)^{2N-1}}{b_0 + b_1(i\omega)^1 + b_2(i\omega)^2 + \dots + b_{2N}(i\omega)^{2N}} \quad (2)$$

where  $N$  is the total number of degrees or the total number of modes of the system;  $p_k = -\sigma_k + i\omega_k$  called the  $k^{th}$  "pole" ( $k^{th}$  extreme – the  $k^{th}$  root of the characteristic polynomial in the denominator of (2);  $p_k^*$  is the complex conjugate of  $p_k$ ;  $\omega_k$  is called the  $k^{th}$  natural frequency;  $A_k$ ,  $\varphi_k$  are called the amplitude and phase angle of the  $k^{th}$  mode;

Call  $h_j = h(\omega_j)$  is the measured value of the FRF corresponding to the frequency  $\omega_j$ . Assuming that  $b_{2m} = 1$ , the error between experimental FRF and theoretical FRF at each frequency point  $\omega$  is determined by the expression:

$$e_j = [a_0 + a_1(i\omega)^1 + \dots + a_{2m-1}(i\omega)^{2m-1}] - h_j [b_0 + b_1(i\omega)^1 + \dots + b_{2m-1}(i\omega)^{2m-1}] - h_j(i\omega)^{2m} \quad (3)$$

For an  $s$  frequency points, construct an error vector of the form:

$$\{E\} = \{e_1 \quad e_2 \quad \dots \quad e_s\}^T$$

From equation (3) deduce:

$$\{E\} = \begin{bmatrix} 1 & (i\omega_1)^1 & (i\omega_1)^2 & \dots & (i\omega_1)^{2m-1} \\ 1 & (i\omega_2)^1 & (i\omega_2)^2 & \dots & (i\omega_2)^{2m-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & (i\omega_s)^1 & (i\omega_s)^2 & \dots & (i\omega_s)^{2m-1} \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{2m-1} \end{Bmatrix} - \begin{bmatrix} h_1 & h_1(i\omega_1)^1 & h_1(i\omega_1)^2 & \dots & h_1(i\omega_1)^{2m-1} \\ h_2 & h_2(i\omega_2)^1 & h_2(i\omega_2)^2 & \dots & h_2(i\omega_2)^{2m-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_s & h_s(i\omega_s)^1 & h_s(i\omega_s)^2 & \dots & h_s(i\omega_s)^{2m-1} \end{bmatrix} \begin{Bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{2m-1} \end{Bmatrix} - \begin{Bmatrix} h_1(i\omega_1)^{2m} \\ h_2(i\omega_2)^{2m} \\ \vdots \\ h_s(i\omega_s)^{2m} \end{Bmatrix} \quad (4)$$

or it can be written shorter:

$$\{E\} = [P]_{(s \times 2m)} \{a\}_{(2m \times 1)} - [T]_{(s \times 2m)} \{b\}_{(2m \times 1)} - \{W\}_{(s \times 1)} \quad (5)$$

The squared sum error is a function that depends on  $\{a\}$  and  $\{b\}$ :

$$J(\{a\}, \{b\}) = \sum_{j=1}^s e_j^* e_j = \{E^*\}^T \{E\} \quad (6)$$

For this error function to be minimized, its partial derivatives with respect to  $\{a\}$ ,  $\{b\}$  must be zero:

$$\frac{\partial J}{\partial \{a\}} = 2 \operatorname{Re}([P^*]^T [P] \{a\}) - 2 \operatorname{Re}([P^*]^T [T] \{b\}) - 2 \operatorname{Re}([P^*]^T \{W\}) = \{0\} \quad (7)$$

$$\frac{\partial J}{\partial \{b\}} = 2 \operatorname{Re}([T^*]^T [T] \{b\}) - 2 \operatorname{Re}([T^*]^T [P] \{a\}) + 2 \operatorname{Re}([T^*]^T \{W\}) = \{0\} \quad (8)$$

Derive the system of equations to determine  $\{a\}$ ,  $\{b\}$ :

$$\begin{bmatrix} \operatorname{Re}([P^*]^T [P]) & -\operatorname{Re}([P^*]^T [T]) \\ -\operatorname{Re}([T^*]^T [P]) & \operatorname{Re}([T^*]^T [T]) \end{bmatrix}_{(4m \times 4m)} \begin{Bmatrix} \{a\} \\ \{b\} \end{Bmatrix}_{(4m \times 1)} = \begin{Bmatrix} \operatorname{Re}([P^*]^T \{W\}) \\ -\operatorname{Re}([T^*]^T \{W\}) \end{Bmatrix}_{(4m \times 1)} \quad (9)$$

Solving the above system of equations, we get the vector  $\{b\} = \{b_0 \ b_1 \ \dots \ b_{2m-1}\}^T$  and deduce that the poles (containing the natural frequencies) are the roots of the polynomial:

$$f(i\omega) = b_0 + b_1(i\omega)^1 + b_2(i\omega)^2 + \dots + b_{2m-1}(i\omega)^{2m-1} + (i\omega)^{2m} \quad (10)$$

## 2.2. The identification method of multiple natural frequencies from multiple measurements

From the equation (1) it can be seen that, if the mode amplitude  $A_k$  is zero, the polynomial (10) will have no  $k^{\text{th}}$  root and the  $k^{\text{th}}$  natural frequency cannot be determined. Therefore, when using a single FRF measurement, it may not be possible to identify all the natural frequencies of the system.

Richardson [9] continued to expand and develop the GRFP method to simultaneously fit many experimental FRF curves.

The equation (9) can write shorter as follows:

$$[U]_{(4m \times 4m)} \{c\}_{(4m \times 1)} = \{V\}_{(4m \times 1)} \quad (11)$$

with  $\{c\} = \begin{Bmatrix} \{a\} \\ \{b\} \end{Bmatrix}$

Combine  $L$  FRF measurements, we have:

$$\begin{bmatrix} [U]_1 \\ [U]_2 \\ \vdots \\ [U]_L \end{bmatrix}_{(4mL \times 4m)} \{c\}_{(4m \times 1)} = \begin{Bmatrix} \{V\}_1 \\ \{V\}_2 \\ \vdots \\ \{V\}_L \end{Bmatrix}_{(4mL \times 4m)} \quad (12)$$

or  $[U_G] \{c\} = \{V_G\} \quad (13)$

The equation (4) is a system of linear algebraic equations with more equations than unknowns, the solution of the system is obtained by the matrix pseudo-inverse procedure as follows:

$$\{c\} = [U_G]^+ \{V_G\} \quad (14)$$

in which the matrix  $[U_G]^+$  is the pseudo-inverse of the matrix  $[U_G]$ .

The solution (14) depends on the number of modes need to identify (m), the number of FRF measurements and the frequency range used in each of those FRFs. After estimating the coefficients  $\{c\}$ , extract the vectors of the coefficients  $\{b\}$  and the natural frequencies of the system are deduced from the solution of the polynomial (10).

### 3. The FE model of the offshore truss structure and simulation of measurement data

#### 3.1. The FE model of the offshore truss structure

The FE model of the offshore truss structure is simulated by SAP2000 software, and has the overall size of the base (11 × 11) m, height 17 m (Figure 1). The upper floor has dimensions of (9 × 16) m; the lower floor has dimensions of (9 × 12.5) m. Dimensions of the main components are given in Table 1. The elements are welded together and the beam-column connection is modeled as rigid joint in SAP2000.

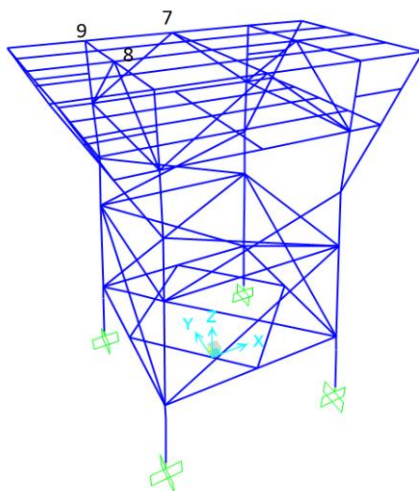


Figure 1. The FE model of the offshore truss structure.

Table 1. Dimensions of main components of the offshore truss structure

No.	Parameter	Dimensions of section (mm)
1	Main column (pipe)	Φ810 × 15/Φ500 × 10
2	Horizontal bracing (pipe)	Φ430 × 10
3	Diagonal bracing (pipe)	Φ300 × 8
4	Major beam	I800 × 400 × 20 × 20
5	Minor beam	I650 × 300 × 15 × 20

### 3.2. Simulation of measurements data in the time domain

A time-based random dynamic load impact on node 9 in the X and Y directions is shown in Figure 2. The time to maintain the dynamic load is 15 (s),  $P_X(t)$  reaches the maximum value of 32.09 kN at  $t = 8.8$  s and  $P_Y(t)$  reached its maximum value of 30.94 kN at  $t = 1.3$  s.

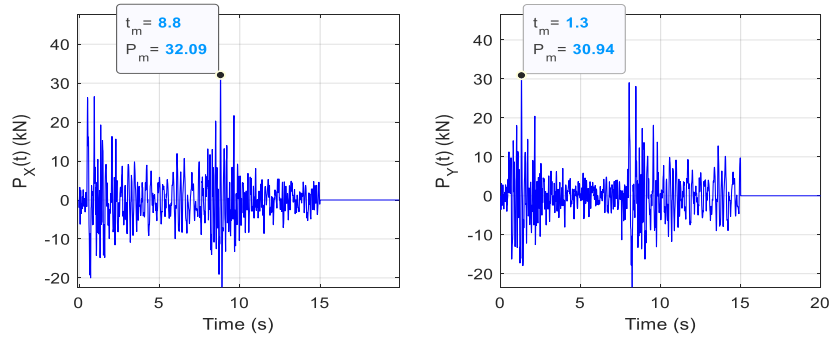


Figure 2. Dynamic loads at node 9.

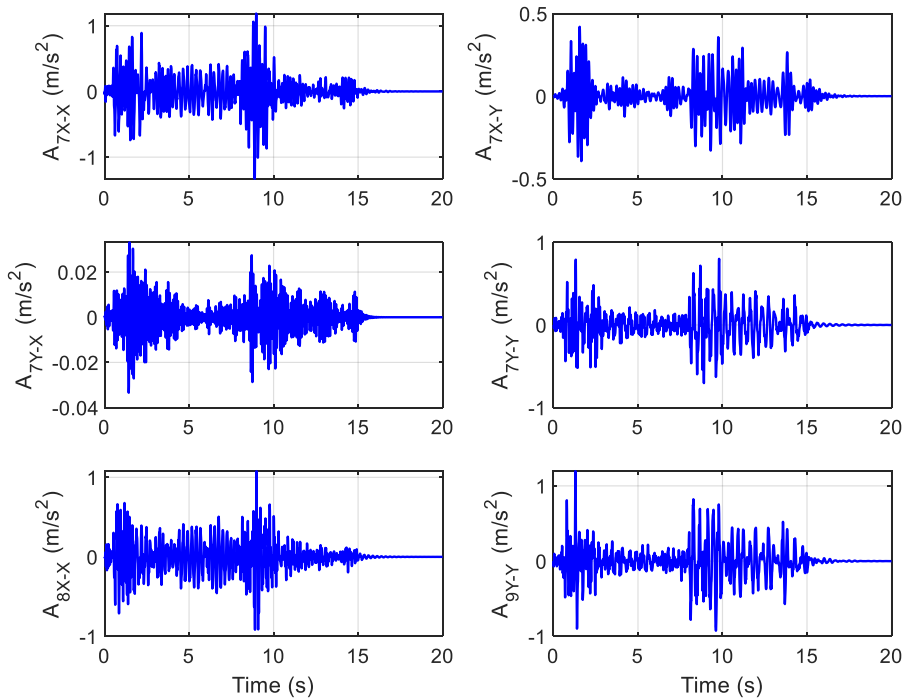


Figure 3. Acceleration time measurement data at nodes 7, 8, 9.

Using the time history analysis method (Newmark), calculate the system subjected to dynamic loads with a time step  $dt = 0.1$  s and a constant damping ratio is 5%. Calculation results of accelerations at nodes 7, 8, 9 in the X and Y directions are shown in Figure 3. The acceleration graph in Figure 3 is assumed to be the simulated

measurement of the structure in the time domain. In the  $A_{7Y-X}$  acceleration graph, two subscript indices represent the acceleration at node 7 in the Y direction under the load in the X direction.

**3.3. Simulation of measurements data in the frequency domain**

Frequency domain measurements are FRFs determined by the following formula:

$$h_{j-k}(\omega) = \frac{X_j(\omega)}{P_k(\omega)} \quad (j,k = 1,2,\dots,n) \tag{15}$$

where  $X_j(\omega)$  is the Fourier transform of the output response  $x(t)$ ;  $P_k(\omega)$  is the Fourier transform of the input excitation  $p(t)$ .

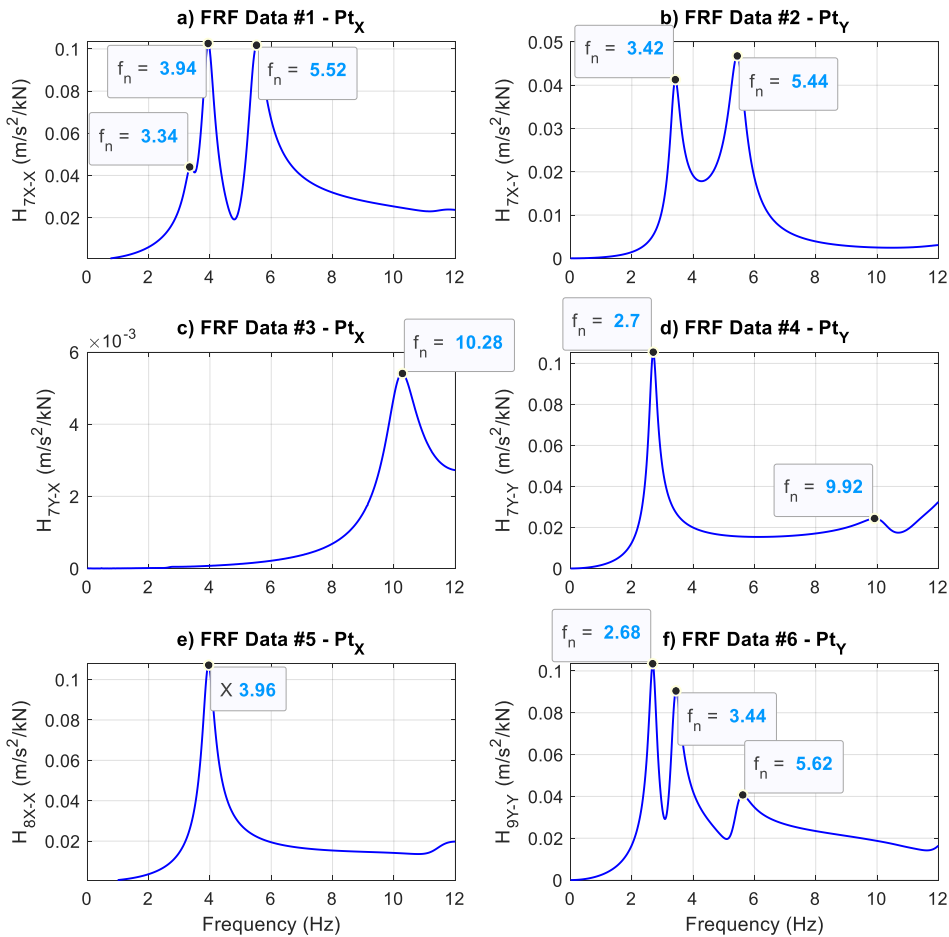


Figure 4. Simulated FRF measurements in the frequency domain and resonance peaks.

Based on the above time domain simulated measurement data, using the fast Fourier transform algorithm in Matlab software, the simulated measurements in the frequency domain are obtained, which are the FRFs shown in Figure 4. In which,

Figure 4a, 4c, 4e show the graph of FRF amplitudes of acceleration when the system is under load  $P_X(t)$  at nodes 7 along the X axis, node 7 along the Y axis, and node 8 along the X axis, respectively. Similarly, Figure 4b, 4d, 4f represent the graph of FRF amplitude of acceleration when the system is under load  $P_Y(t)$  at nodes 7 along the X axis, node 7 on the Y axis, and node 9 on the Y axis, respectively.

#### 4. The results of identification of natural frequencies of the offshore truss structure

##### 4.1. Identification of natural frequencies independently

Using the Peak-picking method in the above section, from the amplitude graph of the FRFs, the resonance peaks are separated and the individual vibrational frequencies are determined as shown in Table 2. The results show that each FRF measurement data can only be separated into 1 to 3 resonance peaks, respectively 1 to 3 natural frequencies. Specifically, each FRF data #3, 5 can only estimate 1 mode, each FRF data #1, 4 can estimate only 2 modes, and each FRF data #1, 6 can estimate 3 modes.

The comparison of the identified natural frequencies with the FE model is shown in Table 2. The error between the results of the identification of the natural frequencies and the FE model is quite small for FRFs with high resonance peaks. When the frequencies are close and the resonance peak amplitude is small, this error increases. Even the peak-picking method cannot estimate high-order natural frequencies (6<sup>th</sup> and 7<sup>th</sup> modes in FE model).

Table 2. Natural frequencies by peak-picking method (Hz)

Mode	By peak-picking method						By FE model
	Data #1	Data #2	Data #3	Data #4	Data #5	Data #6	
1				2.7		2.68	2.69
2	3.34	3.42				3.44	3.399
3	3.94				3.96		3.941
4	5.52	5.44			5.44	5.62	5.457
5			10.28	9.92			10.201
6							11.173
7							11.501

##### 4.2. Identification of natural frequencies of the offshore truss structure from each measurement data in the frequency domain

Using the RFP method to identify the natural frequencies of the system with the considered frequency range in the range (0 ÷ 12) Hz. Choose the largest possible total

number of modes in the above frequency range to be  $N = 10$  and polynomial degree  $m = 10$ . Using each FRF measurement to identify the vibrational modes. Figure 5 illustrates the FRF amplitude graph and the breakdown of recognized modes. In which, Figure 5a shows the measured FRF data #1 is broken down into 4 modes (2, 3, 4, 7). Similar to Figures 5b and 5c perform analysis of FRF data #2 which recognizes modes 2, 4, 6, 7 and FRF data #6 recognizes modes 1, 2, 4, 7. The identification results show that this method can estimate a larger number of modes than the peak-picking method, including high-order modes. However, using only each FRF measurement, it is still not possible to sufficiently estimate the 7 modes in the considered frequency range (0 ÷ 12) Hz.

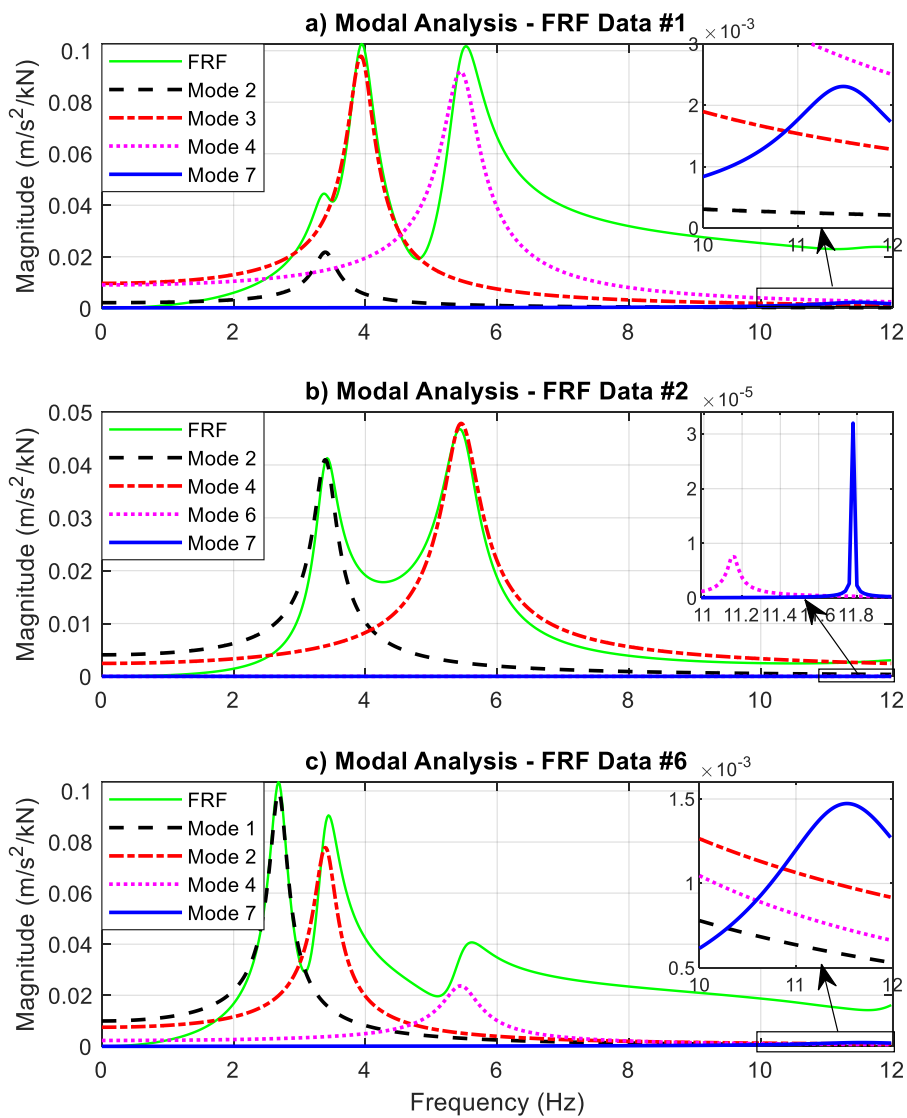


Figure 5. The FRF amplitude graph and the breakdown of recognized modes.

The identification results of natural frequencies by the RFP method from each FRF measurement data are compared with the FE model shown in Table 3. The error between the analytical and the measurement natural frequencies is quite small, even for some modes with small resonance peak amplitudes. However, for modes with resonance peak amplitudes close to zero, the RFP method cannot be identified (shown by the gray box in Table 3).

Table 3. Identification of natural frequencies from each measured data (Hz)

Mode	By RFP method						By FE model
	Data #1	Data #2	Data #3	Data #4	Data #5	Data #6	
1			2.764	2.687		2.687	2.690
2	3.395	3.395				3.396	3.399
3	3.937				3.937		3.941
4	5.451	5.451				5.451	5.457
5			10.195	10.192			10.201
6		11.152		11.157			11.173
7	11.483	11.778			11.48	11.504	11.501

4.2. Identification of natural frequencies from multiple combined measurements

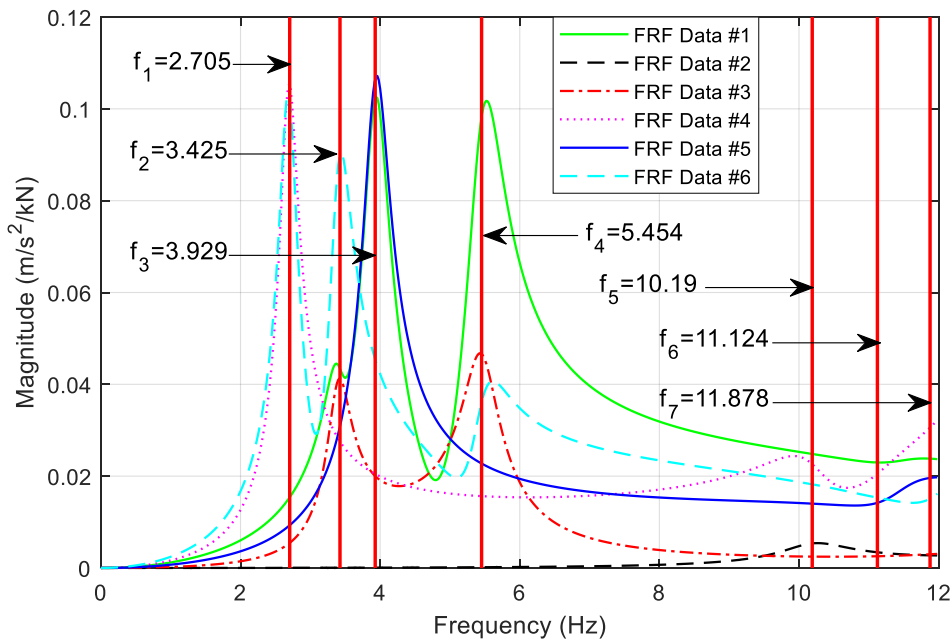


Figure 6. The identification results from multiple combined measurements.

Using a combination of all 6 measurements above to identify the natural frequencies of the system in the frequency range (0 ÷ 12) Hz. Choose the largest possible total number of modes in the above frequency range as  $N = 10$  and polynomial degree  $m = 10$ . The identification results when using the full frequency range (0 ÷ 12) Hz and combining 6 sets of measurement data are shown in Figure 6. In which, all 7 natural frequencies of the system have been identified. Errors compared with FE model are shown in Table 4, low frequencies (from mode 1 to mode 6) have a small identification error (less than 1%), and mode 7 has a recognition error is 3.28%.

Due to the great influence of low-order modes on the structure's response at high-order frequencies, to reduce the error of identification of higher-order modes, it is necessary to select an appropriate frequency range. Specifically, with the offshore truss structure, from the analysis above, there are two groups of separate frequencies, namely group of modes 1 ÷ 4 (within the frequency range 0 ÷ 8 Hz) and group of modes 5 ÷ 7 (in the frequency range 8 ÷ 12 Hz). Perform the identification procedure in the following sequence: (1) divide the initial frequency range into two frequency ranges (0 ÷ 8) Hz and (8 ÷ 12) Hz; (2) for each of the above frequency ranges, using a combination of 6 sets of measurement data to identify the modes of system in the selected frequency range; (3) combine the identification results in 2 frequency ranges to obtain the natural frequencies of the system. The identification results of the natural frequencies of the offshore truss structure according to this procedure are shown in Table 4. Therefore, the natural frequency identification errors of the modes are reduced to less than 1%, including the high-order modes.

Table 4. Identification of natural frequencies from multiple combined measurements

Mode	Use 1 frequency range		Use 2 frequency ranges		$f_n$ by FE model (Hz)
	$f_n$ (Hz)	Error (%)	$f_n$ (Hz)	Error (%)	
1	2.705	0.560	2.684	0.240	2.690
2	3.425	0.760	3.396	0.080	3.399
3	3.929	0.300	3.939	0.060	3.941
4	5.454	0.050	5.451	0.110	5.457
5	10.190	0.110	10.190	0.100	10.201
6	11.124	0.440	11.167	0.060	11.173
7	11.878	3.280	11.475	0.220	11.501

## 5. Conclusions

The article simulates the measurement data of the offshore truss structure in the time and frequency domain. Then, various methods are used to identify the natural frequencies of the structure from single or multiple measurements. The identification results show that, using single measurement data in frequency domain may not be able to sufficiently estimate the natural frequency of structure, especially the high-order modes. Using a combination of multiple measurements allows for more determination of the natural frequencies of the structure. The considered frequency range has an effect on the error of the identification methods. By subdividing the considered frequency range, it is possible to improve the accuracy of the obtained natural frequencies. The results can be applied to the identification of the actual structures from the experimental data.

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## NHẬN DẠNG TẦN SỐ DAO ĐỘNG RIÊNG CỦA KẾT CẤU NHÀ GIÀN TRÊN BIỂN TỪ NHIỀU SỐ LIỆU ĐO MÔ PHÒNG

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**Tóm tắt:** Bài báo trình bày các phương pháp nhận dạng tần số dao động riêng của kết cấu từ một hoặc nhiều số liệu đo trong miền tần số và ứng dụng nhận dạng các tần số dao động riêng của kết cấu nhà giàn. Số liệu đo mô phỏng dao động kết cấu trong miền thời gian được xác định bằng cách phân tích mô hình phần tử hữu hạn kết cấu nhà giàn chịu tải trọng động ngẫu nhiên sử dụng phần mềm SAP2000. Số liệu đo miền tần số thu được bằng biến đổi Fourier nhanh các số liệu đo miền thời gian sử dụng phần mềm Matlab. Phương pháp nhận dạng sử dụng nhiều số liệu đo hàm phản ứng tần số kết hợp có thể thu được nhiều tần số dao động riêng hơn sử dụng một số liệu đo. Một nghiên cứu về dải tần số xem xét cũng được thực hiện cho thấy độ chính xác của các phương pháp nhận dạng so với mô hình phần tử hữu hạn. Bằng cách chia nhỏ dải tần số xem xét, có thể nâng cao độ chính xác của các tần số dao động riêng thu được.

**Từ khóa:** Nhận dạng tần số dao động riêng; phương pháp tỉ số đa thức; kết cấu nhà giàn; phân tích dạng riêng; số liệu đo mô phỏng; miền tần số.

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