NAVIER SOLUTION FOR DEFLECTION AND STRESSES ANALYSIS OF FGM SANDWICH PLATE RESTING ON PASTERNAK ELASTIC FOUNDATION

Thanh Huan Duong[1], Vu Quan Le[1], Thanh Hai Nguyen[1], Cong Binh Dao[2,*], Viet Chinh Mai[2], Van Truong Nguyen[2]

[1]*Vietnam National University of Agriculture, Hanoi, Vietnam*

[2]*Institute of Techniques for Special Engineering, Le Quy Don Technical University, Hanoi, Vietnam*

Abstract

In this article, the first-order shear deformation theory (FSDT) and Navier's solution are used to establish analytical solutions for analyzing the deflection and stresses of FGM sandwich plate structures resting on a Pasternak elastic foundation. The top and bottom layers consist of functionally graded materials (P-FGM) that vary according to the exponential law along the thickness direction, while the core layer is composed of a porous material. The boundary condition of the plate is simply supported on all the edges, and the plate is subjected to a sinusoidal distributed load perpendicular to the mid-surface. The reliability of the model is validated through comparison with findings published in reputable journals. The effects of material parameters, geometric dimensions, foundation coefficients, and face-core-face thickness ratios on deflection and stress components of rectangular FGM sandwich plates are investigated in detail in parametric studies.

Keywords: *Navier's solution; deflection; stresses; sandwich plate; functionally graded materials (FGM); elastic foundation.*

1. Introduction

Functionally graded materials (FGMs) combine different materials to create composites with smoothly varying properties, preventing stress concentration and delamination. They capitalize on the thermal and corrosion resistance of ceramics and the ductility of metals, offering superior properties compared to homogeneous materials. In a sandwich structure, FGM sandwich (FGMSW) plates consist of high-strength face sheets for load-bearing and lightweight cores for structural support, stability, and insulation. FGMSW plates show promise for applications in environments with temperature fluctuations, offering stability, impact and abrasion resistance, and vibration damping. They find use in various constructions such as foundation piles, floor panels, soundproof walls, and ballistic shields for military installations.

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[*] Email: daocongbinh@lqdtu.edu.vn

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Currently, many renowned authors and research groups have focused on exploring the structure of FGMSW plates. These authors have applied various theories and computational methods to enhance understanding of their mechanical properties. Based on the first-order shear deformation theory, Yang et al. [1] conducted a nonlinear static analysis of FGMSW plates on an elastic foundation. Conversely, Kurpa and Shmatko [2] focused on analyzing the free vibration of FGMSW plates. Bessaim et al. [3] utilized higher-order shear deformation theory to investigate the deflection and stress of FGMSW plates. Mahi et al. [4] expanded the scope of research by employing fifth-order hyperbolic shear deformation theory, based on Navier solutions, to analyze FGMSW plates. Another approach was presented by Akavci, who used hyperbolic SDT to analyze the static, free vibration, and instability of FGMSW plates resting on an elastic foundation [5]. Singh et al. [6] investigated the dynamic response of FGMSW plates on a Pasternak foundation using high-order shear deformation theory.

In Vietnam, there are several notable studies such as Nguyen Thanh Tan [7], who analyzed the free vibration and stability of FGMSW plates using the first-order shear deformation theory. Nguyen Huu Khoi [8] analyzed the static behavior of FGMSW plates laying on an elastic foundation using the meshless method and compact higher-order shear deformation theory. Cao Huu Loi [9] analyzed the stability of FGMSW plates with porosities resting on an elastic foundation using the meshless method (MKI). Pham Anh Tu [10] analyzed the free vibration of FGMSW plates with porosities resting on an elastic foundation using the meshless method with moving kriging interpolation (MKI).

The overall study indicates that the mechanical behavior of sandwich plate structures made of materials with variable properties is an intriguing topic that attracts the research interest of scientists. However, to the best of the author's knowledge, there has not been much research on the analysis of deflection and stress of FGM sandwich plates where the core layer is a porous material, and the two surface layers are FGM materials resting on the Pasternak elastic foundation.

Therefore, the aim of this article is to establish the formulations, the main equations of the FGMSW plate resting on the Pasternak foundation based on the first-order shear deformation theory (FSDT). The Navier solution is utilized to determine the displacements and stresses of the FGMSW plate structure with simply supported boundary conditions on all four sides. The reliability of the model is validated through comparisons with the available published literature. The influence of material parameters, geometric dimensions, foundation coefficients and face sheet-core-face sheet thickness ratios on the deflection and stress components of the FGMSW plate is investigated and extensively discussed through specific numerical examples.

2. Formulation of the problem and material properties

2.1. The FGM sandwich plate resting on elastic foundation model

In this article, an FGM sandwich plate model is considered as shown in Fig. 1 with two FG face layers (h_f thickness) and an FG porous core (h_c thickness). The plate has length a, width b and total thickness $h = h_c + 2h_f$. According to the thickness direction, the layers are distinguished according to the coordinates $h_1 = -h/2$, $h_2 = -h_c/2$, $h_3 = h_c/2$ and $h_4 = h/2$. The plate is rested on the Pasternak elastic foundation with two coefficients K_w and K_{si}.

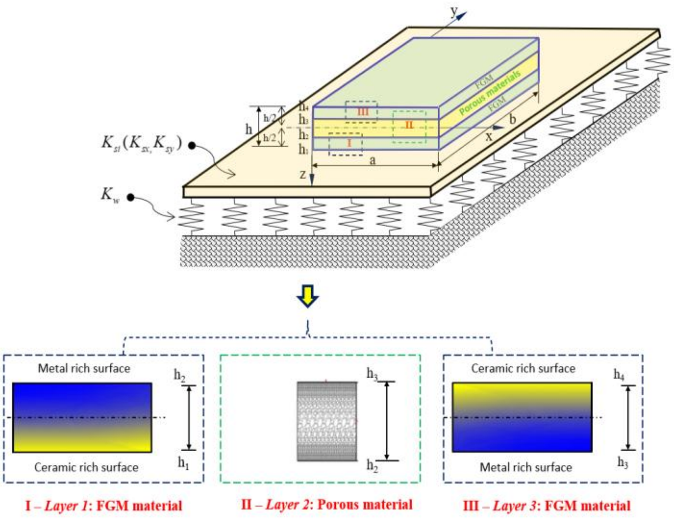


Fig. 1. FGM sandwich plate model resting on elastic foundation.

2.2. Material properties

The Young modulus of the two FGM surface layers is determined according to the following formula (1) [11] :

$$E(z) = E_m V_m + E_c V_c \tag{1}$$

in which E_c and E_m are the elastic modulus of the component materials, ceramic and metal, respectively.

With the assumption that the variation of material properties according to the power law function, the volume ratio of ceramic V_c and metal V_m is calculated as follows:

- *Layer 1* (top layer, $h_1 \le z \le h_2$):

$$V_c = \left(\frac{z - h_2}{h_1 - h_2}\right)^p ;\ V_m = 1 - V_c \tag{2}$$

- *Layer 3* (bottom layer, $h_3 \le z \le h_4$):

$$V_c = \left(\frac{z - h_4}{h_3 - h_4}\right)^p ;\ V_m = 1 - V_c \tag{3}$$

in which p is the volume fraction index and is a non-negative number.

- *Layer 2* (core layer, $h_2 \le z \le h_3$):

To ensure continuous material properties of the plate, the core layer is composed of porous material with a metal base similar to the two surface layers (Non-uniform porosity distribution - Symmetric). The elastic modulus of the core layer is determined as follows [12]:

$$E(z) = E_m \left[1 - e_0 \cos\left(\frac{\pi z}{h_3 - h_2} \right) \right] \tag{4}$$

where e_0 is porosity coefficient of the porous core.

3. Theoretical formulation

3.1. Kinematic relations

According to the first-order shear deformation theory, the displacement field is assumed as follows [13, 14]:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y); \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y); \\ w(x,y,z) &= w_0(x,y). \end{aligned} \tag{5}$$

where w_0, u_0, v_0 are the displacement components at a point on the mid-surface along the x, y, z directions, and θ_x, θ_y are the rotations of the normal to the mid-surface about the *y*, *x* axes, respectively.

The strain field is derived from the displacement field using the displacement - strain relationship in elasticity theory:

$$\begin{aligned} &\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + z\frac{\partial \theta_x}{\partial x};\ \varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} + z\frac{\partial \theta_y}{\partial y}; \\ &\gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right) + z\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right); \\ &\gamma_{xz} = 2\varepsilon_{xz} = \frac{\partial \mathrm{w}}{\partial x} + \frac{\partial u}{\partial z} = \frac{\partial \mathrm{w}_0}{\partial x} + \theta_x;\ \gamma_{yz} = 2\varepsilon_{yz} = \frac{\partial \mathrm{w}}{\partial y} + \frac{\partial v}{\partial z} = \frac{\partial \mathrm{w}_0}{\partial y} + \theta_y \end{aligned} \tag{6}$$

3.2. Constitutive equations

According to Hooke's law, the stress-strain relationship is defined by the following formula (7):

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{7}$$

where $C_{11} = \dfrac{E(z)}{1-v^2}$; $C_{12} = \dfrac{vE(z)}{1-v^2}$; $C_{44} = C_{55} = C_{66} = \dfrac{E(z)}{2(1+v)}$.

3.3. The force and moment resultants

Substitute the strain components from (6) into (7) and then integrate through the thickness of the plate to obtain the relationship between internal forces and strains as follows:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \\ Q_{xz} \\ Q_{yz} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & 0 & 0 \\ A_{12} & A_{11} & 0 & B_{12} & B_{11} & 0 & 0 & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & 0 & 0 \\ B_{12} & B_{11} & 0 & D_{12} & D_{11} & 0 & 0 & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa A_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa A_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{8}$$

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} C_{ij}\left(1, z, z^2\right) dz \tag{9}$$

where $\left(N_{xx}, N_{yy}, N_{xy}\right)$, $\left(M_{xx}, M_{yy}, M_{xy}\right)$ and $\left(Q_{xz}, Q_{yz}\right)$ are the in-plane force components, moment components and shear force components, respectively; $\kappa = 5/6$ is the shear correction factor; A_{ij}, B_{ij}, D_{ij} are the material stiffness matrices and are calculated according to the following formula (9).

3.4. Equations of motion in terms of displacements

Consider a FGM sandwich plate resting on the Pasternak elastic foundation and subjected to a transverse distributed load *q*(*x*,*y*). Sequentially investigating the static equilibrium conditions of the plate element in combination with Eq. (8), we obtain the following static equilibrium equations in terms of displacement [15]:

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}+A_{66}\frac{\partial^2 u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}+B_{11}\frac{\partial^2\theta_x}{\partial x^2}+B_{66}\frac{\partial^2\theta_x}{\partial y^2}+(B_{12}+B_{66})\frac{\partial^2\theta_y}{\partial x\partial y}=0 \quad (10a)$$

$$A_{11}\frac{\partial^2 v_0}{\partial y^2}+A_{66}\frac{\partial^2 v_0}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}+B_{11}\frac{\partial^2\theta_y}{\partial y^2}+B_{66}\frac{\partial^2\theta_y}{\partial x^2}+(B_{12}+B_{66})\frac{\partial^2\theta_x}{\partial x\partial y}=0 \quad (10b)$$

$$A_{44}(\frac{\partial^2 \mathrm{w}_0}{\partial x^2}+\frac{\partial\theta_x}{\partial x})+A_{55}(\frac{\partial^2 \mathrm{w}_0}{\partial y^2}+\frac{\partial\theta_y}{\partial y})+K_{\mathrm{w}}\mathrm{w}_0+K_s(\frac{\partial^2 \mathrm{w}_0}{\partial x^2}+\frac{\partial^2 \mathrm{w}_0}{\partial y^2})+q(x,y)=0 \quad (10c)$$

$$\begin{aligned}&B_{11}\frac{\partial^2 u_0}{\partial x^2}+B_{66}\frac{\partial^2 u_0}{\partial y^2}+(B_{12}+B_{66})\frac{\partial^2 v_0}{\partial x\partial y}+D_{11}\frac{\partial^2\theta_x}{\partial x^2}+D_{66}\frac{\partial^2\theta_x}{\partial y^2}-A_{44}\theta_x\\&+(D_{12}+D_{66})\frac{\partial^2\theta_y}{\partial x\partial y}-A_{44}\frac{\partial \mathrm{w}_0}{\partial x}=0\end{aligned} \quad (10d)$$

$$\begin{aligned}&B_{11}\frac{\partial^2 v_0}{\partial y^2}+B_{66}\frac{\partial^2 v_0}{\partial x^2}+(B_{12}+B_{66})\frac{\partial^2 u_0}{\partial x\partial y}+D_{11}\frac{\partial^2\theta_y}{\partial y^2}+D_{66}\frac{\partial^2\theta_y}{\partial x^2}-A_{55}\theta_y\\&+(D_{12}+D_{66})\frac{\partial^2\theta_x}{\partial x\partial y}-A_{55}\frac{\partial \mathrm{w}_0}{\partial y}=0\end{aligned} \quad (10e)$$

4. Analytical solutions

In this study, the simply supported FGMSW plate with boundary conditions expressed as (11) [16]:

$$\begin{aligned}&w_0(0,y)=0,\ w_0(a,y)=0,\ w_0(x,0)=0,\\&w_0(x,b)=0,\ \theta_y(0,y)=0,\ \theta_y(a,y)=0,\\&\theta_x(x,0)=0,\ \theta_x(x,b)=0,\end{aligned} \quad (11)$$

The displacement components, and the transverse distributed load are assumed to be in the form of a double Fourier series satisfying the boundary conditions (11), and can be given as in (12) [16]:

$$\begin{aligned}&\theta_y(0,y)=0,\ \theta_y(a,y)=0,\ \theta_x(x,0)=0,\\&\theta_x(x,b)=0,\ M_x(0,y)=0,\ M_x(a,y)=0,\\&M_y(x,0)=0,\ M_y(x,b)=0\end{aligned}$$

$$\begin{aligned}&u_0(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}u_{0mn}\cos\alpha x\sin\beta y\ ;\ v_0(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}v_{0mn}\sin\alpha x\cos\beta y\ ;\\&w_0(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}w_{0mn}\sin\alpha x\sin\beta y\ ;\ \theta_x(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\theta_{0xmn}\cos\alpha x\sin\beta y\\&\theta_y(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\theta_{0ymn}\sin\alpha x\cos\beta y\ ;\ q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}\sin\alpha x\sin\beta y\ .\end{aligned} \quad (12)$$

where $\alpha = {m\pi}/{a}; \beta = {n\pi}/{b}$;

$$q_{mn} = \frac{4}{ab}\int_0^a\int_0^b q(x,y)\sin\alpha x\sin\beta y dxdy = \begin{cases} q_0 & \text{for sinusoidally distributed load} \\ \dfrac{16q_0}{mn\pi^2} & \text{for uniformly distributed load} \end{cases}$$

Substituting the displacement components from (12) into the equilibrium equations in terms of displacements (10) and performing mathematical manipulation we obtain the following algebraic system of equations (13).

Solving the system of equations (13), we obtain the coefficients $u_{0mn}, v_{0mn}, w_{0mn}, \theta_{0xmn}, \theta_{0ymn}$. Then, the displacement components, strain components, and desired stress components can be calculated.

$$\begin{bmatrix} S_{11} & S_{12} & 0 & S_{14} & S_{15} \\ S_{21} & S_{22} & 0 & S_{24} & S_{25} \\ 0 & 0 & S_{33} & S_{34} & S_{35} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{bmatrix} \begin{Bmatrix} u_{0mn} \\ v_{0mn} \\ w_{0mn} \\ \theta_{0xmn} \\ \theta_{0ymn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_{mn} \\ 0 \\ 0 \end{Bmatrix} \tag{13}$$

5. Numerical results and discussion

In this section, after validating the accuracy of the solutions and computational programs, numerical examples are conducted to investigate the influence of material parameters, geometric dimensions, foundation coefficients and face sheet-core-face sheet thickness ratios on the deflection and stress of the plate. For convenience, the following nondimensionalizations are used in presenting the numerical results (14) [17]:

$$\begin{aligned} & K_0 = \frac{K_w a^4}{E_0 h^3};\ J_0 = \frac{K_{sx} a^2}{E_0 h^3} = \frac{K_{sy} b^2}{E_0 h^3}; \\ & D_0 = \frac{E_c h^3}{12(1-\nu)}; \\ & \bar{w} = \frac{10hE_0}{a^2 q_0} w\left(\frac{a}{2},\frac{b}{2}\right); \\ & \bar{\sigma}_{xx} = \frac{10h^2}{a^2 q_0}\sigma_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right); \\ & \bar{\sigma}_{xz} = \frac{h}{aq_0}\sigma_{xz}\left(0,\frac{b}{2},0\right); \end{aligned} \tag{14}$$

where the reference values are taken as E_0 = 1.0 GPa.

5.1. Validation study

Consider a square FGM sandwich plate [1-2-1] with a core layer made of isotropic material with $E = E_c = 151$ (GPa), $\nu = 0.3$, and a core thickness twice that of the surface layers; the surface layers (top and bottom layers) are made of FGM material with $E_m = 70$ (GPa), $E_c = 151$ (GPa), $\nu = 0.3$, and the plate is unsupported on an elastic foundation (with a foundation coefficient of zero). The geometric dimensions of the sandwich plate are: $a/h = 10$ and $a/b = 1$, subjected to sinusoidally distributed load with $q_0 = 10^4$ (Pa). Table 1 presents the calculated deflection and the stress components of the [1-2-1] sandwich plate with different volume fraction indices p and compared with the results proposed by A. M. Zenkour [17].

The validation results in Table 1 demonstrate a very high precise between the calculated deflection and stress of the FGM sandwich plate in this paper and the results reported by A. M. Zenkour [17], both utilizing the first-order shear deformation theory (FSDT). The discrepancy between the computational results of the two models seems negligible, indicating the reliability of the analytical solution and the computational program developed in the study. This also serves as the basis for the study to employ the program in conducting further investigation examples.

Table 1. Dimensionless deflection and stress of an FGM sandwich plate resting on elastic foundation subjected to uniformly distributed loading

p	**Model**	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{xz}$
0	A. M. Zenkour [17]	0.1961	1.9758	0.1910
	Present	**0.1961**	**1.9758**	**0.1910**
	Difference (%)	*0.00*	*0.00*	*0.00*
1	A. M. Zenkour [17]	0.2717	1.2810	0.2206
	Present	**0.2717**	**1.2810**	**0.2206**
	Difference (%)	*0.00*	*0.00*	*0.00*
2	A. M. Zenkour [17]	0.3037	1.4358	0.2326
	Present	**0.3037**	**1.4358**	**0.2326**
	Difference (%)	*0.00*	*0.00*	*0.00*
10	A. M. Zenkour [17]	0.3500	1.6584	0.2526
	Present	**0.3500**	**1.6587**	**0.2526**
	Difference (%)	*0.00*	*0.01*	*0.00*

5.2. Parametric study

In this section, the influence of the volume fraction index p of FGMs, porosity coefficient of the porous core e_0, the thickness-to-side ratio a/h, foundation parameters (K_0, J_0) and the face sheet-core-face sheet thickness ratios h_f/h_c on the deflection and stress components of the FGMSW plate are investigated and discussed. The considered plates are made of ceramic and metal constituents with the following mechanical properties: E_c = 151 GPa, E_m = 70 GPa, ν = 0.3. The stress and displacement response of the plates have been analyzed under sinusoidal loading with $q_0 = 10^4$ (Pa).

5.2.1. Effect of the volume fraction index p

Consider a FGM sandwich rectangular plate with side a, b, thickness ratio a/h = 20, aspect ratio a/b = 2, the plate has a configuration of [1-2-1], with a porosity coefficient of e_0 = 0.5 and it is rested on an elastic foundation with foundation coefficients K_0 = 100, J_0 = 100. The calculated deflection at points on the line $x = a/2$ for different volume fraction indices is presented in Fig. 3.

Furthermore, the influence of volume fraction index p on stress $\bar{\sigma}_{xx}$ and $\bar{\sigma}_{yy}$ at the midpoint of the plate ($x = a/2$, $y = b/2$) is also presented in Fig. 4. It can be observed that as the volume fraction index increases, the deflection of the plate also increases. Specifically, the minimum deflection of the plate occurs when p = 0, and the maximum deflection occurs when p = 10. This result is attributed to the fact that when p = 0, both surface layers of the plate are entirely made of ceramics, resulting in the highest stiffness of the plate and, consequently, the smallest deflection.

As the volume fraction index p gradually increases, the proportion of ceramics decreases while the proportion of metal increases, leading to a decrease in the stiffness of the plate and, consequently, an increase in the deflection. Figure 4 illustrates that the distribution of normal stress along the thickness direction of the plate is nonlinear and symmetric about the midplane. This result is consistent with the law of elastic modulus distribution within the plate.

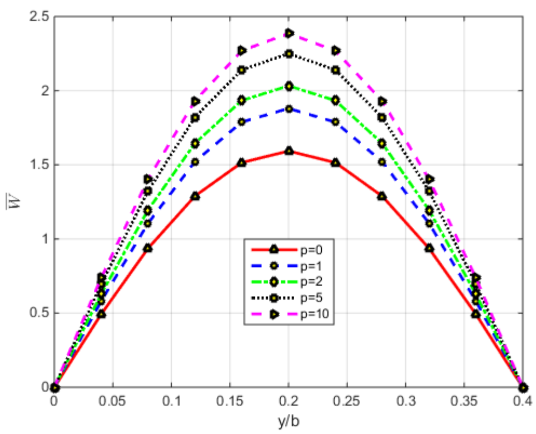
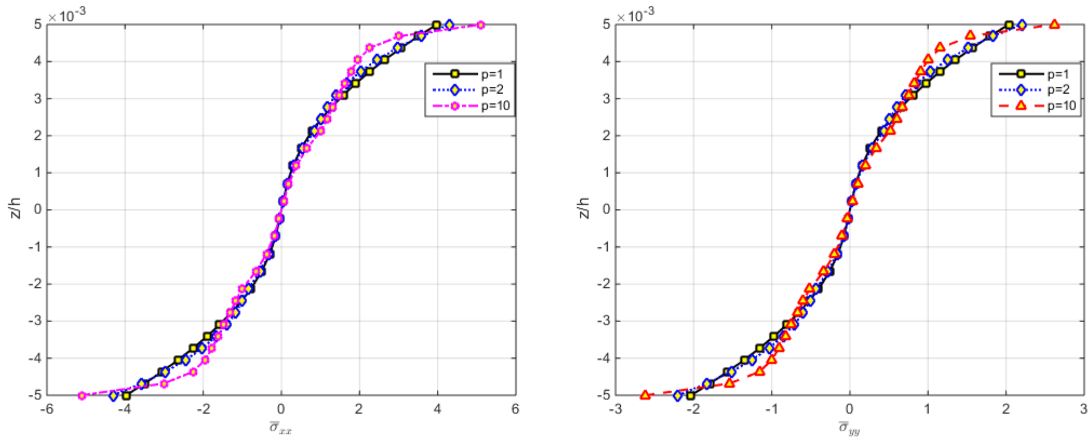


Fig. 3. Effect of volume fraction index p on deflection $\bar{w}$ of FGMSW plates.



a) $\bar{\sigma}_{xx}$ *b)* $\bar{\sigma}_{yy}$

Fig. 4. Effect of volume fraction index p on stress components.

5.2.2. Effect of porosity coefficient of the porous core e_0

The influence of the porosity coefficient on the maximum deflection of the FGMSW plate [1-2-1] is presented in Fig. 5. From the results in Fig. 5, it can be observed that as the porosity increases, the stiffness of the core layer decreases, consequently reducing the overall stiffness of the entire plate, resulting in an increase in the plate's deflection. Significantly, with the surveyed data in this example, when e_0 = 0.7, the deflection of the plate increases approximately threefold compared to when there is no porosity (e_0 = 0).

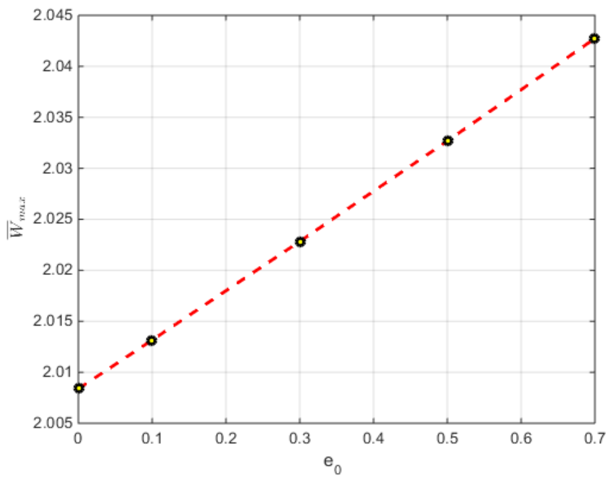


Fig. 5. Effect of porosity coefficient of the porous core e_0 *on FGMSW plate deflection.*

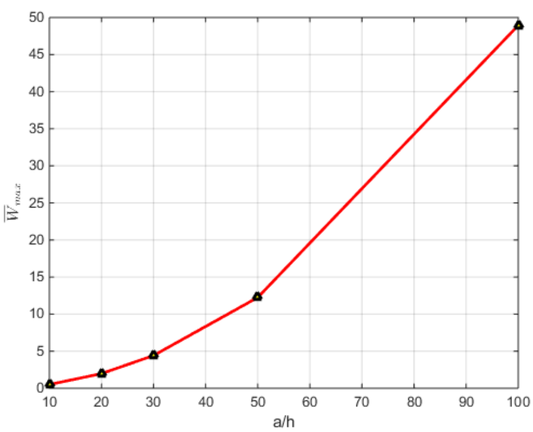
5.2.3. Effect of thickness-to-side ratio a/h

The effect of thickness ratio *a/h* on the deflection of FGMSW plate is presented in Fig. 6a. It is observed that the non-dimensional deflection of the FGM sandwich plate increases rapidly as the thickness ratio *a/h* increases. This can be explained by the fact that as the ratio *a/h* increases, the plate becomes thinner, consequently leading to a larger deflection of the plate. The results demonstrate that the deflection of the plate is very significant when the ratio a/*h* = 100 (corresponding to a thin plate).

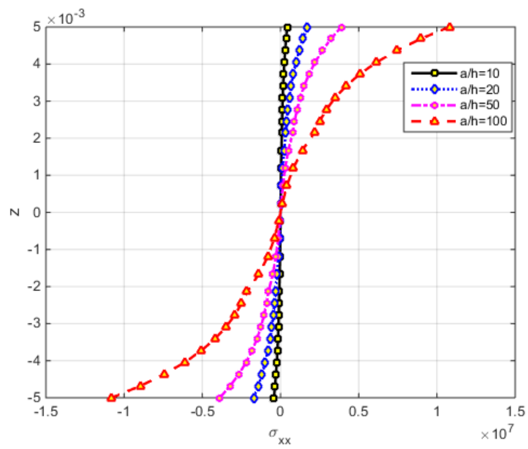
Figure 6b shows the variation of the stress component $\bar{\sigma}_{xx}$ with the ratio *a/h* of the plate. From the graph, it can be observed that the value of the stress component $\bar{\sigma}_{xx}$ increases with the ratio *a/h*. This result aligns with the general principles of mechanics, as the thickness ratio *a/h* increases, indicating a thinner plate, the load-bearing capacity of the plate decreases, leading to higher stress values within the plate.

5.2.4. Effect of foundation parameters

The effect of foundation parameters on deflection and stress of FGMSW plates are considered through two foundation coefficients K_0 and J_0, the results are presented in Fig. 7. It can be seen that the foundation stiffness significantly affects the deflection and stress of the plate. Specifically, as the foundation coefficients K_0 and J_0 increase, both the deflection and stress within the plate decrease. The impact of increasing the foundation coefficient K_0 is more significant compared to J_0.

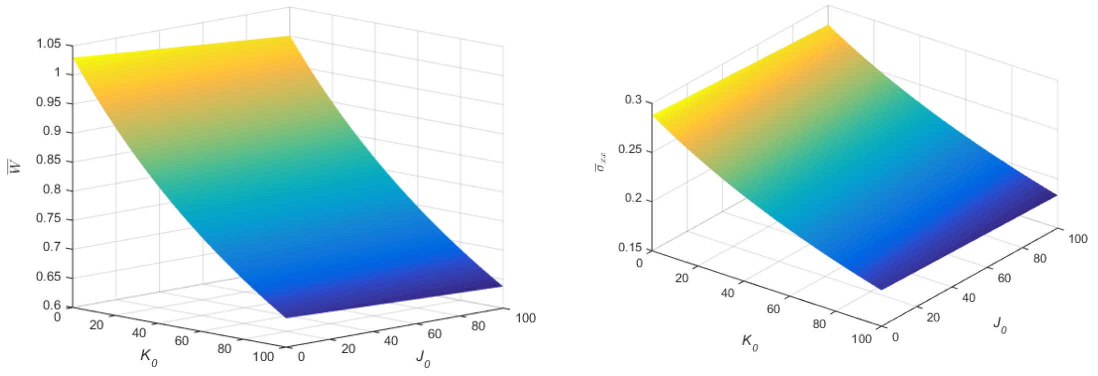


a) Maximum dimensionless deflection $\overline{w}$



b) Stress $\bar{\sigma}_{xx}$

Fig. 6. Variation of deflection and stress with a/h ratio.

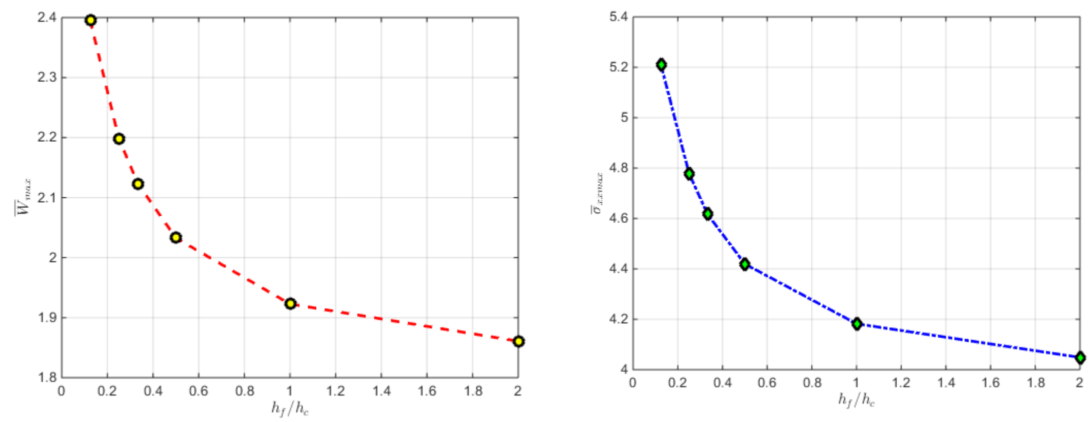


a) Dimensionless deflection $\bar{w}$ *b) Dimensionless stress* $\bar{\sigma}_{xx}$

Fig. 7. Effect of foundation stiffness on deflection and stress of the FGMSW.

5.2.5. Effect of the face sheet-core-face sheet thickness ratios h_f/h_c

Consider a square FGM sandwich plate resting on the Pasternak elastic foundation and subjected to uniformly distributed loading (with parameters such as plate dimensions, loading, and foundation coefficients similar to those in the examples above). The volume fraction indices of FGMs for layers 1 and 3 are $p = 2$, and the porosity coefficient of the core layer material is $e_0 = 0.5$. The influence of varying the ratio coefficient of surface layer thickness (h_f) to core layer thickness (h_c) on the dimensionless deflection and stress of the sandwich plate is presented in Fig. 8.



a) Effect of the face-core-face thickness ratios (h_f/h_c) on maximum deflection in sandwich plates *b) Effect of the face-core-face thickness ratios (h_f/h_c) on stress components* $\bar{\sigma}_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)$ *in sandwich plates*

Fig. 8. Effect of the face-core-face thickness ratios (h_f/h_c) on deflection and stress of FGMSW plates.

It shows that the face-core-face thickness ratios (h_f/h_c) have a significant influence on the dimensionless deflection and stress of the sandwich plate. Particularly, as the ratio h_f/h_c increases (indicating an increase in the thickness of the surface layers h_f), the deflection and stress within the sandwich plate decrease gradually. This study provides valuable insights into the design and arrangement of material layer thicknesses in sandwich plates to meet the desired static mechanical behavior of the plate structure.

6. Conclusion

The present article has given an analytical solution based on the first-order shear deformation plate theory (FSDT) to analyze the deflection and stress components of a simply supported FGM sandwich plate resting on the Pasternak elastic foundation. The reliability of the established theoretical model has been affirmed through validation example. On the other hand, several parametric study examples have been given to assess the influence of material parameters (volume fraction index p, porosity coefficient of the porous core e_0), geometric dimensions of the plate (ratio a/h), foundation stiffness (coefficients K_0, J_0), face-core-face thickness ratios (h_f/h_c)) on the deflection and stress of the FGM sandwich plate structure. The results of this study are anticipated to be a valuable reference for future research on the static mechanical behavior of sandwich plate structures resting on the Pasternak elastic foundation. The findings are expected to serve as a helpful guide for subsequent analyses in this field.

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SỬ DỤNG LỜI GIẢI NAVIER PHÂN TÍCH ĐỘ VÕNG VÀ ỨNG SUẤT KẾT CẤU TẤM SANDWICH FGM ĐẶT TRÊN NỀN ĐÀN HỒI PASTERNAK

Dương Thành Huân[1], Lê Vũ Quân[1], Nguyễn Thanh Hải[1],
Đào Công Bình[2], Mai Viết Chinh[2], Nguyễn Văn Trường[2]
[1]*Học viện Nông nghiệp Việt Nam, Hà Nội, Việt Nam*
[2]*Viện Kỹ thuật công trình đặc biệt, Trường Đại học Kỹ thuật Lê Quý Đôn, Hà Nội, Việt Nam*

Tóm tắt: Trong bài báo này, các tác giả sử dụng lý thuyết tấm bậc nhất FSDT và lời giải Navier để xây dựng mô hình tính toán độ võng và ứng suất kết cấu tấm sandwich có lớp lõi là vật liệu rỗng (porous material), lớp bề mặt là vật liệu có cơ tính biến thiên (FGM) đặt trên nền đàn hồi Pasternak. Tấm có liên kết khớp trên chu vi và chịu tải trọng phân bố hình sin vuông góc với mặt trung bình. Tính chính xác và độ tin cậy của mô hình được xác nhận qua việc so sánh kết quả tính toán với các kết quả đã công bố trên các tạp chí uy tín. Các kết quả số được thực hiện nhằm phân tích ảnh hưởng của tham số vật liệu, kích thước hình học, hệ số nền và tỉ lệ chiều dày các lớp vật liệu đến độ võng và các thành phần ứng suất của tấm sandwich FGM đặt trên nền đàn hồi.

Từ khóa: *Lời giải Navier; độ võng; ứng suất; tấm sandwich; vật liệu FGM; nền đàn hồi.*

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