STATIC BENDING RESPONSE OF COMPOSITE PLATES RESTING ON VARIABLE ELASTIC FOUNDATIONS

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Abstract

This article introduces a finite element approach for examining the static bending behavior of composite plates. The method considers the variable mechanical characteristics of plates and utilizes an elastic foundation with varying stiffness values. The plate's calculation expressions and equilibrium equations are derived using the new style shear deformation theory. The article uses a mesh composed of four-node rectangular components to solve the equilibrium equation. Each node in the mesh has six degrees of freedom. The solution's dependability and convergence are confirmed by comparing it with previously published findings. Based on this premise, the study presents numerical findings that examine how various geometric factors, materials, boundary conditions, and elastic foundations affect the static bending behavior of composite panels. This research is a great resource for engineers, providing excellent guidance for the design, production, and utilization of these structures in real-world applications.

Keywords: *Static bending; variable elastic foundation; shear deformation theory; finite element method.*

1. Introduction

Composite materials with changeable mechanical characteristics, known as Functionally Graded Materials (FGM), consist of ceramic and metal components. The volume ratio of each component gradually and constantly changes from one side to the other throughout the thickness of the structure. This method produces novel materials that exhibit enhanced characteristics in comparison to the original materials, particularly in terms of their capacity to endure substantial loads and elevated temperatures. As a result, structures composed of materials with varying mechanical qualities are used in the production of crucial components such as bomb shelter doors and reactor walls. Scientists from both domestic and foreign origins have shown significant interest in researching the mechanical behavior of constructions composed of materials with varying mechanical characteristics. Reddy [1] used analytical approaches and finite element methods to investigate the static and dynamic reactions of composite panels with temperature-induced variations in mechanical characteristics. Vinh et al. [2] used enhanced theory and the Navier form solution to determine the natural vibration frequency response of FGM

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DOI: 10.56651/lqdtu.jst.v7.n01.836.sce

panels. Dat and colleagues [3] used a four-plate element consisting of four nodes, each with nine degrees of freedom, together with a tendon element of three nodes, each with five degrees of freedom, to replicate the reaction. Analysis of the natural oscillation of ribbed plates composed of materials exhibiting varying mechanical characteristics. Duong and the research team [4] used analytical techniques and Laplace transform to investigate the impact of boundary conditions on the static bending behavior of composite shells enhanced by nanoparticles. The researchers Yu et al. [5] used extended isogeometric analysis to examine how crack faults affect the thermal stability response of FGM panels. Furthermore, the issue of distinct vibration and bending of FGM panels is also highlighted in [6, 7].

Existing literature indicates that the behavior of composite panels with varying mechanical characteristics on an elastic basis with different stiffness parameters is a research challenge that requires investigation. This article aims to address this issue. The solution to this issue is obtained by using the finite element approach.

2. Finite element formulations and solution methods

Consider a plate that has the model shown in Fig. 1. The plate has geometric parameters including length *a*, width *b*, and thickness *h*. The plate is placed in the Oxyz reference coordinate system. The plate rests on an elastic foundation with two coefficients, k_w and k_s.

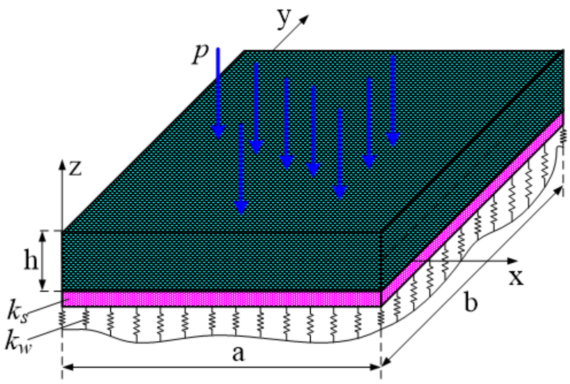


Fig. 1. Model of a composite plate resting on a variable elastic foundation.

Assuming the composite plate is made from two components, ceramic and metal, with corresponding volume ratios of V_c and V_m, the relationship between them is of the form [1]:

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n ; V_m = 1 - V_c \quad (1)$$

where *n* is a non-negative number, called the volume exponent, and subscripts *c* and *m* represent the ceramic and metal constituents, respectively.

The Poisson's ratio ν is assumed to be constant and Young's modulus E varies through the thickness with a power-law distribution [1]:

$$E(\mathrm{z}) = E_m V_m + E_c V_c = E_m + (E_c - E_m)\left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{2}$$

To establish calculation expressions, the article uses improved sinusoidal shear deformation theory [8, 9], the displacement field at any point of the plate is written as follows:

$$\begin{cases} u = -z\dfrac{\partial w_b}{\partial x} - f_z\dfrac{\partial w_s}{\partial x} \\ v = -z\dfrac{\partial w_b}{\partial y} - f_z\dfrac{\partial w_s}{\partial y} \\ w = w_b + w_s \end{cases} \tag{3}$$

wherer u, v, and w displacements along the axes Ox, Oy, and Oz; $f_z = z - g_z$, $g_z = h.\sin\dfrac{z}{h} - z.\cosh\dfrac{1}{2}$.

The normal strain and shear strain components have the form:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -z\dfrac{\partial^2 w_b}{\partial x^2} - f_z\dfrac{\partial^2 w_s}{\partial x^2} \\ -z\dfrac{\partial^2 w_b}{\partial y^2} - f_z\dfrac{\partial^2 w_s}{\partial y^2} \\ -z\dfrac{\partial^2 w_b}{\partial x\partial y} - 2f_z\dfrac{\partial^2 w_s}{\partial x\partial y} \end{Bmatrix} = z\underbrace{\begin{Bmatrix} \varepsilon_{bx} \\ \varepsilon_{by} \\ \gamma_{bxy} \end{Bmatrix}}_{\varepsilon_b} + f_z\underbrace{\begin{Bmatrix} \varepsilon_{sx} \\ \varepsilon_{sy} \\ \gamma_{sxy} \end{Bmatrix}}_{\varepsilon_s} \tag{4}$$

$$\boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \frac{\partial g_z}{\partial z}\begin{Bmatrix} \dfrac{\partial^2 w_s}{\partial x^2} \\ \dfrac{\partial^2 w_s}{\partial y^2} \end{Bmatrix} = \frac{\partial g_z}{\partial z}\boldsymbol{\gamma}_0 \tag{5}$$

The stress and strain relationship is calculated as:

$$\sigma = \frac{E(z)}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \mathrm{C}_b\varepsilon;$$

$$\tau = \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \frac{E(z)}{2(1+\nu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \gamma = C_s \gamma \tag{6}$$

where

$$C_b = \frac{E(z)}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}; \; C_s = \frac{E(z)}{2.(1+\nu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

The article uses a four-node rectangular element, each node has 6 degrees of freedom:

$$\boldsymbol{q}_e = \sum_{i=1}^{4} \left\{ w_{bi}, w_{si}, \frac{\partial w_{bi}}{\partial x}, \frac{\partial w_{si}}{\partial x}, \frac{\partial w_{bi}}{\partial y}, \frac{\partial w_{si}}{\partial y} \right\}^T \tag{8}$$

The two displacement components w_b and w_s are interpolated as follows:

$$\{w_b, w_s\} = \sum_{i=1}^{4} \left\{ R_i \{w_{bi}, w_{si}\} + R_{i+1} \left\{ \frac{\partial w_{bi}}{\partial x}, \frac{\partial w_{si}}{\partial x} \right\} + R_{i+2} \left\{ \frac{\partial w_{bi}}{\partial y}, \frac{\partial w_{si}}{\partial y} \right\} \right\} = \{\boldsymbol{R}_b, \boldsymbol{R}_s\} \boldsymbol{q}_e \tag{9}$$

where R_i are Hermitian functions.

The vectors in expressions (4) - (5) are rewritten as follows:

$$\boldsymbol{\varepsilon}_b = \boldsymbol{A}_1 \boldsymbol{q}_e; \; \boldsymbol{\varepsilon}_s = \boldsymbol{A}_2 \boldsymbol{q}_e; \; \boldsymbol{\gamma}_0 = \boldsymbol{A}_3 \boldsymbol{q}_e \tag{10}$$

where $\boldsymbol{A}_i$ (i = 1-5) are functional differential matrices. To establish the equilibrium equation, this work uses the principle of minimum potential energy. If the influence of the elastic foundation is taken into account, the expression for the variation strain energy of the plate element is:

$$\begin{aligned} \delta\Pi_e &= \int_{V_e} \left(\delta\varepsilon^T \sigma + \delta\gamma^T \tau \right) dV \\ &+ \int_{S_e} \begin{pmatrix} k_w \left(1 - c_n \sin\left(\frac{x}{a}\right)\right)\left(1 - c_n \sin\left(\frac{y}{b}\right)\right) \delta w_z w_z \\ + k_s \left(\frac{\partial w_z}{\partial x} \delta\left(\frac{\partial w_z}{\partial x}\right) + \frac{\partial w_z}{\partial y} \delta\left(\frac{\partial w_z}{\partial y}\right) \right) \end{pmatrix} dxdy \\ &= \delta q_e^T \left(K_e^{plate} + K_e^{found} \right) q_e \end{aligned} \tag{11}$$

where δ is the variation operator, k_w and k_s represent the stiffness characteristics of the elastic foundation, whereas c_n is a coefficient that indicates the change in stiffness of the elastic foundation in the plane of the plate. The stiffness matrix $\boldsymbol{K}_e^{plate}$, $\boldsymbol{K}_e^{found}$ represent the combined stiffness of the composite plate element and the elastic foundation acting on the plate element.

$$\boldsymbol{K}_e^{plate} = \int_{S_e} \int_{-h/2}^{h/2} \left(\boldsymbol{A}_1^T \boldsymbol{C}_b z \boldsymbol{A}_1 + \boldsymbol{A}_1^T \boldsymbol{C}_b z f_z \boldsymbol{A}_2 + \boldsymbol{A}_2^T \boldsymbol{C}_b z f_z \boldsymbol{A}_1 + \boldsymbol{A}_2^T \boldsymbol{C}_b f_z^2 \boldsymbol{A}_2 + \boldsymbol{A}_3^T \boldsymbol{C}_s g_z^2 \boldsymbol{A}_3 \right) dz dx dy \quad (12)$$

$$\boldsymbol{K}_e^{found} = \int_{S_e} \left(\begin{array}{l} k_w \left(1 - c_n \sin\left(\frac{x}{a}\right)\right)\left(1 - c_n \sin\left(\frac{y}{b}\right)\right)\left(R_b + R_s\right)^T \left(R_b + R_s\right) \\ + k_s \left(\frac{\partial \left(R_b + R_s\right)^T}{\partial x} \frac{\partial \left(R_b + R_s\right)}{\partial x} + \frac{\partial \left(R_b + R_s\right)^T}{\partial y} \frac{\partial \left(R_b + R_s\right)}{\partial y} \right) \end{array} \right) dx dy \quad (13)$$

When a plate element is exposed to a load that is perpendicular to the neutral plane of the element, the virtual work of the external force can be determined using the following calculation:

$$\delta W_e = \int_{S_e} \delta \left(w_b + w_s\right)^T \boldsymbol{P}_S dS = \delta \boldsymbol{q}_e^T \int_{S_e} \left(\boldsymbol{R}_b + \boldsymbol{R}_s\right)^T \boldsymbol{P}_S dS = \delta \boldsymbol{q}_e^T \boldsymbol{F}_e \quad (14)$$

where $\boldsymbol{F}_e$ is the external force vector acting on the plate element. From expressions (11), (14), the static equilibrium equation of plate has the form:

$$\sum_e \left(\boldsymbol{K}_e^{plate} + \boldsymbol{K}_e^{found} \right) \boldsymbol{q}_e = \sum_e \boldsymbol{F}_e \quad (15)$$

where the element nodal displacement vector: $\boldsymbol{q}_e = \sum_{i=1}^{4} \left\{ w_{bi}, w_{si}, \frac{\partial w_{bi}}{\partial x}, \frac{\partial w_{si}}{\partial x}, \frac{\partial w_{bi}}{\partial y}, \frac{\partial w_{si}}{\partial y} \right\}$.

Solving equation (15), we will obtain the bending displacement of the plate under the effect of static load. If the edge of the plate is clamped (denoted C), then all degrees of freedom on that edge are 0. If one edge has a simple support (denoted S), it is only necessary to constrain the two degrees of freedom on that edge to be equal to 0 ($w_{bi} = w_{si} = 0$). If the plate has the symbol CCCC, all edges are clamped; if the plate has the symbol SSSS, all edges are subjected to fully simple support.

3. Verification examples

The square plate rests on a two-coefficient elastic foundation, the length of the plate edges *a* = *b*, and the plate thickness *h* = *a*/10, *a*/20. The plate is under the load distributed according to the sinusoidal law $p = P_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$ where P_0 is the amplitude of the load. Material parameters include an elastic modulus of 117 GPa and Poisson's coefficient of 0.33. The two stiffness parameters of the elastic foundation are normalized as follows $K_w^* = \frac{k_w a^4}{A_0}$, $K_s^* = \frac{k_s a^2}{A_0}$ with $A_0 = \frac{Eh^3}{12\left(1 - \nu^2\right)}$ and *n* = 0.

At each plate position, the *x* and *y* coordinates are known. By substituting this coordinate into the load function, one will determine the value of the load. The comparison parameter is the dimensionless maximum displacement computed using the given formula $c_v = \frac{10^2 A_0}{P_0 a^4} w\left(\frac{a}{2}, \frac{b}{2}\right)$. Table 1 presents a comparison between the maximum displacement of the support plate on the elastic foundation and the findings reported in [10, 11]. It is evident that when the mesh is divided into smaller components, the calculation results converge towards the analytical values mentioned in [10, 11]. The article will use a mesh consisting of 196 elements, with 14 elements on each side, to ensure the appropriate level of precision for calculating the subsequent findings.

Table 1. Comparison results of the maximum displacement of a square plate resting on a two-coefficient foundation subjected to sinusoidal loads

(K_w^*, K_s^*)	[10]	[11]	**Present paper**				
			16 elements	**64 elements**	**100 elements**	**196 elements**	**256 elements**
$h = a/10$							
(100,0)	0.212	0.213	0.227	0.216	0.215	0.214	0.214
(0,100)	0.042	0.042	0.045	0.043	0.042	0.042	0.042
(100,100)	0.040	0.040	0.043	0.041	0.041	0.041	0.040
$h = a/20$							
(100,0)	0.206	0.206	0.219	0.209	0.208	0.207	0.207
(0,100)	0.042	0.042	0.043	0.042	0.042	0.042	0.042
(100,100)	0.040	0.040	0.042	0.040	0.040	0.040	0.040

4. Numerical results

Consider a composite plate with variable mechanical properties affected by a uniformly distributed load of intensity P_0 = 0.05 MPa, plate thickness $h = a/10$ to $a/30$. The plate edges are a = 0.5 m and b = 0.5a-2a, and the material properties are E_c = 151 GPa; E_m = 70 GPa; $\nu_m = \nu_c$ = 0.3. The elastic foundation has three parameters k_w, k_s, and c_n with two standardized stiffness parameters as follows $c_w = \frac{12 k_w a^4 \left(1-\nu_m^2\right)}{E_m h_0^3}$, $c_s = \frac{12 k_s a^2 \left(1-\nu_m^2\right)}{E_m h_0^3}$ with $h_0 = a/10$. The parameter calculated and investigated is the displacement along the line $y = x$ normalized as follows $D_p = \frac{10^3 E_m h_0^3}{P_0 a^4 \left(1-\nu_m^2\right)} w.$

The volume exponent index, denoted as n, steadily rises from 0 to 10. The displacement of the plate along the straight-line $y = x$ is calculated and the results are shown in Fig. 2. As the value of n grows, the fraction of metal components in the composite plate also increases, resulting in a reduction in the rigidity of the plate. Consequently, the plate's maximum displacement rises, and the position of the plate's maximum value is not centered owing to the impact of the elastic foundation.

The change in the value of parameter c_n yields different outcomes when computing the displacement along the linear path $y = x$ of the composite plate. These findings are shown in Fig. 3. The maximum value of D_p corresponding to different cases of c_n is shown in Tables 2-3. It is evident that when the parameter c_n is increased, the foundation's stiffness reduces, resulting in a reduction in the overall stiffness of the structure. Consequently, this leads to an increase in the maximum displacement of the plate. Figure 4 displays the plate's deflection in relation to various values of the parameter c_n. These findings indicate that the parameter c_n has an impact on the site where displacement occurs most often. These findings demonstrate that the parameter c_n has an impact on both the maximum displacement value and the distribution of displacement in the plate. Furthermore, it is seen that when the c_n parameter increases, the impact of this parameter on the static displacement distribution of the composite panel gets more pronounced.

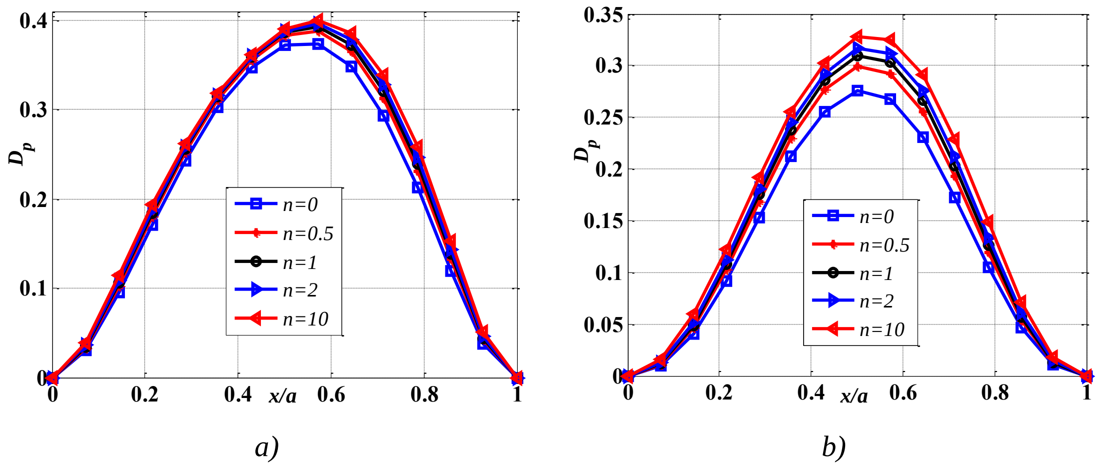
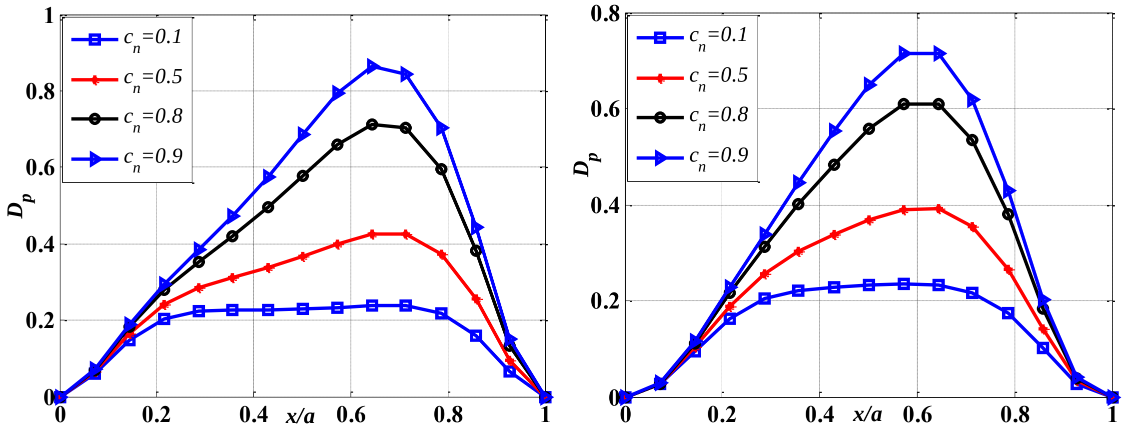


Fig. 2. The displacement D_p along the line $y = x$ depends on n,

$c_w = 5000$, $c_s = 10$, $c_n = 0.5$, $a/b = 1$, $a/h = 10$: (a) SSSS plate; (b) CCCC plate.



a) *b)*

Fig. 3. The displacement D_p along the line y = x depends on c_n,

c_w = 5000, c_s = 10, a/h = 20, a/b = 1, n = 0.5: (a) SSSS plate; (b) CCCC plate.

Table 2. Maximum value of D_p for different values of c_n, SSSS

a/h	***c_n***			
	0.1	0.5	0.8	0.9
10	0.224	0.387	0.554	0.628
20	0.235	0.425	0.713	0.862
30	0.236	0.438	0.771	0.949
40	0.237	0.435	0.782	0.974

Table 3. Maximum value of D_p for different values of c_n, CCCC

a/h	***c_n***			
	0.1	0.5	0.8	0.9
10	0.217	0.299	0.382	0.415
20	0.232	0.391	0.610	0.715
30	0.234	0.410	0.685	0.828
40	0.236	0.422	0.729	0.889

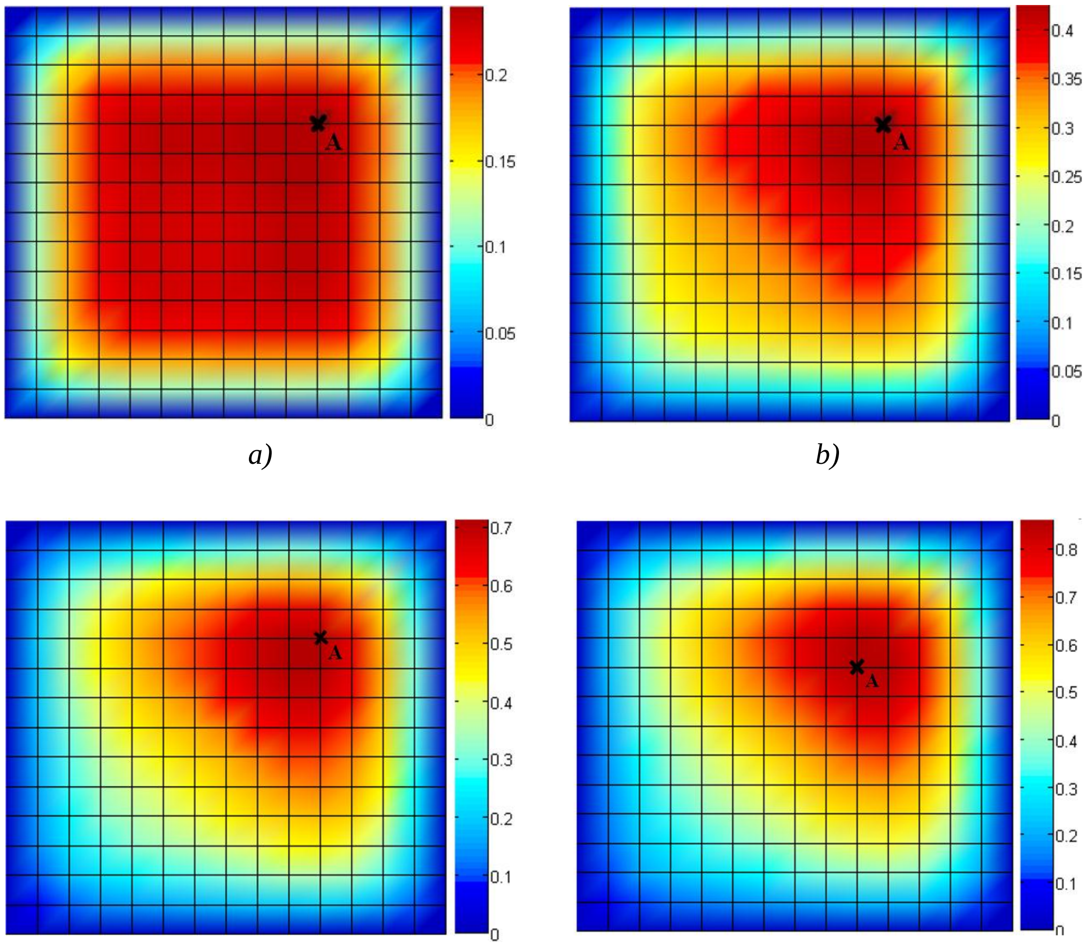
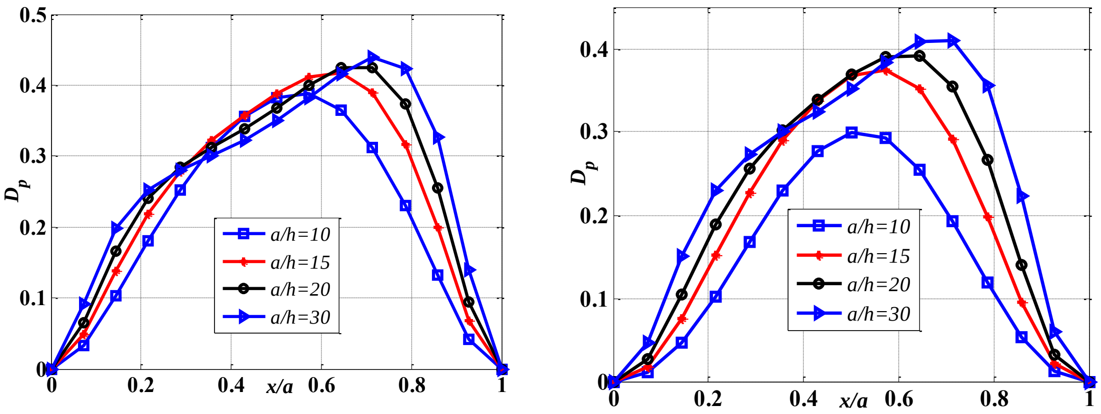


Fig. 4. Displacement D_p of the plate in the plane Oxy,

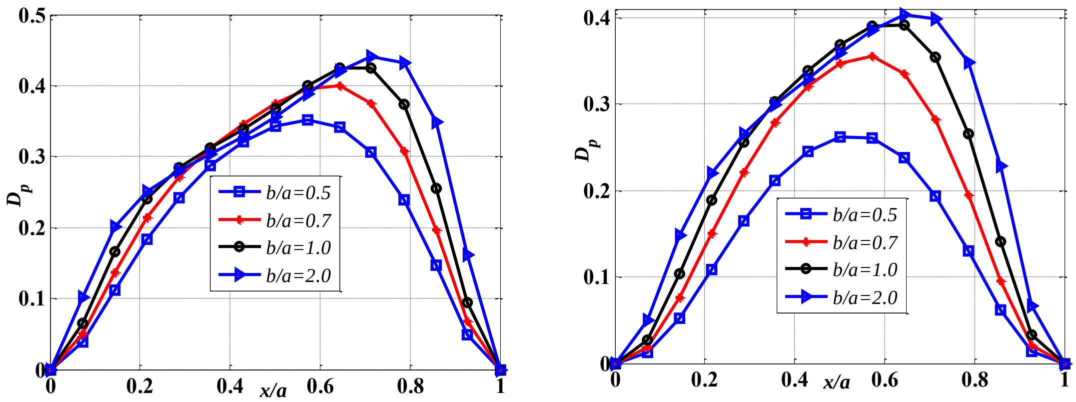
SSSS plate, A is the point with the highest displacement, the value is given in Table 2,
$c_w = 5000$, $c_s = 10$, $a/h = 20$, $a/b = 1$, $n = 0.5$: (a) $c_n = 0.1$; (b) $c_n = 0.5$; (c) $c_n = 0.8$; (d) $c_n = 0.9$.

Figure 5 illustrates the results of the displacement calculation along the $y = x$ line for varying plate thicknesses. Figure 6 shows the displacement calculation results when the ratio of the length of the two sides of the plate is altered. The calculation findings indicate that as the composite plate becomes thinner and the b/a ratio increases, the stiffness of the plate diminishes, leading to an increase in the maximum displacement of the plate. Simultaneously, the elastic basis exerts an impact on the form of the plate's displacement response, resulting in a distinct difference.



a) b)

Fig. 5. The displacement D_p along the line y = x depends on the ratio a/h, c_n = 0.7, c_w = 5000, c_s = 10, a/b = 1, n = 0.5: (a) SSSS plate; (b) CCCC plate.



a) b)

Fig. 6. The displacement D_p along the line y = x depends on the ratio b/a, c_w = 5000, c_s = 10, c_n = 0.5, a/h = 20, n = 0.5: (a) SSSS plate; (b) CCCC plate.

5. Conclusions

The study performed research on the issue of static bending in composite plates with varying mechanical characteristics. This was achieved by using a combination of a novel shear deformation theory and the finite element method. The plates were analyzed on an elastic foundation with variable stiffness. A plate composed of a four-node element, with each node having six degrees of freedom, was analyzed using an equilibrium equation. The equation was solved to determine the static bending displacement of the composite plate. The article has shown the convergence and dependability of the calculation theory by comparing it with published findings. The study additionally performed numerical analyses to determine the impact of certain structural factors, elastic foundation, and boundary conditions on the static bending behavior of composite panels with varying mechanical characteristics. The findings of this research serve as helpful guidelines for the practical design and production of composite structures.

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NGHIÊN CỨU ĐÁP ỨNG UỐN TĨNH CỦA TẤM COMPOSITE TRÊN NỀN ĐÀN HỒI BIẾN ĐỔI

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Tóm tắt: Bài báo trình bày phương pháp mô phỏng hữu hạn để phân tích đáp ứng uốn tĩnh của tấm composite có cơ tính biến thiên tựa trên nền đàn hồi có tham số độ cứng biến đổi. Các biểu thức tính toán và phương trình cân bằng của tấm được thiết lập dựa trên lý thuyết biến dạng cắt kiểu mới, để giải phương trình cân bằng, bài báo sử dụng lưới chia gồm các phần tử chữ nhật bốn nút, mỗi nút có sáu bậc tự do. Sự tin cậy và tính hội tụ của lời giải được kiểm chứng thông qua so sánh với các kết quả đã được công bố. Trên cơ sở đó, bài báo đưa ra một số kết quả số để khảo sát ảnh hưởng của vài tham số hình học, vật liệu, điều kiện biên, nền đàn hồi đến đáp ứng uốn tĩnh của tấm composite. Nghiên cứu này là tài liệu tham khảo có giá trị cho các kỹ sư, phục vụ có hiệu quả khi thiết kế, chế tạo, sử dụng các kết cấu này trong thực tiễn.

Từ khóa: *Uốn tĩnh; nền đàn hồi biến đổi; lý thuyết biến dạng cắt; phương pháp phần tử hữu hạn.*

Received: 08/04/2024; Revised: 18/06/2024; Accepted for publication: 28/06/2024

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