ON A NONLINEAR LANCHESTER COMBAT MODEL OF NETWORK CENTRIC WARFARE TYPE AND AN ANALYSIS OF OPTIMAL FIRE ALLOCATIONS

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Abstract

In this work, we introduce a nonlinear Lanchester model of Network centric warfare (NCW)-type and study a problem of finding the optimal fire allocation for this model. A Blue party B will fight against a Red party consisting of A and R, where A is an independent force and R fights with supports from a supply unit N. Optimal fire allocation will then be sought in the form of piece-wise constant functions so that the remaining force of B is as large as possible. For this model, we also introduce a notion of *threatening rates* which are computed for A, R, N at each stage of the battle. These rates will then be used to derive the optimal fire allocation for B. Several numerical experiments are presented to justify the theoretical findings.

Index terms

Nonlinear Lanchester model, Network centric warfare, fire allocation, optimal problem.

1. Introduction

In 1916, Lanchester [1] introduced a mathematical model for a battle in the form of a system of differential equations. This model has been extended and generalized in many ways, such as guerilla model by Deitchman [2], guerilla model with intelligence by Schaffer [3] and Schreiber [4], counter terrorism model by Kaplan, Kress and Szechtman (KKS) [5]–[9]. There are several problems involving these models, among of which are the problems of optimal fire allocation with number of troops being objective function. This problem has been investigated in various scenarios by Taylor [10], Lin and Mackay [11]. In these works, the role of military supply, however, has not been studied thoroughly. In a combat, the victory of either party is not only decided by the armed forces but also by their supply units. In many historical battles, firepower was

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not only aimed at the direct rivals but also their supply units [12].

In 2017, Kim and his colleagues [13] considered a Lanchester's model where Blue force B fights against Red force R supported by a supply unit N. This model can be denoted by $(B$ *vs* $(R, N))$. Kim considered fire allocations in the form of piece-wise constant functions and derived optimal fire allocation so that number of B's troop is always at its possible maximum. In this paper, we extend Kim's model by considering a model of $(B$ *vs* $\{(R, N), A\})$ where A stands for an independent force. For this model, we also consider the problem of finding optimal fire allocation of B so that its remaining troop at any time is maximal. In Lanchester's model using system of differential equations, the decreasing rate of troops of a force is computed by attrition rate of its rival force multiplied by the rival's number of troops. In our model, attrition rate of R is assumed to be a linear function of the number of N and this supply unit can also be attritted by B. The resulting model is of non-linear Lanchester type. Let us recall that in classical non-linear Lanchester's models, the supply units have not been taken into account but only the armed forces.

In any battle of the type "one against many", the strategy will be intuitively derived as follows: focus all firepower to the entity possessing the "greatest threat". In order to quantify "threats" posed by the entities R, N, A to B, we introduce the notion of "threatening rates" which are computed for each entity R, N, A by involving entities number of troops, their attrition rates against B and B's attrition rates. Fire allocation for B is represented by three non-negative factors whose sum equals to one. This distribution is used to express the strategy of B during the conflict and represented as piece-wise constant function of time. Thus, the fire allocation of B is assumed to be constant for a certain period, which we call "stage". During a certain stage, the fire allocation is kept constant, and the battle moves to a new stage if one (or more) entity is eliminated. The first stage is the period from the beginning of the battle to the instant when one entity gets extinguished - its number of troops reaches zero. The battle moves to the second stage after this instant. In this paper, we limit ourselves in the optimal fire allocation of the first stage. This choice of fire allocation is meaningful since in planning phase of a battle, this choice simplifies the logistic operations. Moreover, capturing the states of the battlefield and altering the fire allocation accordingly is not an easy task. By using the threatening rates, we justify the intuitive strategy mentioned above. Thus, the optimal fire allocation for B is focusing all its firepower to the entity which possesses the greatest threatening rate. Several numerical examples are included to illustrate the theoretical findings.

The rest of the paper is organized as follows. Section 2 is devoted to introduce our model and to investigate the optimization problem for this model. Numerical experiments are presented in Section 3 to illustrate the theoretical results. Conclusion and some possible further developments are discussed in the last section.

2. Main results

2.1. Non-linear Lanchester model of mixed NCW type

Let us consider a battle where B fights against $((R, N), A)$. Our model is called non linear Lanchester of NCW type. NCW stands for "Network Centric Warfare", which is a novel notion of modern warfare. For this notion, the reader is referred to [14]–[17]. A simple diagram for our model is represented in Fig. 1.

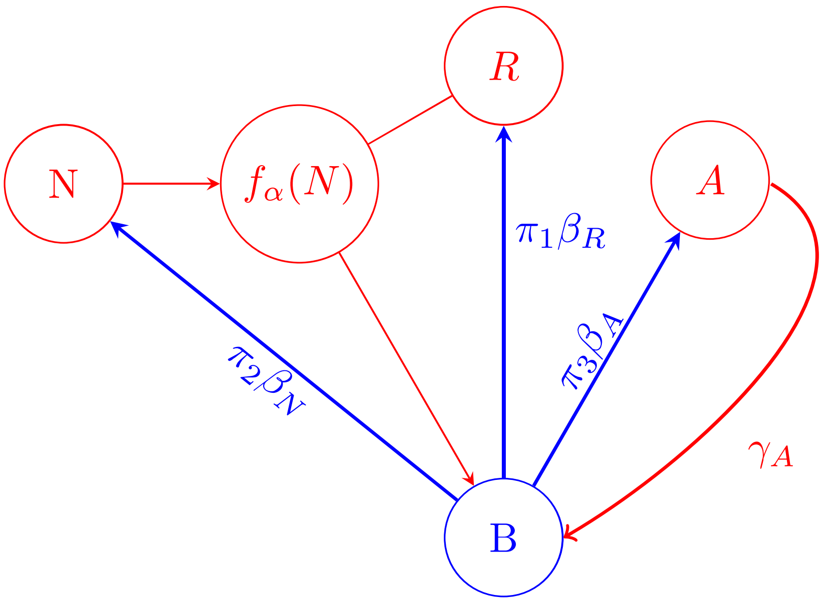

Fig. 1. Diagram of the model.

Before formulating the model, let us denote:

- by β_R attrition rate of B against R,
- by β_A attrition rate of B against A,
- by β_N attrition rate of B against N,
- $f_\alpha(N)$ attrition function of N complementing R^i to B,
- $\Pi = (\pi_1, \pi_2, \pi_3)$ fire allocation of B against R,N,A, respectively,
- α_c^N fully-connected attrition rate of R against B,
- α_d^N disconnected attrition rate of R against B $(\alpha_c^N \leq \alpha_c^N)$,
- and by γ_A attrition rate of A against B.

The fire allocation of B is sought in the set $\{\pi_1, \pi_2, \pi_3 \in [0; 1] : \pi_1 + \pi_2 + \pi_3 = 1\}$. The attrition function of R against B is assumed to be a linear function of N:

$$f_\alpha(N) = \alpha_d^N + \left(\alpha_c^N - \alpha_d^N\right) \frac{N}{N_0}, \tag{1}$$

where N_0 is the initial number of troops of N. It is easily seen from (1) that when $N = N_0$, $f_\alpha(N) = \alpha_c^N$, in other words, R and N are fully connected. When N is totally eliminated by B, $N=0$, R and N are totally disconnected and $f_\alpha(N)=\alpha_d^N$. Our model is a system of differential equations defined as follows:

$$\begin{cases} \dfrac{dB}{dt} = -\left[\alpha_d^N + \left(\alpha_c^N - \alpha_d^N\right)\dfrac{N}{N_0}\right]R - \gamma_A A, \\ \dfrac{dR}{dt} = -\pi_1\beta_R B, \\ \dfrac{dN}{dt} = -\pi_2\beta_N B, \\ \dfrac{dA}{dt} = -\pi_3\beta_A B. \end{cases} \tag{2}$$

2.2. Optimal fire allocation

For our model (2), we consider the problem of maximizing the remaining troops of B at an arbitrary t. Let us introduce the notion of "threatening rates", which are denoted by b_1, b_2, b_3, and computed as follows:

$$\begin{cases} b_1 = \alpha_c^N \beta_R, \\ b_2 = \dfrac{\beta_N\left(\alpha_c^N - \alpha_d^N\right)R_0}{N_0}, \\ b_3 = \gamma_A\beta_A. \end{cases} \tag{3}$$

These quantities represent the corresponding "threats" possessed by R, N, A. Invoking these rates, the optimal fire allocation of B is derived in the following theorem.
Theorem 1. Among all the fire allocations of the form $\Pi = \{\pi_1, \pi_2, \pi_3\}$, $\pi_i's : i = 1,2,3$ are constants, $\pi_1+\pi_2+\pi_3 = 1$ for the first stage, the optimal one for B is as follows:

$$\Pi^* = \begin{cases} (1,0,0) & \text{if} \quad (b_1 > b_2) \wedge (b_1 > b_3), \\ (0,1,0) & \text{if} \quad (b_2 > b_1 > b_3) \vee (b_2 > b_3 > b_1), \\ (0,0,1) & \text{if} \quad (b_3 > b_1 > b_2) \vee (b_3 > b_2 > b_1). \end{cases} \tag{4}$$

Proof: Let $X(t) = \int\limits_0^t B(s)\,ds \Rightarrow X'(t) = B(t)$. We then have:

$$X''(t) = B'(t) = -\left[\alpha_d^N + \left(\alpha_c^N - \alpha_d^N\right)\frac{N}{N_0}\right]R - \gamma_A A. \tag{5}$$

It follows that:

$$\int_0^t dR = -\int_0^t \pi_1\beta_R B(s)\,ds \Rightarrow R(t) - R(0) = -\pi_1\beta_R X(t).$$

From this, we obtain:

$$R = -\pi_1\beta_R X(t) + R_0. \tag{6}$$

By analogous computations, we get:

$$\begin{aligned} N &= -\pi_2\beta_N X(t) + N_0, && (7) \\ A &= -\pi_3\beta_A X(t) + A_0. && (8) \end{aligned}$$

Substituting R, N, A in (6), (7), (8) into (5) yields

$$X''(t) = -C_1 X^2(t) + C_2 X(t) - C_3, \quad (9)$$

where:

$$\begin{aligned} C_1 &= \frac{\pi_1\pi_2\beta_R\beta_N\left(\alpha_c^N - \alpha_d^N\right)}{N_0}, \\ C_2 &= \frac{\pi_2\beta_N\left(\alpha_c^N - \alpha_d^N\right)R_0 + \pi_1\beta_R\alpha_c^N N_0}{N_0} + \gamma_A\pi_3\beta_A, \\ C_3 &= \alpha_c^N R_0 + \gamma_A A_0. \end{aligned}$$

Multiplying both sides of (9) by $dX'(t)$ and integrating both sides, we obtain:

$$\begin{aligned} X'(t) &= B(t) \\ &= \sqrt{-\frac{2}{3}C_1X^3(t) + C_2X^2(t) - 2C_3X(t) + C_4}. \end{aligned}$$

It can be seen that C_1, C_2, C_3 are nonnegative, C_4 is an integral constant.
Since C_3 is constant, in order to maximize $B(t)$, we will seek for condition for which C_1 is minimal as well as C_2 is maximal (if possible). We then need to solve the following multi-objective optimization problem:

$$\begin{cases} \min\limits_{\Pi} C_1, \\ \max\limits_{\Pi} C_2. \end{cases}$$

Let us denote:

$$\begin{aligned} a &= \frac{\beta_R\beta_N\left(\alpha_c^N - \alpha_d^N\right)}{N_0}, \\ b_1 &= \alpha_c^N\beta_R, \\ b_2 &= \frac{\beta_N\left(\alpha_c^N - \alpha_d^N\right)R_0}{N_0}, \\ b_3 &= \gamma_A\beta_A. \end{aligned}$$

The problem now becomes:

$$\begin{cases} \min axy \\ \max\left(b_1x + b_2y + b_3z\right) \end{cases} \text{s.t.} \begin{cases} 0 \le x, y, z \le 1, \\ x + y + z = 1. \end{cases} \quad (10)$$

By weighting method (see [18, Section 3.1]) and setting

$$F_\lambda(x, y, z) = \lambda(axy) - (1-\lambda)(b_1x + b_2y + b_3z),$$

we now obtain a new problem:

$$\min F_\lambda(x,y,z) \text{ s.t. } \begin{cases} 0 \le x,y,z \le 1, \\ x+y+z=1, \\ 0 \le \lambda \le 1. \end{cases}$$

Substituting $x = 1-y-z$, we now get the problem:

$$\min\{\lambda(1-y-z)ay-(1-\lambda)(b_1+(b_2-b_1)y+(b_3-b_1)z)\}$$
$$\text{s.t. } \begin{cases} y,z \ge 0, \\ y+z \le 1, \\ 0 \le \lambda \le 1. \end{cases}$$

Let us consider the following five cases:

1) If $b_2 \ge b_3 \ge b_1$, since $\lambda(1-y-z)ay \ge 0$, it follows:

$$\begin{aligned} \min F_\lambda &\ge -(1-\lambda)(b_1+(b_2-b_1)y+(b_3-b_1)z) \\ &\ge -(1-\lambda)(b_1+(b_2-b_1)(y+z)) \\ &\ge -(1-\lambda)b_2 = F(0,1,0). \end{aligned}$$

2) If $b_2 \ge b_1 \ge b_3$, the problem becomes:

$$\min\{\lambda(1-y-z)ay+(1-\lambda)(b_1-b_3)z-\ (1-\lambda)(b_1+(b_2-b_1)y)\}.$$

Therefore:

$$\min F_\lambda \ge -(1-\lambda)b_2 = F(0,1,0).$$

3) If $b_3 \ge b_2 \ge b_1$, it follows that:

$$\begin{aligned} \min F_\lambda &\ge -(1-\lambda)(b_1+(b_3-b_1)(y+z)) \\ &\ge -(1-\lambda)b_3 = F_\lambda(0,0,1). \end{aligned}$$

4) If $b_3 \ge b_1 \ge b_2$, the problem is now:

$$\min\{\lambda(1-y-z)ay+(1-\lambda)(b_1-b_2)y-\ (1-\lambda)(b_1+(b_3-b_1)z)\}.$$

We obtain that:

$$\min F_\lambda \ge -(1-\lambda)b_3 = F(0,0,1).$$

5) If $b_2 \le b_1$, $b_3 \le b_1$, the problem turns out to be:

$$\min\{\lambda(1-y-z)ay+(1-\lambda)(b_1-b_2)y+(1-\lambda)(b_1-b_3)z-(1-\lambda)b_1\}$$

and, it follows that:

$$\min F_\lambda \ge -(1-\lambda)b_1 = F_\lambda(1,0,0).$$

The proof is now complete. ■

In principle, the battle can be divided into three stages. In the first stage, due to the computed threatening rates b_1, b_2, b_3 the Blue force B will focus all its firepower to of the entities R, N, A. When one of the entities is eliminated, the second stage begins. When one of the remaining two forces is extinguished, the third stage follows. However, if R and A are out of the picture by the end of the second stage, B will be no longer attrited and the battle finishes. As stated above, for the first stage, our results apply. For the second stage, one may use the results by Donghyun Kim et. al. [13] or by Lin and Mackay [11]. Thus, by the first case in our proof, in order that $B(t)$ is always maximal, for the first stage B should focus its firepower to N. After the conclusion of N, the second stage begins, where B concentrates its firepower on R or A due to result of Lin and Mackay. The second case in our proof is explained analogously.

By the third case in our proof, B will focus its firepower on A. The second stage in this case has been considered in [13]. The fourth case is analyzed similarly as the third case. Strategy in the fifth case can be explained as follows: for the first stage, B concentrates on R. B then turns its firepower to A for the second stage. And the battle ends when A is totally annihilated.

3. Numerical experiments

3.1. Case 1

Let us consider equation (2) with coefficients given by Table 1 together with the following initial conditions:

$$B_0 = 170; R_0 = 120;\; N_0 = 20;\; A_0 = 50.$$

Threatening rates are thus computed as: $b_1 = 0.2;\quad b_2 = 0.45;\quad b_3 = 0.04.$

Table 1. Parameters for Case 1

α_c^N	α_d^N	γ_A	β_R	β_N	β_A
0.4	0.15	0.2	0.5	0.3	0.2

It is obvious that $b_2 > b_1 > b_3$, so the optimal fire allocation for the first stage is given by $(0, 1, 0)$. By using results of Lin and Mackay, the optimal strategy for the whole battle is given by:

$$\Pi^* = (0,1,0) \to (1,0,0) \to (0,0,1)\,.$$

This strategy should be interpreted as follows: for the first stage, B will focus all its firepower to N. For the second stage, B will concentrate on R since $b_1 > b_2$. To contrast with the optimal strategy, we use $\Pi_1 = (1,0,0) \to (0,0,1)$. This strategy is explained as: B will focus all its firepower on R in the first stage; for the second stage, it will concentrate on A. The simulation results show that B still win the battle with this strategy. However, its number of troops at any instant is always smaller than itself using the optimal one. Amounts of troops of B during the combats using both strategies are represented in Fig. 2.

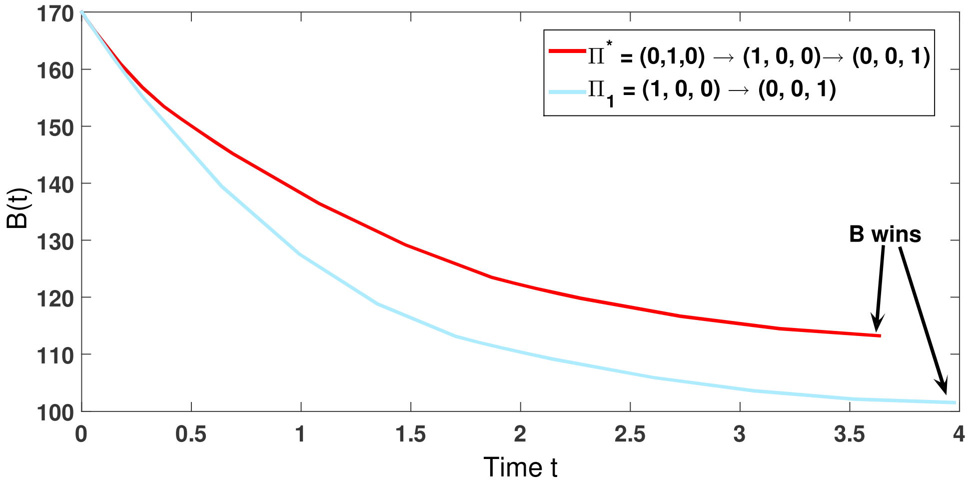

Fig. 2. Processes of battle in Case 1.

3.2. Case 2

We now will investigate model (2) with parameters given by Table 2 together with these initial conditions:

$$B_0 = 170,\ R_0 = 120,\ N_0 = 50,\ A_0 = 50.$$

Threatening rates are thus computed as: $b_1 = 0.2$; $b_2 = 0.12$; $b_3 = 0.04$.

Table 2. Parameters for Case 2

α_c^N	α_d^N	γ_A	β_R	β_N	β_A
0.4	0.15	0.2	0.5	0.2	0.2

Since $b_2 > b_1 > b_3$, the optimal fire allocation for the first stage is $\Pi^* = (1, 0, 0)$. Among three strategies we chose to compare, only $\Pi_3 = (0.7, 0.2, 0.1) \to (0, 0, 1)$ leads to B's victory. However, the number of troops of B is again smaller than one resulted from the optimal strategy. The other two strategies even result in B's failure. Processes and endings of the simulated battles are depicted in Fig. 3.

3.3. Case 3

Now we consider the model with parameters chosen in Table 3 together with initial conditions:

$$B_0 = 170; R_0 = 120;\ N_0 = 60;\ A_0 = 50.$$

The threatening rates are $b_1 = 0.2$; $b_2 = 0.08$; $b_3 = 0.3$. The optimal fire allocation is therefore

$$\Pi^* = (0, 0, 1) \to (1, 0, 0)\,.$$

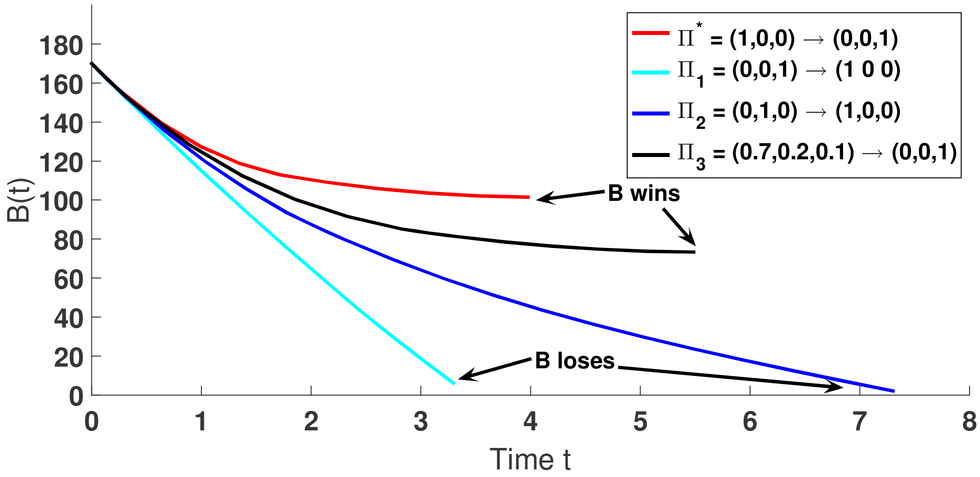

Fig. 3. Results for Case 2.

Table 3. Parameters for Case 3

α_c^N	α_d^N	γ_A	β_R	β_N	β_A
0.4	0.2	0.6	0.5	0.2	0.5

The two contrasting strategies are $\Pi_1 = (1, 0, 0) \to (0, 0, 1)$; $\Pi_2 = (0, 1, 0)$. By Π_1, B wins with more troops lost while Π_2 leads to B's failure. Processes of the battles are given in Fig. 4.

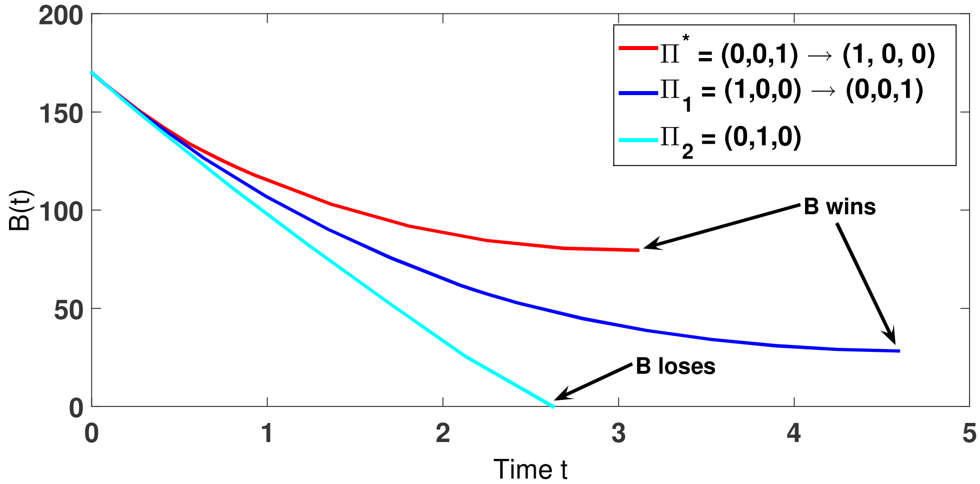

Fig. 4. Results for Case 3.

4. Conclusion

A nonlinear Lanchester model of mixed NCW type has been introduced together with the notion of *threatening rates*. Invoking these rates, threats exposed by rivals have been quantified and these quantities help derive the optimal strategy for Blue force, which decreases the losing of its own troops. Numerical results support the theoretical findings.

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VỀ MỘT MÔ HÌNH TRẬN ĐÁNH LANCHESTER PHI TUYẾN KIỂU NETWORK CENTRIC WARFARE VÀ MỘT PHÂN TÍCH CHO PHÂN BỐ HỎA LỰC TỐI ƯU

Vũ Anh Mỹ, Nguyễn Hồng Nam, Tạ Ngọc Ánh, Nguyễn Thị Hạnh Lê

Tóm tắt

Trong bài báo này, chúng tôi giới thiệu một mô hình Lanchester phi tuyến kiểu Network centric warfare và nghiên cứu bài toán tìm phân bố hỏa lực tối ưu cho mô hình này. Một phe Xanh B chiến đấu với phe Đỏ bao gồm hai lực lượng A và R, trong đó A là một lực lượng tác chiến độc lập còn R thì chiến đấu với sự hỗ trợ của một lực lượng hỗ trợ N. Phân bố hỏa lực tối ưu sẽ được tìm dưới dạng hàm hằng từng khúc sao cho quân số còn lại của B là lớn nhất có thể. Với mô hình này, chúng tôi cũng đưa ra khái niệm *hệ số đe dọa* được tính cho A, R, N tại mỗi giai đoạn của trận đánh. Những hệ số này sẽ được dùng để đưa ra phân bố hỏa lực tối ưu cho B. Một số ví dụ số minh họa được đưa ra để kiểm chứng các kết quả lý thuyết.

Từ khóa

Mô hình Lanchester phi tuyến, chiến tranh mạng trung tâm, phân bố hỏa lực, bài toán tối ưu.