



## **Fraud Pentagon Theory for Detecting Financial Statement Fraudulent at Companies Listed on Vietnam's Stock Market**

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### **Abstract**

*Using the fraudulent pentagon theory of Crowe (2011), this study aims to assess the factors affecting financial statement fraud, namely: (i) pressure may change due to financial stability, (ii) opportunity due to changes in auditors, (iii) rationalization due to auditors' opinions, (iv) capability to be promoted by the duality of CEO and chairman of the board and (v) arrogance by the number of CEOs in the last 3 years. The data used in the study are 88 fraudulent and non-fraudulent companies listed on HOSE 2019. The data is analyzed using Binary logistic regression. Research results show that pressure and rationalization influence financial reporting fraud, while opportunity, capability and arrogance do not.*

**Keywords:** Financial statement, Financial statement fraudulent, Fraud pentagon theory, Vietnam.

## Introduction

In today's global landscape, companies face various challenges such as globalization, rapid technological advancements, swift industrial and business development, as well as complexities in information and data management (Siregar and Tenoyo, 2015). These changes have significantly increased the incidence of financial reporting fraud that companies have to contend with (Han, 2017). To address this issue, researchers have recognized the need for proactive methods to detect and prevent financial reporting fraud, actively minimizing the inherent risks of financial reporting fraud (Wu and Huang, 2013).

In Vietnam, the discrepancy rate in companies' financial statements before and after auditing is notably high. This disparity has been observed in companies listed on the Vietnam stock market (VSM). Taking profit margin as an illustrative example, according to statistics from Vietstock database: (i) From 2012 to the first half of 2015, the adjustment rate of audited net profit was consistently above 70% annually, with the first half of 2015 alone accounting for over 52%; (ii) During the period from 2014 to 2018, the listed construction companies on the VSM reported significantly higher pre-audit profits compared to post-audit profits, with nearly 50% difference. However, this ratio has shown a decreasing trend in subsequent years (47% in 2016, 46% in 2017, and 42% in 2018); (iii) Each company declaring higher profits is also one of the common practices of "beautifying" financial statements. A notable study conducted at the Vietnam Oil and Gas Group's Construction Corporation revealed that the highest error amount in 2016 was VND 65.433 billion. Consequently, the difference in post-audit profits reduced by 65.433 billion VND compared to the initial financial statements, which amounted to VND 157.156 billion (Bui *et al.*, 2021); (iv) According to statistics, the number of entities auditors refused to express opinions on was 38 cases in 2020 and 27 cases in 2021. Companies refused audit opinions will be mandatory delisted; and (v) Vietstock Finance's statistics on Ho Chi Minh City Stock Exchange (HOSE), Hanoi Stock Exchange (HNX), UPCoM up to April 6, 2023, showed a total of 447 non-financial companies.

According to Gullkvist and Jokipii (2013), fraud is intentional deceptive behavior, including financial reporting fraud and embezzlement. A typical form of financial reporting fraud is asset misappropriation, which is crucial as it has real implications, affecting the value and credibility of the company. Therefore, financial reporting fraud is a serious issue. According to Elliot and Willingham (1980), financial reporting fraud is intentional fraudulent behavior perpetrated by management that damages investors and creditors by materially misstating information in financial reports. This action significantly affects the interests of shareholders in particular, the company's value, and the users of financial reports in general, undermining their trust in the stock market. One theory commonly used to evaluate financial reporting fraud is the fraud triangle theory proposed by Cressy in 1953.

Over time, the fraud triangle theory has evolved. In 2004, Wolfe and Hermanson proposed the fraud diamond theory based on the development from the fraud triangle theory by adding a factor believed to significantly influence fraud, which is capability. The

development did not stop there. In 2011, Crowe further refined the theory proposed by Cressy (1953), Crowe (2011) observed that another factor, arrogance, also contributes to the occurrence of financial reporting fraud. The study conducted by Crowe (2011) incorporated the fraud triangle theory as well as capability proposed by Wolfe and Hermanson (2004), thus Crowe's (2011) fraud assessment model includes five indices: pressure, opportunity, attitude, capability, and arrogance. This theory was presented in 2011 and named Crowe's fraud pentagon theory.

## **1. Research methodology**

### ***1.1. Theoretical framework***

According to Priantara (2013), financial reporting fraud is a report or statement that is not truthful or is made carelessly without recognizing or caring whether the report is accurate, affecting users of the report causing them harm. Priantara (2013) also defines financial reporting fraud according to ACFE as a deliberate misrepresentation or presentation of an organization's financials. It can be achieved through intentional errors or omissions of values or disclosures in financial reports to deceive users of financial reports.

#### ***1.1.1. Research theory and explanation of fraudulent behavior***

**Fraud Triangle Theory:** The concept of the fraud triangle originates from research conducted by Cressey (1953), who proposed that fraud could be triggered by three factors: motive/pressure, opportunity, and rationalization. These factors serve as violations of trust, specifically the trust or confidence placed in individuals who then either refuse or violate that trust or belief. Cressey pointed out that this leads perpetrators to lose their resistance to wrongdoing.

**Fraud Diamond Theory:** The concept of the fraud diamond theory is an evolution from the fraud triangle theory, where the fourth factor that promotes fraud, capability, was developed. Wolfe and Hermanson (2004) argue that, in financial reporting fraud schemes, even with any opportunity, pressure, and rationalization, perpetrators must have the ability to recognize and exploit it, thus reducing the likelihood of fraud when capability is lacking.

**Fraud Pentagon Theory:** Crowe (2011) proposed that the fraud pentagon is an adaptation of the fraud triangle and fraud diamond theories, where there is a fifth factor, arrogance. Crowe (2011) argues that there is a fifth factor in the fraud pentagon, as the factors in the fraud triangle and fraud diamond cannot be applied in all situations. With the fifth factor, the fraud pentagon can be used in all situations to detect financial reporting fraud.

#### ***1.1.2. Components of the fraud pentagon***

The first component of the fraud pentagon is Pressure. Pressure is related to the motive leading to financial reporting fraud behavior (Fitriyah and Novita, 2021). Every financial reporting fraud perpetrator must face some type of pressure to engage in fraudulent behavior (Feldman and Halali, 2019). Words become very important related to pressure because pressure is not always a specific action. When perpetrators believe they are under pressure, that belief can lead to financial reporting fraud.

Rationalization refers to a declaration that financial reporting fraud behavior is not a crime. If perpetrators cannot rationalize financial reporting fraud, there is no possibility of committing fraudulent behavior (Shi et al., 2017).

Capability is a characteristic or skill necessary for someone when engaging in financial reporting fraud behavior (Abdullahi and Mansor, 2015). Perpetrators recognize opportunities and then use their capabilities to turn them into reality. Job position, intelligence, ego, pressure, and stress are factors that support capability. Not everyone has the motive, opportunity, and awareness of financial reporting fraud behavior due to insufficient capability to perform or conceal. Capability is a very important factor when it comes to financial reporting fraud.

Research conducted by the Treadway Commission's sponsoring organizations showed that 70% of fraud cases occur due to pressure, greed, and 89% of fraud cases are related to executive directors (Yulianti et al., 2019). The number of executive directors shown in the annual financial reports indicates the level of exclusivity and excellence of the executive directors. Arrogance is a condition where managers with their exclusivity will not comply with internal controls within the company for their position and status reasons (Nindito, 2018).

Research conducted by Umar *et al.* (2020), Handoko and Natasya (2019) shows that pressure influences financial reporting fraud, while research by Wahyuningrum and Iswajuni (2020), Yulianti *et al.* (2019) shows the opposite. Research conducted by Utomo *et al.* (2019), Fitriyah and Novita (2021) indicates that opportunity affects financial reporting fraud, whereas research by Yulianti *et al.* (2019), Indarto and Ghazali (2016) provides evidence that opportunity does not affect financial reporting fraud. Research by Umar *et al.* (2020) shows that rationalization affects financial reporting fraud, while Irwandi *et al.* (2019) show that rationalization does not affect financial reporting fraud. Research by Umar *et al.* (2020) provides evidence that capability affects financial reporting fraud, while Handoko and Natasya (2019), Yulianti *et al.* (2019) present contrasting results and show that arrogance does not affect financial reporting fraud.

### **1.2. Research data**

The data were collected from the financial reports of companies listed on the Ho Chi Minh City Stock Exchange (HOSE) in 2019, excluding financial institutions such as commercial banks, investment funds, financial companies, and insurance companies. The reason is that these financial institutions, in addition to complying with securities laws, also operate under banking regulations and other related laws. Additionally, the financial ratios of companies operating in these fields are very large due to the specific nature of their operations (Nguyen Tien Hung and Vo Hong Duc, 2017).

The research sample consists of fraudulent and non-fraudulent companies (control sample). The study uses a model to determine the level of profit discrepancy based on comparing audited financial statements with unaudited financial statements (prepared by the

company itself). From there, the ratios used to identify discrepancies in financial statements are determined (Nguyen Tien Hung and Vo Hong Duc, 2017; Skousen *et al.*, 2009; Lou and Wang, 2011; Persons, 1995). Companies with fraud are assumed to have a profit discrepancy before and after auditing of 5% or more, while non-fraudulent companies (control sample) are companies with the same industry and a total asset discrepancy before and after auditing of less than 1%.

$$Profit\ discrepancy = \left| \frac{Profit\ after\ auditing - Profit\ before\ auditing}{Profit\ after\ auditing} \times 100\% \right|$$

The study assumes that profit after auditing is the correct profit because it has been accepted by auditors. The study uses absolute values because it does not distinguish between positive discrepancies (overstating actual values) or negative discrepancies (concealing profits) (Nguyen Tien Hung and Vo Hong Duc, 2017; Skousen *et al.*, 2009; Lou and Wang, 2011). In this study, the authors identified 44 pairs of fraudulent and non-fraudulent companies on HOSE.

**Table 1: Data used in the study**

Research Sample	Number
1. Companies with profit discrepancies of 5% or more	78
2. Companies without a control sample, incomplete financial reporting information	34
3. Fraudulent companies (3 = 1 - 2)	44
4. Non-fraudulent companies (control sample)	44

Source: Compiled by the authors.

**1.3. Research model**

The study employs a quantitative approach with a Binary logistic regression model. Based on previous experimental studies, the research model is as follows:

$$Fraud_i = \beta_0 + \beta_1 Pre_i + \beta_2 Opp_i + \beta_3 Rat_i + \beta_4 Cap_i + \beta_5 Arr_i + \epsilon_i$$

Where: (i) *Fraud<sub>i</sub>* is the dependent variable representing the likelihood of financial statement fraud, with a value of 1 (Fraud) or 0 (Non-fraud); (ii) *Pre<sub>i</sub>*, *Opp<sub>i</sub>*, *Rat<sub>i</sub>*, *Cap<sub>i</sub>* and *Arr<sub>i</sub>* are independent variables representing the factors of pressure, opportunity, rationalization, capability, and arrogance, respectively; (iii) *ε<sub>i</sub>* is the error term of the model.

**1.4. Measurement of variables and research hypotheses**

This study is a quantitative investigation examining the influence of the Fraud Pentagon theory on financial statement fraud behavior.

**Table 2: Description of measured variables and research hypotheses**

Variable Name	Definition	Measurement	Previous Research	Hypothesis
<i>Dependent variable</i>				
Fraud	Financial statement fraud	1 if the company commits fraud, 0 otherwise	Umar <i>et al.</i> (2020), Handoko and Natasya (2019), Wahyuningrum and Iswajuni (2020), Yulianti <i>et al.</i> (2019), Indarto and Ghozali (2016)	-
<i>Independent variables</i>				
Pre	Pressure	Profit after tax/Total assets	Feldman and Halali (2019), Umar <i>et al.</i> (2020), Handoko and Natasya (2019), Fitriyah and Novita (2021), Crowe (2011)	+/-
Opp	Opportunity	1 if the company changed independent auditors in the two years prior to fraud, 0 otherwise	Utomo <i>et al.</i> (2019), Fitriyah and Novita (2021), Crowe (2011)	+/-
Rat	Rationalization	1 if the company received a qualified opinion, 0 otherwise	Shi <i>et al.</i> (2017), Umar <i>et al.</i> (2020), Crowe (2011)	+/-
Cap	Capability	1 if the CEO also serves as the chairman of the board, 0 otherwise	Abdullahi and Mansor (2015), Umar <i>et al.</i> (2020), Crowe (2011)	+/-
Arr	Arrogance	Number of CEO turnovers in the last three years	Yulianti <i>et al.</i> (2019), Nindito (2018), Handoko and Natasya (2019), Yulianti <i>et al.</i> (2019), Crowe (2011)	+/-

Source: Authors' proposal.

2. Research results

Table 3: Correlation matrix of variables

	Constant	Pre	Opp	Rat	Cap	Arr
Constant	1.000	-0.417	-0.257	0.002	-0.246	-0.076
Pre	-0.417	1.000	0.084	-0.106	0.102	0.191
Opp	-0.257	0.084	1.000	-0.038	-0.080	-0.018
Rat	0.002	-0.106	-0.038	1.000	-0.042	-0.022
Cap	-0.246	0.102	-0.080	-0.042	1.000	0.070
Arr	-0.076	0.191	-0.018	-0.022	0.070	1.000

Source: Authors' data processing results.

The correlation matrix analysis of the independent variables shows that all correlation coefficients are below 0,75 (the largest being 0.417), indicating the presence of multicollinearity, but not significantly in the model. Conversely, multicollinearity becomes more severe if this coefficient exceeds 0,75 (Neter *et al.*, 1990).

Table 4: Model feasibility

-2 Log likelihood	Cox and Snell R Square	Nagelkerke R Square
107.236 <sup>a</sup>	0.154	0.206

Source: Authors' data processing results.

Table 4 shows the results of testing the feasibility of the model through the R2 coefficients, namely Cox and Snell R2 and Nagelkerke R2. The result indicates that the Nagelkerke R2 coefficient is 0.206. This means that the pentagon variables can explain 20.6% of the behavior of financial statement fraud, and the remaining 79.4% is explained by other variables not used in the study.

Table 5: Results of binary logistic regression analysis

	B	S.E.	Wald	Df	Sig.	Exp(B)
Pre	-11.181	5.567	4.034	1	0.045*	0.000
Opp	-0.339	0.467	0.528	1	0.468	0.713
Rat	2.520	1.108	5.179	1	0.023*	12.435
Cap	-0.475	0.494	0.925	1	0.336	0.622
Arr	0.177	0.624	0.080	1	0.777	1.193
Constant	0.346	0.886	0.152	1	0.696	1.413

Notes: \* indicates statistical significance at 5% level.

Source: Author's data processing results.

The hypothesis testing results for each factor in Table 5 indicate that variables *Pre* and *Rat* have significance levels smaller than  $\alpha$  (5%), meaning they influence financial statement fraud. The three factors *Opp*, *Cap*, and *Arr* have significance levels greater than  $\alpha$ , meaning they do not affect financial statement fraud. The regression equation is as follows:

$$Fraud = 0.346 - 11.181Pre - 0.339Opp + 2.520Rat - 0.475Cap + 0.177Arr$$

Moreover, to assess the accuracy of the model, the author conducted a test of the model's accuracy in classifying the research sample into two groups: fraudulent and non-fraudulent companies. The results show that the model accurately predicts 61.4% of fraudulent companies and 74.5% of non-fraudulent companies, with an average prediction rate of 68.2%.

**Table 6: Model accuracy**

Observed		Fraud		Percentage Correct
		0	1	
Fraud	0	33	11	75.0
	1	17	27	61.4
Overall Percentage				68.2

Source: Author's data processing results.

### 3. Discussions and policy implications

The results of testing the first hypothesis indicate that pressure (*Pre*) influences financial statement fraud. This result aligns with findings from previous studies by Feldman and Halali (2019), Umar *et al.* (2020), Handoko and Natasya (2019), Fitriyah and Novita (2021), and Crowe (2011), but contrasts with the research findings of Wahyuningrum and Iswajuni (2020) and Yulianti *et al.* (2019). Financial stability pressure may encourage company leadership to engage in financial statement fraud. A high return on assets (*ROA*) is indicative of a company with substantial assets, growth prospects, and investor attractiveness. Conversely, a decrease in *ROA* may be interpreted as poor asset management by the board, creating pressure to engage in financial statement fraud to maintain the illusion of asset growth and competent investment decisions.

The results of testing the second hypothesis show that opportunity (*Opp*) does not influence financial statement fraud. This result is consistent with the research findings of Yulianti *et al.* (2019) and Indarto and Ghozali (2016), but contrasts with the findings of Utomo *et al.* (2019), Fitriyah and Novita (2021), and Crowe (2011). Changing auditors is often aimed at reducing the risk of detecting financial statement fraud by the previous auditor. Additionally, changing auditors may be an attempt to eliminate evidence or audit trails uncovered by the previous auditor. Companies changing auditors may be suspected of financial statement fraud. The previous auditor may have detected fraudulent behavior, leading company leadership to change auditors to remove audit trails in subsequent periods.

Ultimately, company boards may expect that changing auditors will result in new auditors being unaware of past financial statement fraud.

The results of testing the third hypothesis indicate that rationalization (*Rat*) influences financial statement fraud. This result is consistent with the research of Shi et al. (2017) and Crowe (2011), while it contradicts the findings of Irwandi *et al.* (2019) and Umar *et al.* (2020). Auditor opinions may describe a company's financial status, including indications of whether the company is engaged in financial statement fraud. Auditor opinions are statements made by auditors regarding the reasonableness of a company's financial statements. The significance level determined by auditors is sometimes overlooked, leading auditors to issue opinions that are not fully acceptable. However, opinions that are not fully acceptable but are ultimately issued by auditors are viewed by company leadership as non-material errors or not indicative of financial statement fraud. Some overlooked opinions due to immaterial errors still come with supplementary explanations, often referred to as unqualified audit opinions. This underscores that auditor opinions are not entirely devoid of significant errors or financial statement fraud indicators.

The results of the hypothesis testing for the fourth hypothesis indicate that capability (*Cap*) does not influence financial statement fraud. This result aligns with the findings of studies conducted by Handoko and Natasya (2019), Yulianti and colleagues (2019), but contradicts the results of the research by Abdullahi and Manso (2015), Umar and colleagues (2020), and Crowe (2011). This discrepancy may stem from the primary objective of changing company directors, which is to replace less competent directors with more capable ones to enhance the company's operational efficiency. Ultimately, as the company's operational effectiveness improves, it may attract more investors. Additionally, every company has a policy of rotating directors every five years, three years, or even annually. This is a common practice across most companies and varies depending on each company's policies. Overall, when there is a rotation of the board of directors, the policies set by the preceding board may be adjusted or altered. It is expected that this policy change can minimize the occurrence of financial statement fraud in the company.

The results of the hypothesis testing for the fifth hypothesis show that exclusivity (*Arr*) does not affect financial statement fraud. This finding is consistent with the research conducted by Yulianti and colleagues (2019). The number of managing directors displayed in the company's latest financial report cannot be considered as a form of indicating financial statement fraud. The number of managing directors shown in the company's latest financial report may be a way to showcase the achievements the managing director has attained during their tenure. Other accomplishments achieved by the managing director reflect their effective and reliable operational performance. Furthermore, the number of managing directors displayed in the company's latest financial report may also indicate the high level of confidence of the managing director. The confidence the managing director gains based on their successes and achievements, as well as their diligent work, demonstrates their capability to drive company growth and attract investors.

This research outcome reveals that among the five independent variables representing the five facets of the pentagon of fraud used in this study, only two variables have an impact on financial statement fraud, while the remaining three variables do not influence financial statement fraud. This could be explained by the fact that the sample used in the study was conducted only in 2019 and solely on the HOSE exchange. Additionally, the number of variables representing the pentagon of fraud is limited. Therefore, future research should aim to increase the sample size and extend the study to both the HNX and HOSE, while also enhancing the number of variables representing the pentagon of fraud, thereby improving the accuracy of the research findings.

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### Article history

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