Communications in Physics, Vol. 17, No. 4 (2007), pp. 209-212

GENERALIZED q-DEFORMATION OF VIRASORO ALGEBRA

LUU THI KIM THANH
Hanoi Pedagogical University No. 2

Abstract. *We find an extension of the q-deformed Virasoro algebra. This deformation includes on an equal footing the usual q-deformed oscillators and the "quons" of infinite statistics. Various representations of Virasoro algebra, both differential and oscillator representation, are considered. A new realization of the generalized q-deformed centerless Virasoro algebra is constructed by introducing a so-called power raising operator.*

I. INTRODUCTION

Virasoro algebra plays a crucial role in string theory [1,2] of elementary particles, which has attracted a great deal of interest over the last decades. This algebra may be viewed as infinite-dimensional conformal algebra [3] and is closely related to the Korteweg-de Vries (KdV) equation. In particular the q-Virasoro algebra generates the sympletic structure which can be used for a description of the discretization of the KdV equation [4, 5].

In this paper we would like to consider a version of generalized q-deformation of Virasoro algebra. This generalization includes on an equal footing the usual q-deformed oscillator [6] and the "quons" of infinite statistics [7]. The aim of this paper is to consider a new realization of centerless Virasoro algebra, as well as its super-extension. In Sec. II we consider the various representations of Virasoro algebra, both differential and oscillator reprsentation. In Sec. 3 we associated generalized q-deformation of Virasoro algebra. A new realization of generalized q-deformed centreless Virasoro algebra is constructed by introducing a so-called power raising operator.

II. REALIZATION OF CENTRELESS VIRASORO ALGEBRA [2, 4, 5]:

The centreless Virasoro algebra consists of generators L_n, $n \in z$, satisfying the commutation relation:

$$[L_n, L_m] = (n-m)L_{n+m}. \tag{1}$$

The simplest differential realization of this algebra is to identify:

$$L_n \equiv x^{-n+1}\frac{\partial}{\partial x}. \tag{2}$$

Indeed, the equation (1) can be verified straightforwardly, using the commutation relation

$$\left[x, \frac{\partial}{\partial x}\right] = -1. \tag{3}$$

It can also be shown that instead of (2) one can use the more general expression for L_n:

$$L_n = \left(x\frac{\partial}{\partial x} + c_1 n + c_2 \right) x^{-n}, \tag{4}$$

where c_1 and c_2 are arbitrary constants. The expression (2) corresponds to the value $c_1 = c_2 = 0$.

The super-Virasoro algebra consists of the generators L_n and G_r, satisfying the commutation relations:

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m}, \\ [L_n, G_r] &= (\frac{1}{2}n - 1)G_{n+r}, \\ [G_r, G_s] &= 2L_{r+s}, \end{aligned} \tag{5}$$

where $r \in z + \frac{1}{2}$ for Neuveu-Schwarz sector, and $r \in z$ for Ramond sector.

Let θ be Grassmann variable with

$$\theta^2 = 0, \quad \frac{\partial^2}{\partial \theta^2} = 0, \quad \left\{ \theta, \frac{\partial}{\partial \theta} \right\} = 1. \tag{6}$$

Then it can be checked in direct manner that the operators

$$\begin{aligned} L_n &\equiv \left(x\frac{\partial}{\partial x} + n - \frac{1}{2} n\theta \frac{\partial}{\partial \theta} \right) x^{-n} \\ G_r &\equiv \left\{ \theta \left(x\frac{\partial}{\partial x} + r \right) + \frac{\partial}{\partial \theta} \right\} x^{-r} \end{aligned} \tag{7}$$

realize the superagebra (5).

In the (undeformed) oscillator, formalism the oscillator a and its hermitian conjugate a^+ obey the commutatin relation:

$$\left[a, a^+ \right] = 1 \tag{8}$$

Then the generators

$$L_n \equiv (a^+)^{-n+1} a \tag{9}$$

realize the Virasoro algebra (1).

Instead of (9) we can also use a more general expression for L_n, namely:

$$L_n = (a^+)^{-n} \left(a^+ a + c_1 n + c_2 \right) \tag{10}$$

For the realization of super-Virasoro algebra, in addition to the bosonic oscillators a and a^+, fermionic oscillators b and b^+ with the anticommutators

$$\left\{ b, b^+ \right\} = 1; \quad b^2 = {b^+}^2 = 0 \tag{11}$$

are introduced.

Now it can be checked that the generators

$$\begin{aligned} L_n &\equiv (a^+)^{-n+1} a - \frac{n}{2} b^+ b (a^+)^{-n} \\ G_r &\equiv b^+ (a^+)^{-r+1} a + b (a^+)^{-r} \end{aligned} \tag{12}$$

form the superalgebra (5).

Finally, another version of realizing the Virasoro algebra is constructed on basis (a, M) with the commutator

$$[M, a] = -a^2 \tag{13}$$

Note that the operator M acts as power raising operator. From (13) it is easy to prove that

$$[M, a] = -na^{n+1} \tag{14}$$

for arbitrary n.

The Virasoro generators can be now identified with

$$L_n \equiv Ma^{n-1}. \tag{15}$$

To extend the formalism to super-Virasoro algebra, we introduce also the fermionic oscillators b, b^+ satisfying (11) and

$$[M, b] = \left[M, b^+\right] = 0. \tag{16}$$

With the basis (a, b, M) we can construct the supergenerators as follows:

$$\begin{aligned} L_n &= Ma^{n-1} + \frac{n}{2}b^+ba^n \\ G_r &= Ma^{r-1}b + a^r b \end{aligned} \tag{17}$$

III. REPRESENTATION OF GENERALIZED q-DEFORMED VIRASORO ALGEBRA

Consider now the generalized q-deformed Virasoro algebra based on the generalized q-oscillator algebra [8], with oscillator a and a^+ obey the commutation relation:

$$aa^+ - qa^+a = q^{cN} \tag{18}$$

The usual q-deformatiom

$$aa^+ - qa^+a = q^{-N}, \tag{19}$$

corresponds to the value $c = -1$, and the "infinite statistics"

$$aa^+ = 1, \tag{20}$$

corresponds to $c = 0$, $q = 0$.

In this section we propose a version of quantum deformation of Virasoro algebra in the framework of power raising formalism described in the previous section.

Instead of (13) we now assume that

$$[M, a]_{(q^c,q)} = -a^2. \tag{21}$$

Here we use the notation:

$$[A, B]_{(\alpha,\beta)} \equiv \alpha AB - \beta BA. \tag{22}$$

From (18) it is easy to prove that

$$[M, a^n]_{(q^{nc},q^n)} = -[n]_q^{(c)} a^{n+1}, \tag{23}$$

for arbitrary n, where the general notation

$$[x]_q^{(c)} = \frac{q^x - q^{cx}}{q - q^c}, \tag{24}$$

is used.

Now we can show that the expression (15) of L_n satisfies the following generalized q-deformation of Virasoro algebra:

$$[L_n, L_m]_{q^{n-m}, q^{c(n-m)}} = [n-m]_q^{(c)} L_{n+m}. \tag{25}$$

From these we reconver the result:

$$[M, a^n]_{(q^{-n}, q^n)} = -[n]_q a^{n+1} \tag{26}$$

$$[L_n, L_m]_{q^{n-m}, qm-n)} = [n-m]_q L_{n+m} \tag{27}$$

for usual q-deformation of Virasoro algebra.

When $c = -1$, in the limit of "infinite statistics", $c = 0$, $q = 0$, we have:

$$\begin{aligned} M &= -a \\ L_n L_m &= L_{n+m} \end{aligned} \tag{28}$$

ACKNOWLEDGMENTS

We would like to thank Prof. Dao Vong Duc for helpful discussions and valuable comments.

REFERENCES

[1] M. Chaichian, P. P. Kulish and J. Lukierski, *Phys. Lett. B* **237** (1990) 401.
[2] N. Aizawa and H. Sato, *Phys. Lett. B* **256** (1991) 185.
[3] A. Jannussis, G. Brodimas and R. Mignanit, *J. Phys. A: Math. Gen.* **24** (1991)
[4] M. Chaichian, Z. Popowicz, P.Presnajder, *Phys. Lett. B* **249** (1990) 63.
[5] T. L. Curtwright and C. K. Zachos, *Phys. Lett. B* **243** (1990) 237.
[6] A. J. Macflance, *J. Phys. A* **22** (1989) 4581.
[7] M. Jimbo, *Intern. J. Mod. Phys.* **4** (1989) 3759.
[8] Dao Vong Duc, Enslapp. A-494/94

Received 02 August 2007.