# INERTIAL OPTIMIZATION ALGORITHM FOR SOLVING PROBLEM OF AIRCRAFT MANEUVERING TO REACH ALTITUDE AND ATTACK TARGET

**Pham Kim Quy, Pham Ngoc Cong, Dang Van Hieu***

University of Air Force

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** Tác giả chính:*

*dv.hieu83@gmail.com*

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***Title:***

*Inertial Optimization Algorithm for Solving Problem of Aircraft Maneuvering to Reach Altitude and Attack Target*

***Từ khóa:***

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***Keyword:***

*Optimal control problem, Inertial optimization algorithm, Nonlinear programming, Global collocation, Equation of aircraft motion*

**TÓM TẮT:** *Trong bài báo này, chúng tôi nghiên cứu mô hình máy bay cơ động, lấy độ cao, tấn công một mục tiêu cố định, trong đó hàm mục tiêu là độ cao tối thiểu dọc theo quy đạo. Mô hình được thiết lập dưới dạng bài toán điều khiển tối ưu. Bài báo giới thiệu phương pháp trùng khớp toàn cục nhằm chuyển mô hình bài toán liên tục sang bài toán quy hoạch phi tuyến (NLP) cỡ lớn. Thuật toán tối ưu quán tính được đề xuất để giải bài toán NLP thu được. Một vài thử nghiệm số được thực hiện trên chiến đấu cơ đa năng Su-30MK2. Các kết quả thử nghiệm đã định dạng quỹ bay tối ưu: bay định mức, leo cao, bổ nhào. Điều này gợi ý giải pháp khi xây dựng các kế hoạch bay và thực hiện các nhiệm vụ thực tế.*

**ABSTRACT:** *In this paper, we study a model of an aircraft maneuvering to reach altitude and attack a suggested target, where the object is the minimum altitude along the trajectory. We establish the model as an optimal control problem, and then propose the global collocation method to transform continuous problem model to large-scale nonlinear programming (NLP). We introduce an inertial optimization algorithm for solving the obtained NLP. Some numerical experiments are implemented for the multi-role fighter aircraft Su-30MK2. The experimental results have shaped the form of optimal trajectory: level flight, climb, dive. This suggests the solution when we construct flight plans and perform practical tasks.*

## 1. Introduction

Many problems in science and engineering require choosing the best solution among all possible solutions [1, 2, 5, 11]. In the field of aviation, one of the challenging problems is to determine optimal trajectories, in the sense that, of a flight vehicle in real three-dimensional space [11]. The optimization denpends on each task [4, 7, 10], for example, in flight operations, some optimization problems are often interested such as fuel optimization, flight time optimization, distance optimization to target or flight altitude optimization [2]. In the atmosphere, a flying object of mass $m$ is subjected to the gravitational force $mg$, thrust force $T$ (when the propulsion system is operating) and aerodynamic force $A$ (usually, including lift force $L$ and drag force $D$). These forces vary according to the position of the object in space as well as the control activities of users. Trajectory optimization here is the problem of controlling an object, through thrust and aerodynamic forces, from its initial position $P_i$ to the final position $P_f$ so that a function of the state variable reaches maximum or minimum. This optimization function is often called the performance index. The acting performance index is based on the constraints of motion, the limits of controls as well as the specifications of the flying object.

The equations of motion of flying objects in the atmosphere are often very complex, described in form of a system of differential equations (or, dynamical system) [11]. It is not easy to find the analytical solutions of flight dynamical system, except for simple cases. Therefore, numerical solutions are often more interested. The commonly used methods for solving optimal control problems are well-known as indirect methods and direct methods [5, 6, 9]. The first method requires a first-order optimization necessary condition which is based on variational calculus. Following this approach, we convert the original problem into a Hamiltonian boundary-value problem (BVP). After that the BVP is discretized and solved numerically by methods of ordinary differential equations. Thus, the indirect method performs by the manner, firstly the "optimization" step, and then "discretization" step. Meanwhile, the direct method performs the opposite process, that is, the discretization of the original problem is firstly implemented and then using optimization algorithms to solve numerically the resulting problem. The problem after discretizing is actually a large-scale nonlinear programming [1].

It is also emphasized that the first method using the first-order optimality necessary condition often has to pay a high cost because of the complexity of the flight dynamics and many constraints which are not easy to process. Therefore, the second method, i.e. the direct method, is more attractive and promising where we can use the well-known optimization programmings to solve [3, 9].

## 2. Flight Model

In this section, we focus on studying the optimal trajectory for an aircraft when performing the mission: maneuvering and climbing to attack a fixed target on the ground or at sea. This means that the aircraft maneuvers from the original position $P_i(x_i, h_i)$ (at the initial time $t_i$, flight path angle $\gamma_i$, velocity $V_i$) to the terminal position $P_f(x_f, h_f)$ at the final time $t_f$ where the aircraft reaches the flight path angle $\gamma_f$ and the velocity $V_f$ to fire or drop bomb, enough to destroy the fixed target $M$ (see, Figure 1).

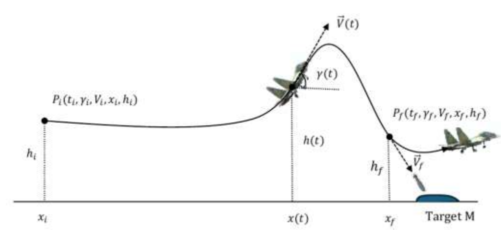


FIGURE 1. Model of considered problem

During the maneuver, from the initial position $P_i$ to the terminal position $P_f$, the fighter aircraft may not be necessary to have to fly so high, and also by any reason, for example, minimizing the aircraft's exposure to anti-air defences, the aircraft is not allowed to fly so low. This means that the flight altitude should be as low as possible, but the impact must be obtained by a final dive which is enough to destroy the target. Then, the quantity minimized for the aircraft here is the integrated altitude [4]

$$J = \int_{t_i}^{t_f} h(t)dt, \tag{2.1}$$

where $h(t)$ is the altitude of aircraft at the actual time $t$ ($t_i \leq t \leq t_f$) with $t_i$ as the initial time and $t_f$ as the final time. This minimum must take into acount, among others, the flight dynamical constraints (i.e., equation of motion), the limits of the specifications and controls of aircraft, such as thrust and angle of attack, the final dive conditions.

The equation of motion (dynamical system) of aircraft in the aforementioned model [11] can be written as

$$\begin{aligned}\dot{\gamma} =& \frac{T-D}{mV}\sin\alpha \\ &+ \frac{L}{mV}\cos\alpha - \frac{g}{V}\cos\gamma,\end{aligned} \tag{2.2}$$

$$\begin{aligned}\dot{V} =& \frac{T-D}{m}\cos\alpha \\ &- \frac{L}{m}\sin\alpha - g\sin\gamma,\end{aligned} \tag{2.3}$$

$$\dot{x} = V\cos\gamma, \tag{2.4}$$

$$\dot{h} = V\sin\gamma, \tag{2.5}$$

where $g$ is gravitational constant, $\gamma$ is the angle of flight path; $V$ is the velocity; $x$, $h$, $m$ are the horizontal position, altitude and mass of airplane, respectively. These quantities are considered as state variables. The thrust $T$ and the angle of attack $\alpha$ are the control variables, while $L$ and $D$ are the lift and drag forces, given by

$$L = \frac{1}{2}\rho S C_L V^2, \tag{2.6}$$

$$D = \frac{1}{2}\rho S C_D V^2, \tag{2.7}$$

where $S$ is the reference area of airplane; the density $\rho$ is the function of altitude $h$; the lift coefficient $C_L$ and drag coefficient $C_D$ are functions of angle of attack $\alpha$, formed by

$$\rho = \begin{cases} \rho_0\left(1-\frac{h}{44300}\right)^{4.256} & \text{if } 0 \leq h \leq 11000, \\ \rho_{11}e^{-\frac{h-11000}{6340}} & \text{if } h > 11000, \end{cases} \tag{2.8}$$

and

$$C_D = A_1\alpha^2 + A_2\alpha + A_3, \tag{2.9}$$

$$C_L = B_1\alpha + B_2. \tag{2.10}$$

The forces $T$, $L$, $D$ and the angles $\alpha$, $\gamma$ are described in Figure 2. The lift coefficient $C_L$ varies linearly with the angle of attack $\alpha$. For numerical experiments, we consider the multi-role fighter aircraft SU30MK2 and its parameters [8] are given in Table 2, Sect. 6.

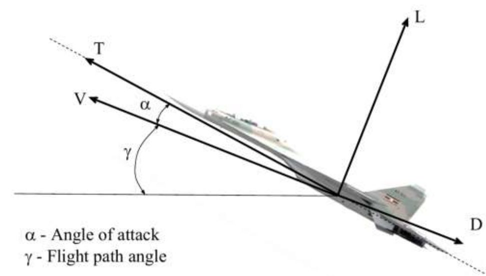


FIGURE 2. Describe the parameters of Su-30MK2

Regarding the limits of specifications and controls, we consider the following constraints:

- State constraints

$$V_{\min} \leq V \leq V_{\max}, \tag{2.11}$$

$$h_{\min} \leq h \leq h_{\max}. \tag{2.12}$$

The minimum velocity $V_{\min}$ is always an important parameter for any aircraft. This ensures the maneuverability of flight and avoids the stall of the aircraft. The theoretical minimum velocity [11] for an aircraft is found as follows:

$$V_{\min} = \sqrt{\frac{2W}{\rho S C_{L\max}}}. \tag{2.13}$$

- Control constraints

$$T_{\min} \leq T \leq T_{\max}. \tag{2.14}$$

For a particular aircraft, Su-30MK2, the boundary of state and control constraints are given in Table 3, Sect. 6.

- Boundary conditions.

The initial and final conditions for four state variables are described in Table 1

TABLE 1. Boundary conditions

| Initial condition | Final condition |
|---|---|
| $\gamma(t_i) = \gamma_i$ | $\gamma(t_f) = \gamma_f$ |
| $V(t_i) = V_i$ | $V(t_f) = V_f$ |
| $x(t_i) = x_i$ | $x(t_f) = x_f$ |
| $h(t_i) = h_i$ | $h(t_f) = h_f$ |

## 3. Mathematical model

This section focuses on formulating the considered problem in Sect. 3 under an optimal control problem. Following that we put:

- State variablev $\mathbf{x}$:

$$\mathbf{x} = [x_1, x_2, x_3, x_4]^\top = [\gamma, V, x, h]^\top. \tag{3.1}$$

- Control variable $\mathbf{u}$:

$$\mathbf{u} = [u_1, u_2]^\top = [\alpha, T]^\top. \tag{3.2}$$

- Function $\mathbf{f}$:

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = [f_1(\mathbf{x},\mathbf{u}), f_2(\mathbf{x},\mathbf{u}), f_3(\mathbf{x},\mathbf{u}), f_4(\mathbf{x},\mathbf{u})]^\top, \tag{3.3}$$

where

$$\begin{aligned} f_1(\mathbf{x},\mathbf{u}) =& \frac{T-D}{mV}\sin\alpha \\ &+ \frac{L}{mV}\cos\alpha - \frac{g}{V}\cos\gamma, \end{aligned} \tag{3.4}$$

$$\begin{aligned} f_2(\mathbf{x},\mathbf{u}) =& \frac{T-D}{m}\cos\alpha \\ &- \frac{L}{m}\sin\alpha - g\sin\gamma, \end{aligned} \tag{3.5}$$

$$f_3(\mathbf{x},\mathbf{u}) = V\cos\gamma, \tag{3.6}$$

$$f_4(\mathbf{x},\mathbf{u}) = V\sin\gamma. \tag{3.7}$$

Then, the dynamical system (3.2) - (3.5) can be written as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}). \tag{3.8}$$

The boundary of control and state variables

$$\begin{aligned} [\gamma_{\min}, V_{\min}, x_{\min}, h_{\min}]^\top =: \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max} \\ := [\gamma_{\max}, V_{\max}, x_{\max}, h_{\max}]^\top, \end{aligned} \tag{3.9}$$

$$\begin{aligned} [\alpha_{\min}, T_{\min}]^\top =: \mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max} \\ := [\alpha_{\max}, T_{\max}]^\top. \end{aligned} \tag{3.10}$$

The boundary condition

$$\mathbf{x}(t_i) = \mathbf{x}_i := [\gamma_i, V_i, x_i, h_i]^\top, \tag{3.11}$$

$$\mathbf{x}(t_f) = \mathbf{x}_f := [\gamma_f, V_f, x_f, h_f]^\top. \tag{3.12}$$

Consider the function $E : \mathbb{R}^4 \to \mathbb{R}$ defined by

$$\mathbf{x} = [x_1, x_2, x_3, x_4]^\top \in \mathbb{R}^4 \mapsto E(\mathbf{x}) = x_4 \in \mathbb{R}.$$

Then, $E(\mathbf{x}(t)) = h(t)$ and our considering problem is formulated in a mathematical form as

$$\min_{\mathbf{u}} J = \int_{t_i}^{t_f} E(\mathbf{x}(t))dt \tag{3.13}$$

subject to

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}),\ t_i \leq t \leq t_f, \tag{3.14}$$

$$\mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max}, \tag{3.15}$$

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}. \tag{3.16}$$

$$\mathbf{x}(t_i) = \mathbf{x}_i, \tag{3.17}$$

$$\mathbf{x}(t_f) = \mathbf{x}_f. \tag{3.18}$$

## 4. Proposed method

In this section, we introduce a direct method for solving numerically problem (4.13) with constraints (4.14) - (4.18), namely the global collocation method (GCM). This method uses the global orthogonal polynomials to discrete state and control variables where a function is approached by the sum of finitely smooth functions. The collocation points (nodes) are the solutions of Chebyschev polynomials which satisfy the following property:

$$T_{n+1}(\tau) = 2\tau T_n(\tau) - T_{n-1}(\tau),\ T_0(\tau) = 1,$$

$T_1(\tau) = \tau$, $\tau \in [-1,1], n = 1,2,\cdots$

Since the Chebyschev nodes $\tau_i \in [-1,1]$, we use a transformation between the GCM domain $\tau \in [-1,1]$ and the physical domain $t \in [t_i, t_f]$, given by

$$t = \frac{t_f + t_i}{2} + \frac{t_f - t_i}{2}\tau. \quad (4.1)$$

Hence, the dynamical system (4.14) becomes

$$\dot{\mathbf{x}}(\tau) = \tfrac{t_f - t_i}{2}\mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau)),\ -1 \le \tau \le 1. \quad (4.2)$$

Now, let us fix a natural number $N \ge 1$. The Chebyschev nodes $\{\tau_i\}_{i=0}^N$ are the extreme points of the polynomial $T_N(\tau)$ on the interval $[-1,1]$. Denote $\{L_k\}_{k=0}^N$ by the system of basical Lagrange polynomials which are constructed from the Chebyschev nodes $\{\tau_i\}_{i=0}^N$, i.e.,

$$L_k(\tau) = \frac{\omega_k(\tau)}{\omega_k(\tau_k)},\ k = 0,1,\cdots,N, \quad (4.3)$$

where

$$\begin{aligned}\omega_k(\tau) =& (\tau - \tau_0)(\tau - \tau_1)\cdots \\ &\cdots(\tau - \tau_{k-1})(\tau - \tau_{k+1})\cdots(\tau - \tau_N).\end{aligned}$$

Actually, we have

$$L_k(\tau) = \tfrac{2}{Nc_k}\textstyle\sum_{p=0}^N \frac{T_p(\tau_k)T_p(\tau)}{c_p},\ k = 0,1,\cdots,N, \quad (4.4)$$

where $c_0 = c_N = 2$ and $c_p = 1$ for all $p = 1,2,\cdots,N-1$. Note that

$$L_k(\tau_j) = \delta_{kj} = \begin{cases} 1 & \text{if} \quad k = j, \\ 0 & \text{if} \quad k \neq j. \end{cases} \quad (4.5)$$

Discreting the state variable

$$\begin{aligned}\mathbf{x}(\tau) &= [x_1(\tau), x_2(\tau), x_3(\tau), x_4(\tau)]^\intercal \\ &\approx \mathbf{x}^N(\tau) = [x_1^N(\tau), x_2^N(\tau), x_3^N(\tau), x_4^N(\tau)]^\intercal\end{aligned}$$

and the control variable

$$\begin{aligned}\mathbf{u}(\tau) &= [u_1(\tau), u_2(\tau)]^\intercal \\ &\approx \mathbf{u}^N(\tau) = [u_1^N(\tau), u_2^N(\tau)]^\intercal\end{aligned}$$

by

$$\begin{aligned}x_i(\tau) &\approx x_i^N(\tau) \\ &= \sum_{k=0}^N x_i(\tau_k)L_k(\tau),\ i = 1,2,3,4, \quad (4.6)\\ u_j(\tau) &\approx u_j^N(\tau) \\ &= \sum_{k=0}^N u_i(\tau_k)L_k(\tau),\ j = 1,2. \quad (4.7)\end{aligned}$$

By the relation (5.5), we have

$$x_i(\tau_k) = x_i^N(\tau_k)$$

and

$$u_j(\tau_k) = u_j^N(\tau_k), \forall i,j,k. \quad (4.8)$$

Approxing the derivative

$$\begin{aligned}\dot{x}_i(\tau_p) &\approx \dot{x}_i^N(\tau_p) \\ &= \sum_{k=0}^N x_i(\tau_k)\dot{L}_k(\tau_p) \\ &= \sum_{k=0}^N x_i(\tau_k)C_{pk}, \quad (4.9)\end{aligned}$$

where $C = [C_{pk}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is the Chebyshev differentiation matrix, defined by

$$C_{pk} = \begin{cases} -\frac{2N^2+1}{6} & \text{khi} \quad n = p = 0, \\ \frac{2N^2+1}{6} & \text{khi} \quad n = p = N, \\ -\frac{t_k}{2(1-t_k^2)} & \text{khi} \quad 1 \le p = k \le N-1, \\ \frac{(-1)^{p+k}c_p}{c_k(t_p - t_k)} & \text{khi} \quad p \neq k. \end{cases} \quad (4.10)$$

Set

$$\begin{aligned}X_i =& [x_{ik}]_k^\intercal,\ U_j = [u_{jk}]_k^\intercal \\ F_i =& [f_i(\mathbf{x}(\tau_0), \mathbf{u}(\tau_0)), \cdots, f_i(\mathbf{x}(\tau_N), \mathbf{u}(\tau_N))]^\intercal,\end{aligned}$$

and

$$\begin{aligned}X &= [X_1, X_2, X_3, X_4]^\intercal, \\ U &= [U_1, U_2]^\intercal, \\ F &= [F_1, F_2, F_3, F_4]^\intercal.\end{aligned}$$

Then, the set of variables in NLP is

$$y = [X, U]^\intercal.$$

The continuous constraint (4.14) becomes the following algebraic one

$$\mathbf{C}y = \frac{t_f - t_i}{2}\mathbf{F}(y), \qquad (4.11)$$

where $\mathbf{C} = diag(C,C,C,C,O,O)$
and $\mathbf{F}(y) = [F,0,0]^\intercal$.

Let $A = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$
with $a_{11} = a_{N+1,N+1} = 1$ and $a_{ij} = 0$ otherwise. Put $\mathbf{A} = diag(A,A,A,A,O,O)$ and let $\mathbf{b}$ be a vector in $\mathbb{R}^{6N}$, given by

$$\mathbf{b}(1) = \gamma_i,\ \mathbf{b}(N) = \gamma_f,\ \mathbf{b}(N+1) = V_i,$$
$$\mathbf{b}(2N) = V_f, \mathbf{b}(2N+1) = x_i,$$
$$\mathbf{b}(3N) = x_f,\ \mathbf{b}(3N+1) = h_i,\ \mathbf{b}(4N) = h_f,$$

and $\mathbf{b}(i) = 0$ otherwise. Then, the boundary conditions (4.17) - (4.18) become the equality constraint

$$\mathbf{A}y = \mathbf{b}. \qquad (4.12)$$

The optimal function is

$$\min_y J = \frac{t_f - t_i}{2}\sum_{k=0}^{N} w_k E(\mathbf{x}^N(\tau_k)),$$

where $w_k$ are the GCM weights. Summarize that we have transferred the optimal control problem (4.13) - (4.18) to the following nonlinear programming (NLP):

$$\min_y J = \frac{t_f - t_i}{2}\sum_{k=0}^{N} w_k E(\mathbf{x}^N(\tau_k)) \qquad (4.13)$$

subject to

$$\begin{cases} \mathbf{C}y = \frac{t_f - t_i}{2}\mathbf{F}(y), \\ \mathbf{A}y = \mathbf{b} \\ y_{\min} \le y \le y_{\max}. \end{cases} \qquad (4.14)$$

For solving problem (5.13) with constraint (5.14), we use the inertial optimization algorithm (IOA) proposed in [3]. Algorithm in [3] was designed for solving a more general problem, namely equilibrium problem. For convenience, we represent the algorithm for optimization problem. We set

$$\Omega = \Big\{ y \in \mathbb{R}^{6(N+1)} : \mathbf{C}y = \frac{t_f - t_i}{2}\mathbf{F}(y),$$
$$\mathbf{A}y = \mathbf{b}, \quad y_{\min} \le y \le y_{\max} \Big\}.$$

Then, optimization problem (5.13) can be shortly rewritten as

$$\min_{y\in\Omega} J(y). \qquad (4.15)$$

**Inertial Optimization Algorithm (IOA)** [3]
**Step 0:** Take $y^0, y^1 \in \mathbb{R}^{6(N+1)}$, $Err > 0$, two sequences $\{\xi_k\} \subset [0, \frac{1}{3})$, $\{\lambda_k\} \subset (0, +\infty)$.
Set $w^k = y^k + \xi_k(y^k - y^{k-1})$. Compute
**Step 1:**
$y^{k+1} = \arg\min \left\{ J(y) + \frac{1}{2\lambda_k}\left\|y - w^k\right\|^2 : y \in \Omega \right\}$. **Step 2:** If $\frac{\|y^{k+1} - y^k\|}{\|y^k\|} \le Err$ then Stop. Else, go back to Step 1.

The sequence $\{y^k\}$ generated by IOA converges to a solution of problem (5.15). The analysis in details can be found in [3]. The term $w^k = y^k + \xi_k(y^k - y^{k-1})$ in Step 1 of IOA is called the inertial term.

## 5. Numerical results

In this section, we perform a numerical experiment on the multi-role fighter aircraft Su-30MK2 which was used by many countries around the world for training and combat, providing air superiority, air-to-air, air-to-ground, operating well in complex weather conditions, day and night. The physical parameters of Su-30MK2 are given in Table 2 [8], while the boundary of controls and states is in Table 3. All the programs are written in Matlab R2017b using the Optimization Toolbox and run on a PC Desktop Intel(R) Core(TM) i5-7200U CPU @ 2.50GHz, RAM 2.70 GHz.

TABLE 2. Physical parameters of SU-30MK2

| Quantity | Value | Unit | Quantity | Value | Unit |
|---|---|---|---|---|---|
| $m$ | 25000 | kg | $g$ | 9.81 | $m.s^{-2}$ |
| $S$ | 62.037 | $m^2$ | $\rho_0$ | 1.225 | $kg.m^{-3}$ |
| $A_1$ | - 0.1719 | | $\rho_{11}$ | 0.3636 | $kg.m^{-3}$ |
| $A_2$ | 0.1606 | | $B_1$ | 5.8535 | |
| $A_3$ | 0.0059 | | $B_2$ | 0.3565 | |

TABLE 3. The boundary of state and control constraints

| Quantity | Value | Quantity | Value | Unit |
|---|---|---|---|---|
| $\gamma_{\min}$ | $-\frac{\pi}{2}$ | $\gamma_{\max}$ | $\frac{\pi}{2}$ | rad |
| $V_{\min}$ | 85 | $V_{\max}$ | 580 | m.s$^{-1}$ |
| $x_{\min}$ | 0 | $x_{\max}$ | 3000000 | m |
| $h_{\min}$ | 1000 | $h_{\max}$ | 17300 | m |
| $T_{\min}$ | 4730 | $T_{\max}$ | 246000 | N |

For solving problem (5.15), we use algorithm IOA, where tolerance $Err = 10^{-8}$, the starting point $y^0 = y^1$ generated randomly in Matlab, the inertial term $\xi_k = 0.1$ and the stepsize $\lambda_k = 1$ for all $k$. The number of nodes in method GCM is $N = 20$.

*Experiment 1.* We first consider the initial and final conditions as in Table 4. We take different final flight path angles, namely $\gamma_f = -30^0, -45^0, -60^0$, while others are fixed. The experimental results are shown in Figures 3 - 8.

TABLE 4. Boundary condition with different flight path angles

| Initial condition | Final condition |
|---|---|
| $\gamma_i = 0^0$ | $\gamma_f = -30^0, -45^0, -60^0$ |
| $V_i = 250$ m/s | $V_f = 300$ m/s |
| $x_i = 0$ m | $x_f = 10000$ m |
| $h_i = 1000$ m | $h_f = 1200$ m |

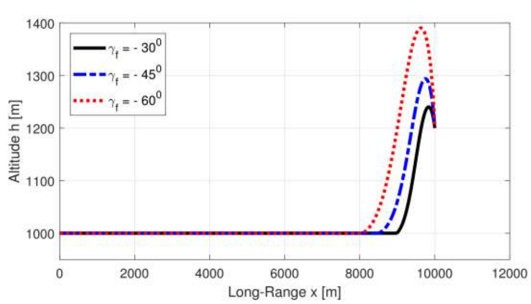


FIGURE 3. Altitude and Range (Exp. 1: Changing Final Flight Path Angle)

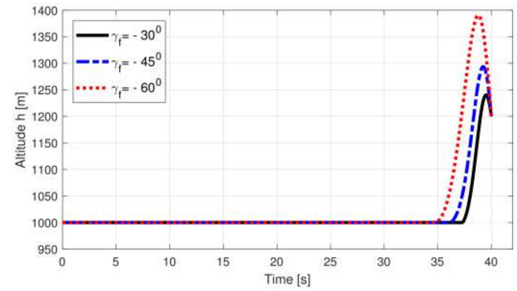


FIGURE 4. Altitude and Time (Exp. 1: Changing Final Flight Path Angle)

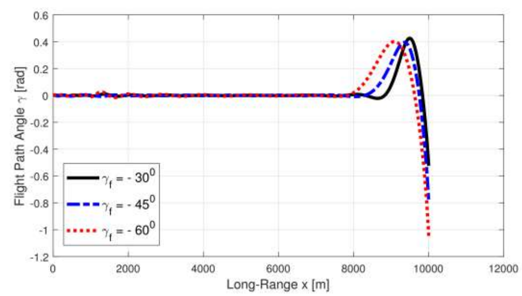


FIGURE 5. Flight Path Angle (Exp. 1: Changing Final Flight Path Angle)

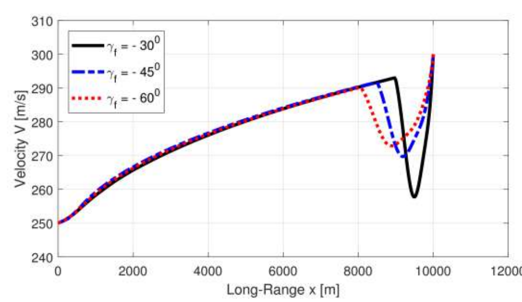


FIGURE 6. Velocity (Exp. 1: Changing Final Flight Path Angle)

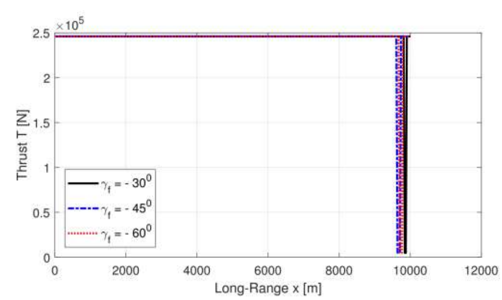


FIGURE 7. Thrust (Exp. 1: Changing Final Flight Path Angle)