The propagation of the SH wave in layered concentric cylindrical structure

Đỗ Xuân Tùng

Abstract

The present work deal with the propagation of a horizontally polarised shear (SH) wave in an infinitely long cylindrical structure comprised of three concentric isotropic layered media. The model has been formulated in cylindrical co-ordinates and an analytical approach is employed to achieve the closed form of the dispersion equation. The present analysis highlights the influence of the wave number on the phase velocity of the shear wave propagating in the embraced structure. Numerical computations have been carried out to accomplish the graphical demonstration unravelling some important peculiarities associated with the propagation characteristics of the shear wave in the considered cylindrical structure.

Key words: *propagation, cylindrical structural, SH wave, isotropic*

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1. Introduction

The dynamical problems on the propagation of horizontally polarized shear waves (SH waves) in anisotropic media have great geophysical importance because they help to investigate the structure of the earth. The horizontally polarised shear (SH) wave, a type of seismic surface wave is a useful indicator for possible fluid pathways because with the increase in permeability of the medium, velocity of shear wave propagation through it decreases. The study of propagation of the shear wave is useful in assessing hydrological properties of the medium, in particular the oceanic basement rocks. Also, the shear wave can detect and characterize the permeable zones which is very useful in geophysical exploration [1],[2].

Nowadays, the study of wave propagation in cylindrical structured media for its dynamic behaviour became a subject of great interest in many fields such as seismology, geophysics, and some engineering streams including mechanical, aerospace and geotechnical engineering, etc. Such a cylindrical structure occurs practically in various engineered form like pipes, aircrafts, submarines, missiles, rockets; boreholes and power transmission shafts are typical cylindrical structures [3],[4],[5].

Therefore, the main purpose of this paper is to consider the propagation of the SH-wave in a triple layered concentric finite long cylindrical structure.

2. Basic equations

In the present work, we have considered the propagation of the SH-wave in an infinitely long horizontal cylindrical structure which is constituted by three concentric isotropic media with different width. In many respects, surface wave propagation in elastic solid layered cylindrical structure is analogous to that in a rectangular elastic layered structure. Let a, b, c be the radii of innermost, intermediate and outermost media with $0 < a < b < c$, respectively, whereas $h_1(=b-a), h_2(=c-b)$ be the width of intermediate and outermost layered media. Introducing the cylindrical coordinate (r,θ,z) of a point inside the model with the z-axis being along the axis of the cylinder as shown in Fig.1. The direction of wave propagation over the cylindrical surface is symmetric about the axis of the cylinder, consequently along the rotating angle θ. The propagation of the shear wave over the cylindrical surface is symmetric about the axis of the cylinder, so that the displacement may be assumed to be independent of z and characterized as

$$u_z = u_z(r,\theta) \tag{1}$$

The stress σ_{ij} are related to the displacement component u_z by the following relations

$$\sigma_{rz} = \mu u_{z,r}; \sigma_{\theta z} = \frac{1}{r}\mu u_{z,\theta} \tag{2}$$

The equations of motion have the form [6], [7]

$$\sigma_{rz,r} + \frac{1}{r}\sigma_{\theta z,\theta} + \frac{\sigma_{rz}}{r} = \rho \ddot{u}_z \tag{3}$$

The equation of motion for the shear wave propagation about the cylindrical surface can be obtained from eq. (1), (2), (3) as

$$\frac{1}{r^2}\mu u_{z,\theta\theta} + \mu u_{z,rr} + \frac{1}{r}\mu u_{z,r} = \rho \ddot{u}_z \tag{4}$$

3. Formulation of the problem

We consider a model which is constituted by three concentric isotropic media with elastic constant of the cylindrically isotropic material and they are defined as

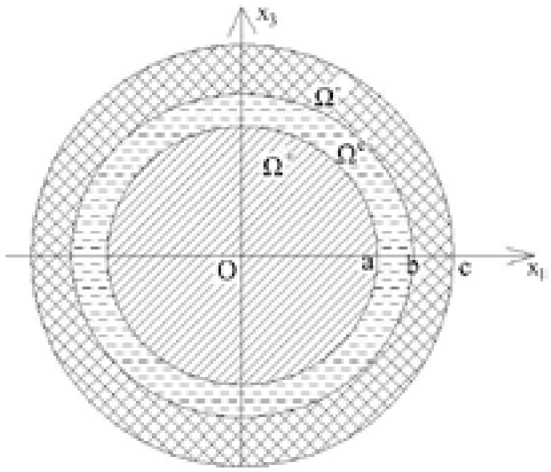

Figure 1. Geometry of the problem

$$\mu,\rho=\begin{cases}\mu^{+},\rho^{+} & in \quad \Omega^{+}\\ \mu^{e},\rho^{e} & in \quad \Omega^{e}\\ \mu^{-},\rho^{-} & in \quad \Omega^{-}\end{cases} \tag{5}$$

Therefore, the motion of SH-wave in three concentric isotropic homogeneous media is given

$$\begin{cases} r^2u_{z,rr}+u_{z,\theta\theta}+ru_{z,r}=\dfrac{r^2}{\beta_+^2}\ddot{u}_z, 0<r<a\\ r^2u_{z,rr}+u_{z,\theta\theta}+ru_{z,r}=\dfrac{r^2}{\beta_e^2}\ddot{u}_z, a<r<b\\ r^2u_{z,rr}+u_{z,\theta\theta}+ru_{z,r}=\dfrac{r^2}{\beta_-^2}\ddot{u}_z, b<r<c\end{cases} \tag{6}$$

where $\beta_+^2=\dfrac{\mu^+}{\rho^+};\beta_-^2=\dfrac{\mu^-}{\rho^-};\beta_e^2=\dfrac{\mu^e}{\rho^e}$.

Using the transformation $u_z=U_z(r)e^{i\omega t}\cos(n\theta)$ [4], [5], [9], the system equations (6) take the form

$$\begin{cases} r^2U_{z,rr}+rU_{z,r}+(\dfrac{r^2\omega^2}{\beta_+^2}-n^2)U_z=0, 0<r<a\\ r^2U_{z,rr}+rU_{z,r}+(\dfrac{r^2\omega^2}{\beta_e^2}-n^2)U_z=0, a<r<b\\ r^2U_{z,rr}+rU_{z,r}+(\dfrac{r^2\omega^2}{\beta_-^2}-n^2)U_z=0, b<r<c\end{cases} \tag{7}$$

where n is positive integer.

The solutions of (7) for innermost layer medium $0<r<a$ (the shear wave dies out with increase in depth as we approach towards the origin $r\to 0)$ may be written as [1], [2]

$$U_z=A_1J_n\left(\frac{\omega r}{\beta_+}\right) \tag{8}$$

The solutions of $(7)_2$ and $(7)_3$ for intermediate layer $a<r<b$ and outermost layer medium $b<r<c$ can be obtained as [8], [9]

$$U_z=A_2H_e^{(1)}\left(\frac{\omega r}{\beta_e}\right)+A_3H_e^{(2)}\left(\frac{\omega r}{\beta_e}\right), a<r<b$$

$$U_z=A_4H_n^{(1)}\left(\frac{\omega r}{\beta_-}\right)+A_5H_n^{(2)}\left(\frac{\omega r}{\beta_-}\right), b<r<c \tag{9}$$

where $A_i,(i=\overline{1,5})$ arbitrary constants, $J_n;H_n^{(1)};H_n^{(2)}$ are Bessel's function, Hankel's functions of first and second kind of order n, respectively.

Recalling the inversion formula of the finite transformation, the displacement components for three respective layer media may further be written as [8],[9]

$$\begin{cases} u_z=A_1J_n\left(\dfrac{\omega r}{\beta_+}\right)e^{i\omega t}\cos\left(n\theta\right), 0<r<a\\ u_z=\left(A_2H_e^{(1)}\left(\dfrac{\omega r}{\beta_e}\right)+A_3H_e^{(2)}\left(\dfrac{\omega r}{\beta_e}\right)\right)e^{i\omega t}cos\left(n\theta\right), a<r<b\\ u_z=\left(A_4H_n^{(1)}\left(\dfrac{\omega r}{\beta_-}\right)+A_5H_n^{(2)}\left(\dfrac{\omega r}{\beta_-}\right)\right)e^{i\omega t}cos\left(n\theta\right), b<r<c\end{cases} \tag{10}$$

4. The dispersion equation of SH wave

The following conditions concerned with the continuity of stresses u_z and displacement u_z at the interfaces r = a and r = b as well as the free stress $\sigma_{rz}=0$ at the outermost surface r = c. Using the relation stress σ_{rz} to the displacement component u_z in (2) and taking into account (10), we have five equations for five constants $A_1...A_5$, namely

$$\begin{aligned} &A_1J_n\left(\frac{\omega r}{\beta_+}\right)=A_2H_e^{(1)}\left(\frac{\omega r}{\beta_e}\right)+A_3H_e^{(2)}\left(\frac{\omega r}{\beta_e}\right), at \quad r=a\\ &A_2H_e^{(1)}\left(\frac{\omega r}{\beta_e}\right)+A_3H_e^{(2)}\left(\frac{\omega r}{\beta_e}\right)=\\ &=A_4H_n^{(1)}\left(\frac{\omega r}{\beta_-}\right)+A_5H_n^{(2)}\left(\frac{\omega r}{\beta_-}\right), at \quad r=b\\ &A_1\frac{\mu^+}{\beta_+}J_n^{'}\left(\frac{\omega r}{\beta_+}\right)=\\ &=\frac{\mu_e}{\beta_e}\left[A_2H_e^{'(1)}\left(\frac{\omega r}{\beta_e}\right)+A_3H_e^{'(2)}\left(\frac{\omega r}{\beta_e}\right)\right], at \quad r=a\\ &\frac{\mu_e}{\beta_e}\left[A_2H_e^{'(1)}\left(\frac{\omega r}{\beta_e}\right)+A_3H_e^{'(2)}\left(\frac{\omega r}{\beta_e}\right)\right]=\\ &=\frac{\mu^-}{\beta_-}\left[A_4H_n^{'(1)}\left(\frac{\omega r}{\beta_-}\right)+A_5H_n^{'(2)}\left(\frac{\omega r}{\beta_-}\right)\right], at \quad r=b\\ &\frac{\mu^-}{\beta_-}\left[A_4H_n^{'(1)}\left(\frac{\omega r}{\beta_-}\right)+A_5H_n^{'(2)}\left(\frac{\omega r}{\beta_-}\right)\right]=0, at \quad r=c \end{aligned} \tag{11}$$

where prime (') appearing in the superscript denotes the derivative of the quantity with respect to r. Eliminating arbitrary constants $A_1...A_5$ from system (11), we get the dispersion relation of SH wave propagating in a cylindrical structure constituted by three concentric isotopic layered media with different width, which includes Bessel's functions of first and second kind along with their derivatives.

The numerical calculation has been carried out for illustrating the theoretical results obtained in the preceding sections. The following data [2] have been taken into account

+) For the innermost layer medium Ω^+:

$$\mu^+ = 18.32\times10^{10} N/m^2; \rho^+ = 4700 kg/m^3$$

+) For the innermost layer medium Ω^-:

$$\mu^- = 3.23\times10^{10} N/m^2; \rho^- = 2802 kg/m^3$$

+) For the intermediate layer medium Ω^e:

$$\mu^e = 6.248\times10^{10} N/m^2; \rho^e = 3155 kg/m^3$$

The dispersion curve describes the variation of dimensionless phase velocity $x=c/\beta_+$ against dimensionless wave number $ep = k.h$ (h=c-a) for the SH wave propagating in three concentric cylinders have been plotted in Fig.2. It can be observed that the dimensionless phase velocityx decreases rapidly as the value of the dimensionless wave number ep increases from 0.6 to 0.9. In comparison with the model of Kumar [2], the domain of ep in which the dimensionless phase velocity exists is smaller.

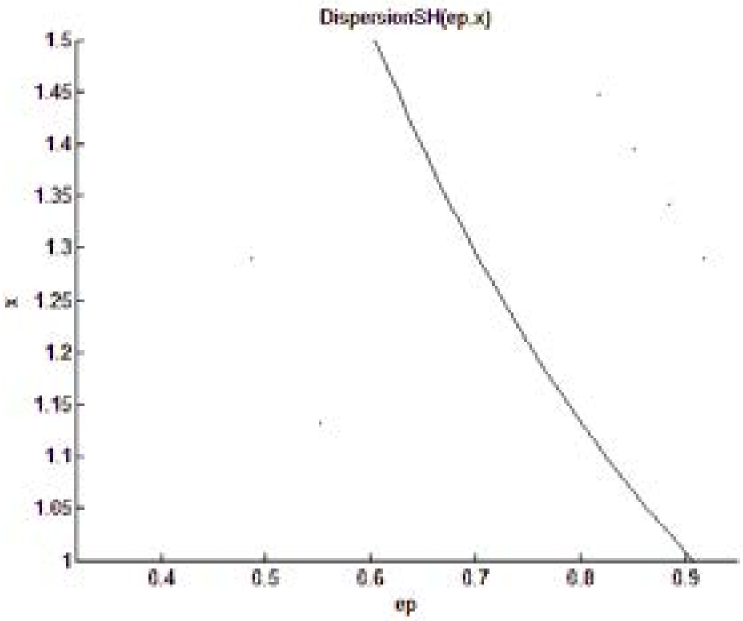

Figure 2. Variation of the dimensionless phase velocity with dimensionless wave number

5. Conclusions

This paper deals with the propagation of the shear wave in an infinitely long horizontal cylindrical structure which is comprised of three isotropic elastic concentric media with distinct radii. The dispersion equation has been obtained based on Bessel's and Hankel's functions. The effects of dimensionless wave number on the propagation of the shear wave have been accomplished by numerical simulation and depicted graphically./.

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