

**COMPUTER SIMULATION OF RADIATION FROM DIELECTRIC-WEDGE ANTENNA BY THE GUIDED-MODE EXTRACTED INTEGRAL EQUATIONS**  
**TÍNH TOÁN MÔ PHÒNG THUỘC TÍNH BỨC XẠ TỪ ANTEN ĐIỆN MÔI HÌNH NÊM BẰNG CÁC PHƯƠNG TRÌNH TÍCH PHÂN TÁCH MỘT DẪN**

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**ABSTRACT**

*Radiation properties of a dielectric-wedge antenna fed by a dielectric slab waveguide of the same material is accurately analyzed by using new boundary integral equations that are called guided-mode extracted integral equations. The derived boundary integral equations in this paper can be solved numerically by using the boundary-element method. Computer simulation results are confirmed by the use of energy conservation law. Typical results of reflected and scattered powers, as well as radiation patterns are presented. With certain conditions, it is shown that almost 100% of incident power is radiated.*

**TÓM TẮT**

*Bài toán bức xạ từ một anten điện môi hình nêm tiếp nguồn bởi một phiến dẫn sóng điện môi có cùng vật liệu được phân tích một cách chính xác bằng các phương trình tích phân biên mới được gọi là các phương trình tích phân tách một dẫn. Các phương trình tích phân biên được xây dựng từ bài toán này có thể được giải trực tiếp bằng phương pháp phần tử biên thông thường. Các kết quả mô phỏng được kiểm tra thông qua định luật bảo toàn năng lượng. Một số kết quả điển hình của công suất phản xạ và công suất tán xạ, cũng như giản đồ bức xạ được trình bày. Với một số điều kiện nhất định, các kết quả tính toán chỉ ra rằng gần như 100% năng lượng đưa tới anten được bức xạ ra ngoài không gian, đồng nghĩa với tỷ số sóng đứng rất thấp.*

**I. INTRODUCTION**

The problem of radiation from tapered dielectric waveguides has been attracted a great deal of attention for a long time due to their potential uses in millimeter-wave and integrated optical devices [1,2]. For the case of the structure to be an antenna, the coupling efficiency to the radiation field must be affected. By tapering the end section of a dielectric waveguide along its axis, a guided surface wave field gets transformed into a radiation field which is characterized by maximum intensity in the forward direction. Tapering the dielectric waveguide, as opposed to suddenly truncating it, will significantly reduce the Voltage Standing Wave Ratio (VSWR) on the uniform part of the waveguide and improve the radiation characteristics of the tapered part, thus resulting in an antenna of improved performance over a wide frequency band [3].

Tapered dielectric antennas have been known for over fifty years, but have not been

widely used as microwave antennas. Meanwhile, an accurate antenna theory has not been available for their design, most likely because the taper geometry does not allow for a convenient representation in a separable geometry. However, with the development of low-cost silicon and the availability of solid state energy sources, interest in tapered dielectric antennas has been grown.

Recently, rigorous solutions for the integrated dielectric-wedge antennas (DWA) have become available [3,4]. In [3], the field solution was obtained for the case that the fundamental TE and TM guided wave of the slab is incident upon and excites the wedge antenna. Solution of the problem was accomplished by modeling the tapered region by the staircase approximation. In [4], the local mode theory together with Schelkunoff equivalence principle was used to obtain the solution also for the fundamental TE and TM guided wave, in which the reflection coefficient is regarded as negligible.

As can be seen, unfortunately, since both the above-mentioned studies have based on the approximation solution, it seems to be impossible to calculate accurately the reflection coefficient of the incident guided-mode. Let us note that the power-reflected coefficient plays an important role in such application as ground penetrating radar (GPR) [5]. So, the technique that gives accurate reflection coefficient for the problem of DWA is the important subject.

In this paper, the radiation properties of a DWA fed by the slab waveguide of the same material are accurately analyzed by using the boundary-element method (BEM) based on the guided-mode extracted integral equations (GMEIEs). By treating this problem, we can easily understand the advantages of GMEIEs compared with other techniques proposed before. Because the method in this paper does not employ any approximation, the results are accurate in principle. The numerical results of computer simulations are presented, in which, the reflection coefficient, the reflected and radiated powers as well as the radiation pattern are calculated numerically for the incidence of fundamental TE guided wave. The results are compared with those reported in the literature, and are confirmed by the law of energy conservation.

## II. BOUNDARY INTEGRAL EQUATIONS

The theory for derivation of the new boundary integral equations (BIE) is presented here for the case of TE guided wave. For the TM case, since the analysis follows an analogous procedure, only the end formulas are shown.

A semi-infinite dielectric slab waveguide of thickness  $W$  feeding a DWA of length  $L_w$  of the same lossless material is depicted in Figure 1. The fundamental even TE guided wave of the infinite dielectric slab waveguide is assumed to be incident in the  $+z$  direction from  $z = -\infty$ . This mode does not experience cutoff, and hence is propagated also by very thin dielectric slabs. Because of this excitation and the geometry, the field is independent of  $y$  and is TE everywhere with field components  $E_y$ ,  $H_x$ , and  $H_z$ . A time

dependence of  $\exp(+j\omega t)$  is assumed and suppressed, where  $\omega$  is radian frequency.

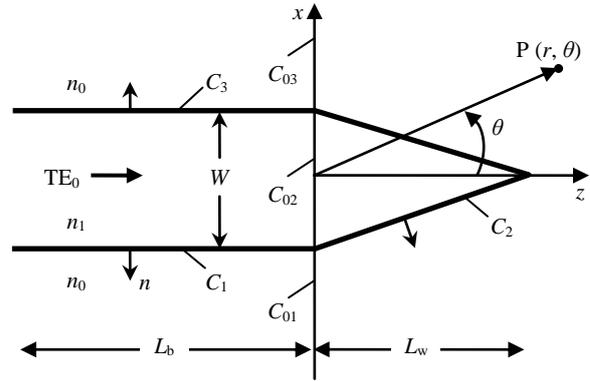


Fig. 1 DWA under consideration

In order to derive the BIEs, first we denote the  $y$ -component of the electric field by  $E(x)$ . The guided incidence and reflection waves are denoted by  $E^{(-)}(x)$  and  $RE^{(+)}(x)$ , respectively, where  $R$  is reflection coefficient. The essential idea of the new BIEs used in this paper is truncation of the semi-infinite longitudinal boundaries by assistance of the virtual boundary  $C_{01} + C_{02} + C_{03}$ , as shown in Figure 1. Let us note that the virtual boundary can be placed at any position on the left-hand side (LHS) of the wedge antenna part.

We first consider the case in which an observation point  $x$  is in the region surrounded by the boundary  $C = C_1 + C_2 + C_3$ . From Maxwell equations and Green's theorem, the well-known BIE for the total electric field  $E(x)$  is given by

$$E(x) = \int_C \left[ G_1(x/x') \frac{\partial E(x')}{\partial n'} - E(x') \frac{\partial G_1(x/x')}{\partial n'} \right] dl', \quad (1)$$

where  $\partial/\partial n'$  denotes the derivative with respect to the unit normal vector to the boundary  $C$ , as shown in Figure 1. In Eq. (1),  $G_1(x|x')$  represents Green's function in free space, whose refractive index is given by  $n_1$ , and is express as

$$G_1(x/x') = -\frac{j}{4} H_0^{(2)}(n_1 k_0 |x - x'|), \quad (2)$$

with  $H_0^{(2)}(x)$  denotes the zeroth-order Hankel function of second kind. As can be seen, it is difficult to solve the BIE (1) directly by using

the BEM or MoM because of that the BIE (1) has semi-infinite integral boundaries  $C_1$  and  $C_3$ . To overcome this difficulty, we use the previously proposed idea [6], [7] that: Even though the total electric fields near the wedge antenna are very complicated, only the reflected guided wave can survive at points far away from the antenna. Therefore we decompose the total electric fields on the boundary  $C_1 + C_3$  into the field components as

$$E(x) = E^-(x) + RE^+(x) + E^C(x), \quad (3)$$

and we call the field  $E^C(x)$  the disturbed field. In Eq. (3),  $R$  is the reflection coefficient. We also express the total electric fields on the boundary  $C_2$  by the same notation,  $E^C(x)$ , with the disturbed field. It is possible to consider that the disturbed field in (3) will vanish at points far away from the aperture.

Substituting field components on the boundaries into Eq. (1), we obtain an integral equation that includes the semi-infinite line integrals of the guided wave along the boundary  $C_1 + C_3$  as follows:

$$E(x) = \int_C \left[ G_1(x|x') \frac{\partial E^C(x')}{\partial n'} - E^C(x') \frac{\partial G_1(x|x')}{\partial n'} \right] dl' - U_1^-(x) - RU_1^+(x), \quad (4)$$

where

$$U_1^\pm(x) = \int_{C_{oi}} \left[ G_1(x|x') \frac{\partial E^\pm(x')}{\partial n'} - E^\pm(x') \frac{\partial G_1(x|x')}{\partial n'} \right] dl', \quad (5)$$

Using the asymptotic expression for the Hankel function we can obtain the field  $E^C(r, \theta)$  ( $r \rightarrow \infty$ ) except for guided waves by the cylindrical coordinates  $(r, \theta)$  as

$$\frac{1}{2} E^C(r, \theta) = -\frac{j}{4} \left( \frac{2j}{\pi n_1 k_0 r} \right)^{\frac{1}{2}} \exp(-jn_1 k_0 r) B_1(\theta), \quad (6)$$

where

$$B_1(\theta) = \int_C \left[ g_1(\theta|x') \frac{\partial E^C(x')}{\partial n'} - E^C(x') \frac{\partial g_1(\theta|x')}{\partial n'} \right] dl' - Ru_1^+(\theta) - u_1^-(\theta), \quad (7)$$

$$u_1^\pm(\theta) = \int_{C_{oi}} \left[ g_1(\theta|x') \frac{\partial E^\pm(x')}{\partial n'} - E^\pm(x') \frac{\partial g_1(\theta|x')}{\partial n'} \right] dl', \quad (8)$$

$$g_1(\theta|x') = \exp(jn_1 k_0 (z' \cos \theta + x' \sin \theta)), \quad (9)$$

The coefficient  $B(\theta)$  represents a kind of radiation coefficient. Since it is impossible for the radiation field to exist at points far away from the tilted end facet at  $\theta = \pi$ , we can set

$$B_1(\pi) = 0 \quad (10)$$

Since condition (10) gives a linear relation for unknown  $R$ , by substituting Eq. (7) into Eq. (10), we can express the reflection coefficient in the waveguide in term of the boundary integral of the field  $E^C(x)$  on the boundary  $C$ . Substituting the expression of  $R$  into the original integral equation (4), we can obtain a new type of BIE as

$$\frac{1}{2} E^C(x) = \int_C \left[ P_1(x|x') \frac{\partial E^C(x')}{\partial n'} - E^C(x') \frac{\partial P_1(x|x')}{\partial n'} \right] dl' - S_1(x), \quad (11)$$

where

$$P_1(x|x') = G_1(x|x') - g_1(\pi|x') \frac{U_1^+(x)}{u_1^+(\pi)} \quad (12)$$

$$S_1(x) = U_1^-(x) - u_1^-(\pi) \frac{U_1^+(x)}{u_1^+(\pi)}. \quad (13)$$

As can be seen, the guided waves  $E^\pm(x)$  have been extracted from the original BIE (1), so that we call BIE (11) the guided-mode extracted integral equation.

By using the same procedure as that used in the above derivation of BIE (11) for

the case in which the observation point  $x$  in the surround region, we can easily derive another BIE in the same form as BIE (11).

Once the fields on all the boundaries have been obtained, the reflection coefficient  $R$  can be evaluated numerically, and fields at any points can also be calculated by boundary integral representations. So far, we have discussed the case of TE guided wave incidence. For the TM case, we can derive the same integral equations as those for TE guided wave incidence when we adopt the  $y$ -component of magnetic field  $H(x)$  instead of the  $y$ -component of the electric field  $E(x)$ .

### III. NUMERICAL RESULTS AND DISCUSSION

In order to verify the accuracy of the present method, we use the law of energy conservation to check the simulated results. As shown in Table 1, the results satisfy the law of energy conservation within accuracy of 1% well, also do not depend on the normalized waveguide boundary discretization size. The parameters for simulation are:  $n_1 = 2.56$ ,  $n_0 = 1$ ,  $k_0W = 1.256$ ,  $k_0L_W = 16$ , where  $k_0$  is the wave number in free space.

Table 2 compares the directivity  $D_{\max}$  and reflection coefficient  $|\Gamma|$  for  $n_1 = 1.6$ . The results show an excellent agreement with those appeared in the literature [8].

Typical radiation patterns are presented in Figure 2(a)-(d). It can be observed that by increasing the length of wedge antenna and/or decreasing the waveguide width and/or decreasing the refractive index of the waveguide, the narrow radiation beam can be obtained. This feature is very important for the GPR application, in which narrower radiation beam gives higher radar resolution, also with higher throughput deeper penetration into the ground can be performed.

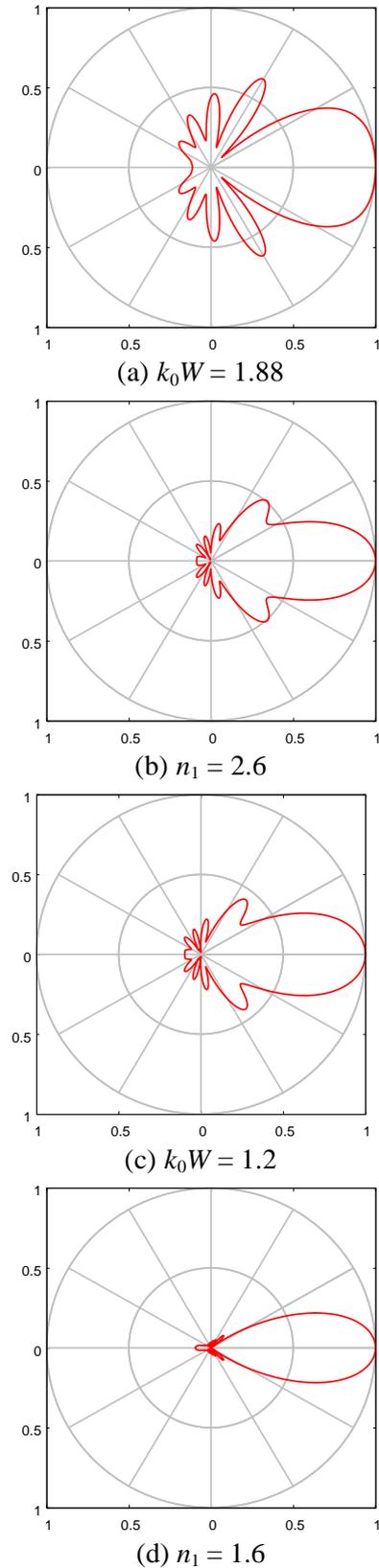


Fig. 2 Typical radiation patterns of the dielectric-wedge antenna

Table 1. Dependence of power reflection coefficient  $\Gamma_R$ , normalized scattering power  $\Gamma_S$ , and their total  $\Gamma_{total}$  for fundamental TE guided wave incidence on the normalized discretization size  $\Delta_b$ .

$\Delta_b$	$\Gamma_R$	$\Gamma_S$	$\Gamma_{total}$
0.30	0.017	0.978	0.995
0.32	0.017	0.977	0.994
0.34	0.018	0.979	0.997
0.36	0.016	0.976	0.992
0.38	0.016	0.975	0.991

Table 2: Comparison of directivity  $D_{max}$  and reflection coefficient  $|\Gamma|$ .

$L_w/\lambda_0$	$D_{max}$ (dB)		$ \Gamma $	
	[8]	Present method	[8]	Present method
1	7.519	7.437	0.0429	0.0495
5	9.347	9.157	0.0023	0.0027
10	10.254	10.098	0.0015	0.0017

#### IV. CONCLUSIONS

In conclusion, the BIEs that are called GMEIEs have applied to investigation of radiation properties of the dielectric-wedge antennas. The presented results were evaluated from the viewpoint of the law of energy conservation, as well as by comparing with those reported in the literature. It is apparent that the method in this paper is suitable for the basic theory of computer-aided design (CAD) software for dielectric waveguide circuits, dielectric antennas.

In the formulation of integral equations used in this paper, we do not employ any approximations. The new BIEs can be solved numerically by the conventional BEM. It is easy to extend the GMEIEs to more complicated dielectric antennas. Since the theory is based on the exact theory, the solution can be accurate.

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