

**THE FINITE ELEMENT METHOD  
FOR SIMULATING LIGHTNING-TRANSIENT VOLTAGE  
OF GROUNDING GRID WITH THE NON-UNIFORM PARAMETERS**  
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CỦA LƯỚI NỐI ĐẤT VỚI THAM SỐ KHÔNG ĐỒNG NHẤT

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**ABSTRACT**

*In this paper, we use a non-uniform transmission line model, in which the per – unit length parameters ( $L(x,t), G(x,t), C(x,t)$ ) are dependent on space and time in [5], to simulate lightning-transient voltage of grounding systems under lightning strikes. In this model, the electromagnetic coupling between different parts of the grounding systems is taken into account, but the skin effect and soil ionization along the electrodes are not considered. The simulation results are obtained by solving the telegrapher's equations based on the time-domain finite element method (FETD). The modeling results compared with the literature data of [5] and [3] have proved the primacy and effectiveness of the proposed method.*

Keywords: Grounding system, time-domain finite element method (FETD), time-domain finite different method (FDTD).

**TÓM TẮT**

*Trong bài báo này, chúng tôi sử dụng mô hình đường dây truyền tải không đồng nhất, trong đó các thông số trên đơn vị dài ( $L(x,t), G(x,t), C(x,t)$ ) thay đổi theo vị trí và thời gian, để mô phỏng điện áp quá độ sét của các hệ thống nối đất do sét gây ra. Trong mô hình này, ảnh hưởng tương hỗ của trường điện từ giữa các phần của hệ thống nối đất được tính đến, tuy nhiên hiệu ứng bề mặt và sự ion hóa của đất dọc theo chiều dài điện cực được bỏ qua. Các kết quả mô phỏng thu được bằng cách giải hệ phương trình truyền sóng dựa trên phương pháp phần tử hữu hạn trên miền thời gian. Kết quả tính toán trên thanh và lưới nối đất được so sánh với các tài liệu tham khảo cho thấy tính ưu việt và hiệu quả của phương pháp được đề xuất.*

**I. INTRODUCTION**

The grounding system which contains a network of vertical and horizontal conductors is one of the most important parts of power systems such as the grounding of power plant, substations, towers, etc. Therefore, calculating and modeling of the lightning-transient behaviour of grounding systems under lightning strikes are the important tasks for designing and operating electrical systems. Many numerical models have been developed since 1970's. Generally, we can classify them into three following categories (the accuracy increases along the order).

- Circuit approach
- Transmission line approach
- Electromagnetic field approach

The last model [3] gives the most accurate results, but it needs a very long time to calculate. Whereas the second one gives quite accurate results with a shorter time. Many numerical methods have been used in these models like time-domain finite different method (FDTD) [4,5].

In this paper, we focus on two points as i) the first one is to simulate the lightning-transient voltage of some standard-designed grounding grids [2]; ii) the second one is to study the effect of vertical rods which are installed along the perimeter and the diagonal of grounding grids. The time-domain finite element method and the nonuniform transmission line model are used to simulate the transient response of these grounding grids under lightning strikes.

## II. NON-UNIFORM TRANSMISSION LINE MODEL

In the transient analysis of transmission line model, the distribution of voltage and current along an electrode satisfies the telegrapher's equations as follows

$$\begin{cases} -\frac{\partial v(x,t)}{\partial x} = Ri(x,t) + L(x,t)\frac{\partial i(x,t)}{\partial t}, \\ -\frac{\partial i(x,t)}{\partial x} = G(x,t)v(x,t) + C(x,t)\frac{\partial v(x,t)}{\partial t}. \end{cases} \quad (1)$$

The equations can be presented by a equivalent circuit in Fig.1. In this,  $R$ ,  $L(x,t)$ ,  $G(x,t)$  and  $C(x,t)$  are per – unit length parameters of grounding electrode (per – unit length resistance, inductance, conductance and capacitance of the electrode, respectively).

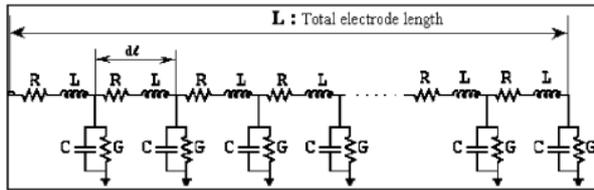


Fig. 1 Equivalent circuit of a grounding electrode

In this model, the per – unit length parameters are the functions of space and time. The value of these parameters can be calculated using formulae in [5].

## III. TIME-DOMAIN FINITE ELEMENT METHOD (FETD)

First, this method requires the electrode to be subdivided into many finite elements. Each element has two voltage points of  $v$  and a current point of  $i$  called interpolation nodes. So, we can write  $v$  and  $i$  in the forms as

$$\begin{cases} v(x,t) = \sum_{i=1}^M \Phi_i(x)V_i(t), \\ i(x,t) = \sum_{j=1}^N \Psi_j(x)I_j(t). \end{cases} \quad (2)$$

where  $M$  and  $N$  are the number of nodes  $v$  and  $i$ , respectively, and  $\Phi_i$  and  $\Psi_i$  are shape functions which interpolate the voltage and current within each element. Substituting (2) into (1) and handling on the process in [1], we

have the final formulae used to calculate the voltage and current on every point of the electrode at every time step as follows

$$I_j^{n+\frac{1}{2}} = \frac{1}{R + \frac{L(x,t)}{\Delta t}} \left[ \frac{L(x,t)}{\Delta t} I_j^{n-\frac{1}{2}} - \sum_{a=1}^2 \frac{d\Phi_a}{dx} V_a(t) \right], \quad (3)$$

$$V_i^{n+1} = \frac{1}{G(x,t) + \frac{C(x,t)}{\Delta t}} \left[ \frac{C(x,t)}{\Delta t} V_i^n - \sum_{a=1}^2 \frac{d\Psi_a}{dx} I_a(t) \right]. \quad (4)$$

where

- $I_j^{n-\frac{1}{2}}, I_j^{n+\frac{1}{2}}$  are current values at  $j^{th}$  point, at  $t = \left(n - \frac{1}{2}\right)\Delta t$  and  $t = \left(n + \frac{1}{2}\right)\Delta t$
- $V_i^n, V_i^{n+1}$  are voltage values at  $i^{th}$  point, at  $t = n\Delta t$  and  $t = (n+1)\Delta t$
- $V_a$  is voltage value at points close to current point  $j$
- $I_a$  is current value at points close to voltage point  $i$

Using (3) and (4), we can calculate the current and voltage at every point on the electrode and at every time step. With the same procedure, the transient response of a grounding grid can be completely estimated.

## IV. NUMERICAL RESULTS

### 4.1 Verification of the proposed method

To verify the accuracy of the time-domain finite element method, we simulate lightning-transient response of some horizontal conductors and some grounding grids, and then compare the results with the ones obtained by FDTD in [5] using the same model. After that, we make another simulation for a grounding grid and compare the results with the ones in [3] which are obtained by using electromagnetic field approach.

#### 1. Horizontal grounding conductors [5]

In Fig.2., the radius of conductors is 7.5 mm. These conductors are buried at 0.5 m in the soil with  $\rho = 100 \Omega m$ ,  $\epsilon_r = 50$  and  $\mu_r = 1$ . The lightning current used in this simulation is a fast impulse one,  $1/5\mu s$  wave shape with the double-exponential function

$I_s(t) = 12935 \cdot (e^{-190099t} - e^{-2922879t})(A)$ . It is injected at the end of the conductors.

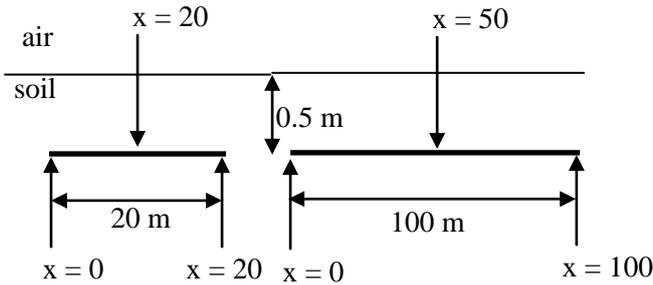


Fig. 2 Model of 20 m and 100 m horizontal conductors

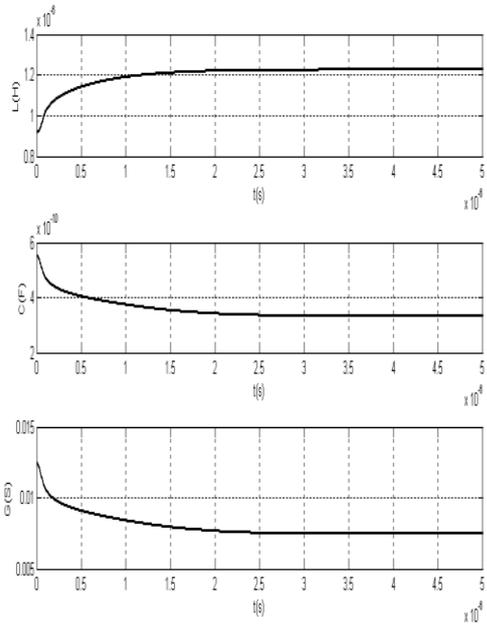


Fig. 3 The value of per-unit length parameters  $L$ ,  $C$  and  $G$  in the first segment of 20 m conductor in a interval from 0 to  $0.5\mu s$

As shown in Fig.3., the per – unit length parameters ( $L(x,t)$ ,  $G(x,t)$  and  $C(x,t)$ ) used in this model are function of time and space due to the coupling effect between any two segments on the conductor. The further the impulse current pervades, the more coupling effect increases. This phenomenon increases the per – unit length inductance, and decreases the per – unit length capacitance and conductance as in Fig.3. Therefore the results obtained in this model are able to describe the fundamental nature of the transient phenomenon of grounding systems.

The comparison of the lightning-transient voltages at different points using FETD and the ones using FDTD are illustrated in Fig. 4. and Fig .5., respectively. We recognize that the results of the two methods are the same. This proves that the time-domain finite element method used in this model is suitable for estimating transient behaviour of any grounding systems.

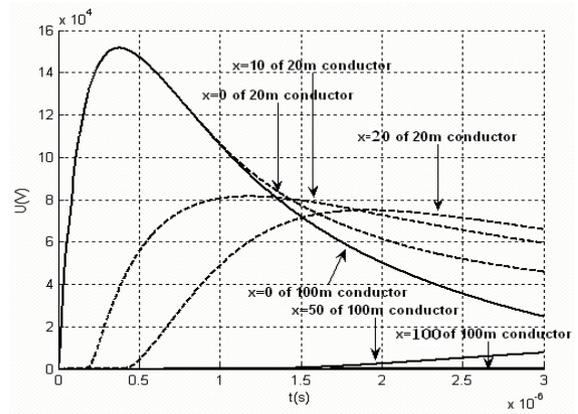


Fig. 4 Transient voltage at different positions of 20m and 100m conductor using FETD

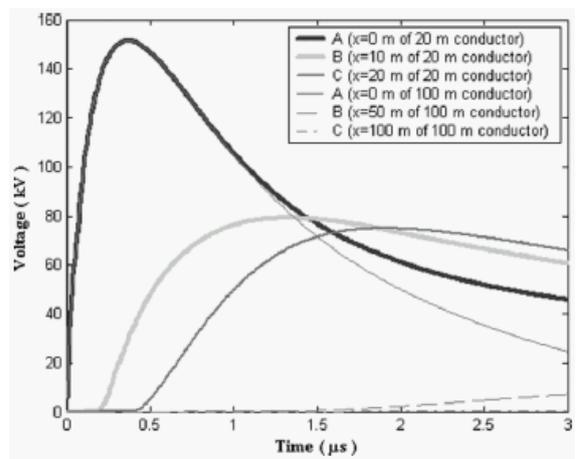


Fig. 5 Transient voltage at different positions of 20m and 100m conductor using FDTD in [5]

2. Grounding grid [5]

As shown in Fig. 6., the radius of conductors is 7 mm. These grid is buried at 0.5 m in the soil with  $\rho = 1000 \Omega m$ ,  $\epsilon_r = 9$  and  $\mu_r = 1$ . The lightning current used in this case is the double-exponential function  $I_s(t) = 1(e^{-27000t} - e^{-560000t})(A)$ . It is injected at the corner of the grounding grids.

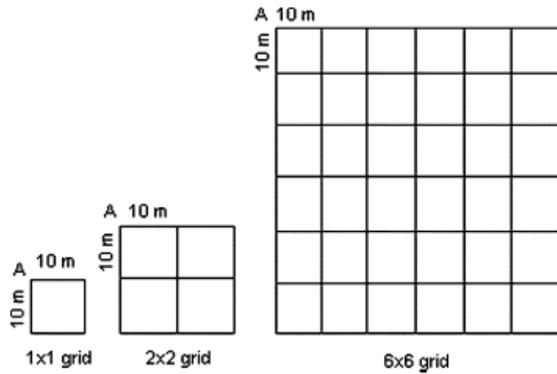


Fig. 6 Grounding grid 1x1, 2x2 and 6x6

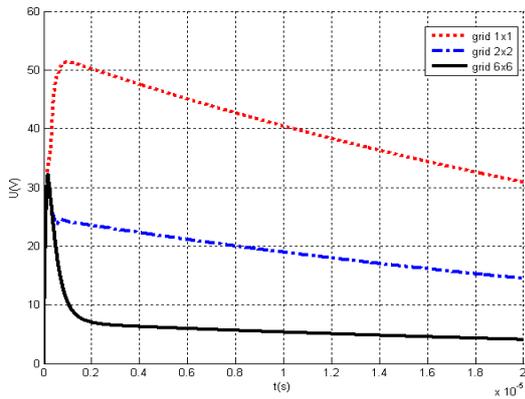


Fig. 7 Transient voltage at the injected position on grids 1x1, 2x2 and 6x6 using FETD

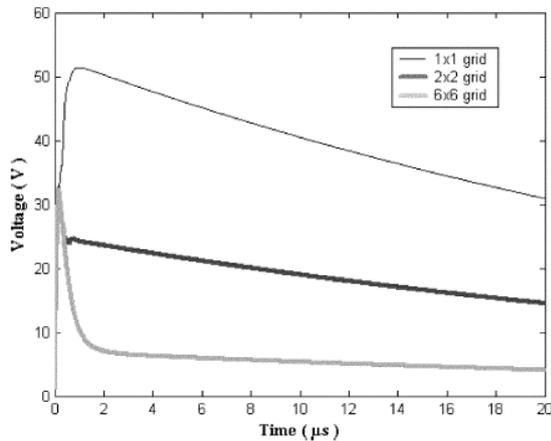


Fig. 8 Transient voltage at the injected position on grids 1x1, 2x2 and 6x6 using FDTD in [5]

Comparing the results in Fig. 7. and Fig.8, we can come to the same conclusion that the FETD are suitable in simulation the transient phenomenon on the grounding grids.

### 3. Grounding grid [3]

In this simulation, the radius of conductors is 7 mm. This grid 6x6, as in Fig. 6.

is buried at 0.5 m in the soil with  $\rho = 100 \Omega m$ ,  $\epsilon_r = 36$  and  $\mu_r = 1$ . The lightning current used in this case is 1/50  $\mu s$  wave shape one with the double-exponential function  $I_s(t) = 1.0167(e^{-14200t} - e^{-5073000t})(kA)$ . It is injected at the corner of the grounding grid.

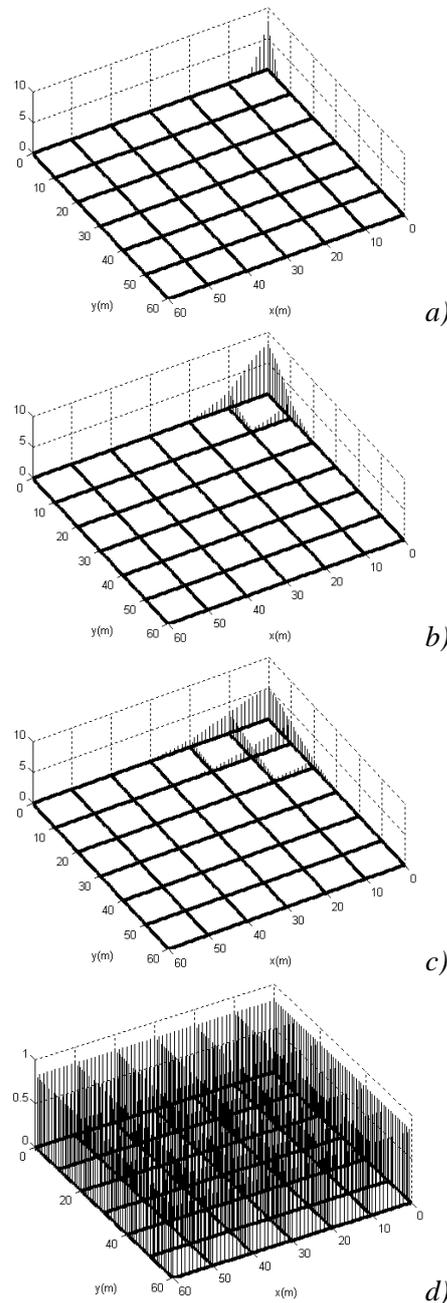


Fig. 9 Distribution of transient voltage on the grounding grid at: a) 0.1; b) 0.5; c) 1 and d) 10  $\mu s$

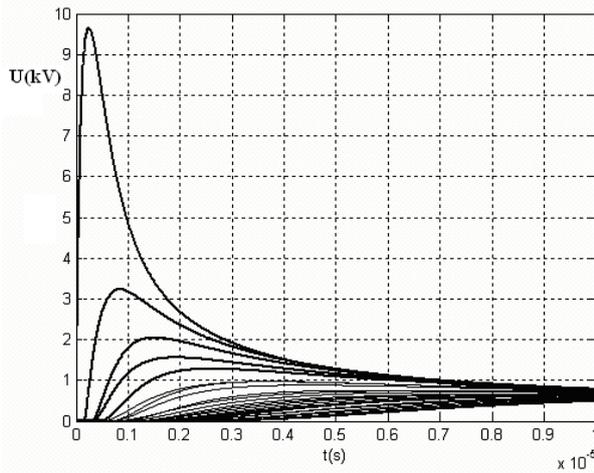


Fig. 10 Transient voltage at different intersection-points on the grounding grid

The lightning-transient voltages at the different intersection-points on the grounding grid are presented in Fig.10. It has been seen that the further intersection-points are away from the injected position of the impulse current, the smaller the transient voltages are. And after  $10\mu\text{s}$ , the transient voltages at different points become equally. This voltage value, which is called the stable voltage, is about 0.8 kV. The distribution of transient voltage on a large grounding grid can be globally observed in Fig.9. The results are similar as the ones that are obtained from a different model – the electromagnetic field approach as in [3].

**4.2 Investigating the effect of vertical rods on the grounding grid**

In order to perform this purpose, we use a impulse current,  $I_s(t) = 1(e^{-27000t} - e^{-560000t})(A)$ , with the rise time is about  $0.36\mu\text{s}$  and the decay time ( from  $I_s = 0\%$  to  $50\%$  of the maximum value) is about  $26.8\mu\text{s}$ . This impulse current is injected at the corner (position 1) and the center (position 13) of three grounding grids as in Fig. 11. The radius of horizontal conductors is 7 mm, the length and radius of vertical conductors (rods) are 3 m and 15 mm, respectively. These grids are buried at the depth of 0.5m in the soil with  $\rho = 1000\Omega\text{m}$ ,  $\epsilon_r = 9$  and  $\mu_r = 1$ .

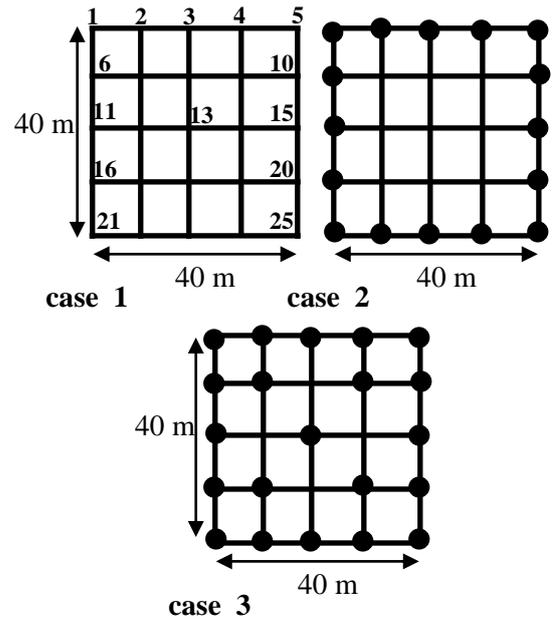


Fig. 11 Grounding grids[4]: case 1: without rods, case 2: with rods along the perimeter, case 3: with rods along the perimeter and diagonal.

Table.1. Comparison of peak values

Peak voltage(V)	Corner	Center
Case 1	32.2339	18.9398
Case 2	28.8184	18.9295
Case 3	28.8177	18.2063

As shown in Fig.12.-.13., installing rods along the perimeter and diagonal increases the ability of grounding grids to dissipate the lightning current into the ground. Thus, the transient voltage at the injected position on the grounding grids with rods in their perimeter and diagonal is smaller than the one of the grounding grid without rods. However, there is a small difference in transient voltage at the injected position between the one with rods in the perimeter and the one with rods in both the perimeter and the diagonal as TABLE.1. Therefore, to decrease the transient voltage on the grounding grids in the economic way, we just need to install the rods along the perimeter of the grounding grids.

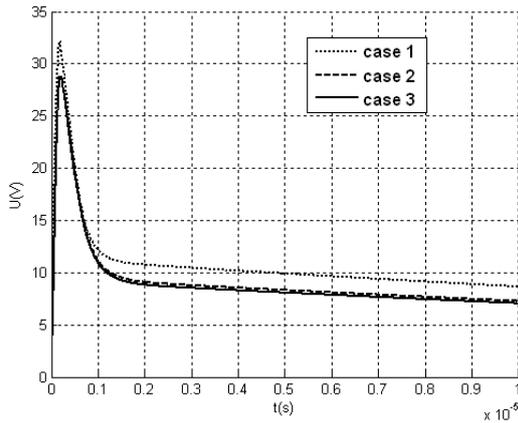


Fig. 12 Lightning-transient voltage at position 1 on the three grounding grids when the impulse current is injected at this position

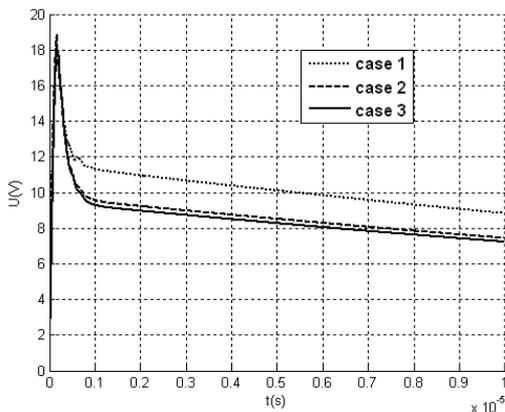


Fig. 13 Lightning-transient voltage at position 13 on the three grounding grids when the impulse current is injected at this position

## V. CONCLUSIONS

In our work, the time domain finite element (FETD) method used in the non-uniform transmission line model of horizontal conductor and grounding grid gives the highly accurate results when comparing the results with the ones of the time-domain finite different method in [4] and the analytical method in electromagnetic field model in [3].

The vertical grounding conductors which are installed along the perimeter of grounding grids can increase the ability of these grids in dissipating the lightning current into the ground. If we continue to install these rods along the diagonal of these grids, the above effect will only be considerable when the lightning current is injected at the center position.

In this paper, the soil ionization phenomenon is not included. The greater the lightning current is, the more powerfully this phenomenon occurs. In order to take into account this phenomenon in the non-uniform transmission line model, we have to compute the electric field along the conductors of the grounding grids and compare it with the critical value of soil ionization to determine the expanse of the radius of these conductors. Then, the per – unit length parameters on these conductors must be calculated again using the “new” radius. This problem will be presented in the next paper.

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