THE AUTOPILOT UNDERWATER VEHICLE FOR DEPTH CONTROL
MỘT THIẾT BỊ LÁI TỰ ĐỘNG ĐIỀU KHIỂN ĐỘ SÂU CỦA TÀU NGẦM

Pham Huu Duc Duc
University for Economic and Technical Industries

ABSTRACT

Model Predictive Control (MPC) is one of the modern control methods, which has many advantages. Nowadays, this control method has been being researched by scientists in applying it to controllers, which have high intelligence level. This article presents the Underwater Vehicle dynamic Model (UVM). Based on which, the equation of state is built to define one of the Underwater Vehicle position parameters, which is its depth. In the result, a new solution applying MPC to the Autopilot Underwater Vehicle (AUV) in case of adjusting its depth without the operator is proposed. The aim of this method is to improve the level of AUV intelligence. The results of simulation in the Matlab software shows that this solution is advantageous and feasible.

TÓM TẮT

Điều khiển mô hình dự báo là một trong những phương pháp điều khiển hiện đại có nhiều ưu điểm. Hiện nay, phương pháp này đang được các nhà khoa học nghiên cứu để ứng dụng vào các bộ điều khiển có độ thông minh cao. Bài báo này giới thiệu mô hình động học của tàu ngầm. Từ mô hình này xây dựng phương trình trạng thái xác định một trong những thông số để định vị vị trí tàu ngầm, đó là độ sâu của nó. Trên cơ sở đó đề xuất một giải pháp mới là ứng dụng phương pháp điều khiển mô hình dự báo vào thiết bị lái tự động trang bị cho tàu ngầm, khi cần điều chỉnh độ sâu của nó, không cần có người điều khiển. Mục đích của giải pháp là làm tăng thêm mức độ thông minh của thiết bị lái tự động trên tàu ngầm. Các kết quả mô phỏng trên Matlab sẽ minh chứng cho tính đúng đắn và khả thi của giải pháp này.

I. INTRODUCTION

Model Predictive Control (MPC) is one of the modern control methods, which has many advantages. Nowadays, this control method has been being researched by scientists in apply it to controllers, which have high intelligence level. This article presents the Underwater Vehicle dynamic Model (UVM) in section 2. In section 3, based on UVM, a new solution applying MPC in the Autopilot Underwater Vehicle (AUV) in case of adjusting its depth without the operator is proposed. Section 4 covers conclusion.

II. THE UNDERWATER VEHICLE DYNAMIC MODEL

UVM is depicted in figure 1 [1]. The simple form of equation of motion is obtained with body axes coincident with the principles axes of inertia, and the origin at the center of mass center of gravity (CG), in this case, the equations in the dimensionless are:

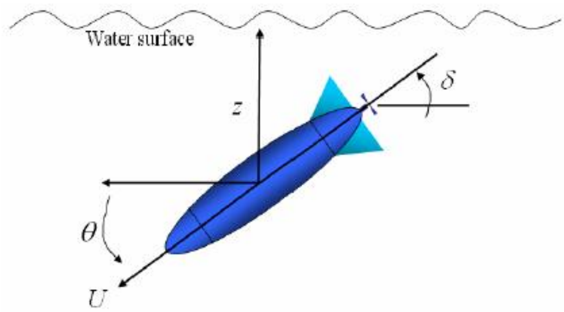

Fig. 1 The UVM.

$$\begin{aligned}
X &= m[\dot{u} + qw - vr]; \\
Y &= m[\dot{v} + ru - pw]; \\
Z &= m[\dot{w} + pv - qu]; \\
K &= I_x\dot{p} + (I_z - I_y)qr; \\
M &= I_y\dot{q} + (I_x - I_z)rp; \\
N &= I_z\dot{r} + (I_y - I_x)pq \qquad (1)
\end{aligned}$$

The 6 DOF components of the rigid body dynamic equations of motion of the submerged vehicle are:

$$X = m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})];$$

$$Y = m[\dot{v} + ur - wq + x_G(pq + \dot{r}) - y_G(p^2 + r^2) + z_G(qr - \dot{p})];$$

$$Z = m[\dot{w} - uq + vp + x_G(pr - \dot{q}) + y_G(qr + \dot{p}) - z_G(p^2 + q^2)];$$

$$K = I_x\dot{p} + (I_z - I_y)qr + I_{xy}(pr - \dot{q}) - I_{yz}(q^2 - r^2) - I_{xz}(pq + \dot{r}) + m[y_G(\dot{w} - uq + vp) - z_G(\dot{u} + vr - wp)];$$

$$M = I_y\dot{q} + (I_x - I_z)pr - I_{xy}(qr - \dot{p}) - I_{yz}(pq - \dot{r}) - I_{xz}(p^2 - r^2) + m[x_G(\dot{w} - uq + vp) - z_G(\dot{u} - vr - wq)]; \quad (2)$$

$$N = I_z\dot{r} + (I_y - I_x)pq - I_{xy}(p^2 - q^2) - I_{yz}(pr + \dot{q}) + I_{xz}(qr - \dot{p}) + m[x_G(\dot{v} + ur - wp) - y_G(\dot{u} - vr - wq)];$$

where X, Y, and Z are surge, sway, and heave force; K, M, and N are roll, pitch, and yaw moment; p, q, and r are roll, pitch, and yaw rate; u, v, and w are surge, sway, and heave velocity; x, y, and z are body fixed axes in positive forward, positive starboard, and positive down; I_x, I_y and I_z is vehicle mass moment of inertia around the x-axis, around the y-axis, and around the z-axis; x_G, y_G, and z_G are longitudinal position, athwart position, and vertical position of center of gravity; ϕ, θ, and ψ are roll, pitch, and yaw angle.

We can further simplify equations (2) by assuming that y_G is small compared to the other terms. After several steps of linearization as in [2], vertical motion equations become:

$$\dot{\theta} = q;$$

$$(m - Z_{\dot{w}})\dot{w} - (mx_G + Z_{\dot{q}})\dot{q} = Z_wUw + (m + Z_q)Uq + U^2Z_\delta\delta;$$

$$(-M_{\dot{w}} - mx_G)\dot{w} + (I_y - M_{\dot{q}})\dot{q} = -(z_GW - z_B)\theta + M_wUw + (M_q - mx_G)Uq - M_\delta U^2\delta;$$

$$\dot{z} = -U\theta + w; \quad (3)$$

It can be rewritten in the matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (m - Z_{\dot{w}}) & -(mx_G + Z_{\dot{q}}) & 0 \\ 0 & (-mx_G - M_{\dot{w}}) & (I_y - M_{\dot{q}}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{w} \\ \dot{q} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & Z_wU & (m + Z_q)U & 0 \\ -(z_GW - z_BB) & M_wU & (M_q - mx_G)U & 0 \\ -U & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ w \\ q \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ Z_\delta U^2 \\ M_\delta U^2 \\ 0 \end{bmatrix} \delta; \quad (4)$$

Then, the UVM dynamic equation typically represented using the notation:

$$\dot{x} = Ax + Bu,$$

with state vector: $x = [\theta \quad w \quad q \quad z]^T$ and the control input $u = \delta$, U is velocity of the UVM. Can be rewritten in the form:

$$\begin{bmatrix} \dot{\theta} \\ \dot{w} \\ \dot{q} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ a_{21}z_{GB} & a_{22}U & a_{23}U & 0 \\ a_{31}z_{GB} & a_{32}U & a_{33}U & 0 \\ -U & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ w \\ q \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ b_1U^2 \\ b_2U^2 \\ 0 \end{bmatrix} \delta \quad (5)$$

where:

$$Dv = (m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - (mx_G + Z_{\dot{q}})(mx_G + M_{\dot{w}});$$

$$z_{GB} = z_G - z_B;$$

$$a_{21} = -\frac{(mx_G + Z_{\dot{q}})W}{Dv};$$

$$a_{22} = \frac{(I_y + M_{\dot{q}})Z_w + (mx_G + Z_{\dot{q}})M_w}{Dv};$$

$$a_{23} = \frac{(I_y - M_{\dot{q}})(m + Z_q) + (mx_G + Z_{\dot{q}})(M_q - mx_G)}{Dv};$$

$$a_{31} = -\frac{(mx_G - Z_{\dot{q}})W}{Dv};$$

$$a_{32} = \frac{(m - Z_{\dot{w}})M_w + (mx_G + M_{\dot{w}})M_\delta}{Dv};$$

$$a_{33} = \frac{(m - Z_{\dot{w}})(Mq - mx_G) + (mx_G + M_{\dot{w}})(m - Z_q)}{Dv};$$

$$b_1 = \frac{(I_y - M_{\dot{q}})Z_\delta + (mx_G + Z_{\dot{q}})M_\delta}{Dv};$$

$$b_2 = \frac{(I_y - Z_{\dot{w}})M_\delta + (mx_G + M_{\dot{q}})Z_\delta}{Dv};$$

III. APPLICATION OF MPC IN THE AUTOPILOT UNDERWATER VEHICLE FOR DEPTH CONTROL

Block diagram apply MPC to UVM in figure 2, where r(t) is the desired depth (or setpoint), $\delta(t)$ is the control signal (or control signal input), v(t) is the measured disturbance, d(t) is the unmeasured disturbance, z(t) is the depth (or UMV output), n(t) is noise and y is the measured output

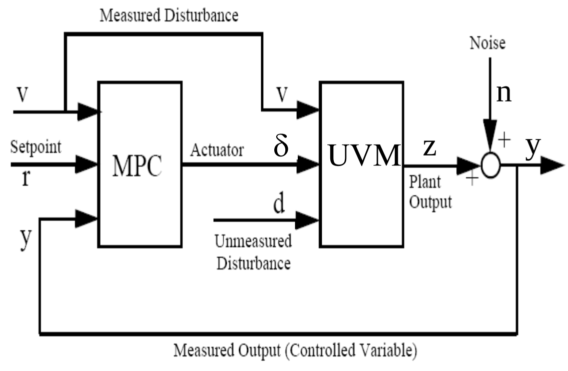

Fig. 2 Block diagram apply MPC to UVM

The problem is to find the control signal $\delta(t)$, which makes the depth z(t) follow very closely to the desired depth r(t). The below part presents the solution that applies MPC to adjust the depth of UVM.

3.1. Optimizing of AUV Standard Form

MPC action at time *k* is obtained by solving optimization problem:

$$\min_{\Delta\delta(k|k),\ldots,\Delta\delta(m-1+k|k),\,\varepsilon} \left\{ \sum_{i=0}^{p-1} \left(\sum_{j=1}^{n_y} \left| w^y_{i+1,j}(y_j(k+i+1|k) - r_j(k+i+1|k)) \right|^2 + \sum_{j=1}^{n_\delta} \left| w^{\Delta\delta}_{i,j} \Delta\delta_j(k+i|k) \right|^2 + \sum_{j=1}^{n_\delta} \left| w^{\delta}_{i,j}(\delta_j(k+i|k) - \delta_{j\,target}(k+i|k)) \right|^2 \right) + \rho_\varepsilon \varepsilon^2 \right\} \quad (6)$$

$$w^\delta = \text{blkdiag}(R_\delta, \ldots, R_\delta);$$

$$w^y = \text{blkdiag}(Q, \ldots, Q);$$

$$w^{\Delta\delta} = \text{blkdiag}(R_{\Delta\delta}, \ldots, R_{\Delta\delta}); \quad (7)$$

where: $\rho_\varepsilon = 10^5 \max\left(w^{\Delta u}_{i.j}, w^u_{i.j}, w^y_{i.j}\right)$; the block-diagonal matrices repeat p times, i.e., once for each step in the prediction horizon, the subscript (.)j denotes the j-th component of a vector, (k+i|k) denotes the value predict for time (k+i) based on the information available at time k, and r(k) is the current sample of the output reference.

Subject to:

$$\delta_{j\min}(i) - \varepsilon V^\delta_{j\min}(i) \le \delta_j(k+i|k) \le \delta_{j\max}(i) + \varepsilon V^\delta_{j\max}(i);$$

$$\Delta\delta_{j\min}(i) - \varepsilon V^{\Delta\delta}_{j\min}(i) \le \Delta\delta_j(k+i|k) \le \Delta\delta_{j\max}(i) + \varepsilon V^{\Delta\delta}_{j\max}(i);$$

$$y_{j\min}(i) - \varepsilon V^y_{j\min}(i) \le y_j(k+i+1|k) \le y_{j\max}(i) + \varepsilon V^y_{j\max}(i); \quad (8)$$

where: $\Delta\delta(k+h|k) = 0$; $h = m, \ldots, p-1$; $i = 0, \ldots, p-1$; ε is the slack variable $\varepsilon \ge 0$;

with respect to the sequence of input increments $[\Delta\delta(k|k),\ldots,\Delta\delta(m-1+k|k)]$ and to the slack variable ε, and by setting $\delta(k)=\delta(k-1)+\Delta\delta(k|k)$, where $\Delta\delta(k|k)$ is the first element of the optimal sequence; $\delta_{j\min}$, $\delta_{j\max}$, $\Delta\delta_{j\min}$, $\Delta\delta_{j\max}$, $y_{j\min}$, $y_{j\max}$ are lower/upper bounds on the corresponding variables. The Equal Concern for the Relaxation (ECR) vectors $V_{\min}^{\delta}$, $V_{\max}^{\delta}$, $V_{\min}^{\Delta\delta}$, $V_{\max}^{\Delta\delta}$, $V_{\min}^{y}$, $V_{\max}^{y}$ have nonnegative entries which represent the concern for relaxing the corresponding constraint; the larger V, the softer the constraint. V=0 means that the constraint is the hard one which cannot be violated. By default, all input constraints are hard: ($V_{\min}^{\delta}=V_{\max}^{\delta}=V_{\min}^{\Delta\delta}=V_{\max}^{\Delta\delta}=0$) and all output constraints are soft ($V_{\min}^{y}=V_{\max}^{y}=1$). As hard output constraints may cause infeasibility of the optimization problem (for instance, because of unpredicted disturbances, model mismatch, or just as numericalround off), a warning message is produced if $V_{\min}^{y}$, $V_{\max}^{y}$ be small values and automatically adjusted. Note that also ECRs can be time varying.

3.2. Alternative Cost Function

There is an option to use the following quadratic objective instead of the standard one (6):

$$J(\Delta\delta,\varepsilon)=\sum_{i=0}^{p-1}[y(k+i+1|k)-r(k+i+1)]^T Q[y(k+i+1|k)$$
$$-r(k+i+1)]+\Delta\delta(k+i|k)^T R_{\Delta\delta}\Delta\delta(k+i|k)$$
$$+[\delta(k+i|k)-\delta_{target}(k+i)]^T R_{\delta}[\delta(k+i|k)$$
$$-\delta_{target}(k+i)]+\rho_{\varepsilon}\varepsilon^2 \qquad (9)$$

where $\delta_{target}(k+i)$ is the desired depth for the control signal.

3.3. The Unconstrained MPC

The optimal solution is computed analytically in case of the unconstrained MPC:

$$\zeta=-K^{-1}\Delta\delta\left(\begin{bmatrix} r(1)\\ \ldots\\ r(p)\end{bmatrix}^T K_r+\begin{bmatrix} v(0)\\ \ldots\\ v(p)\end{bmatrix}^T K_v+\delta(-1)^T K_{\delta}\right.$$
$$\left.+\begin{bmatrix}\delta_{target}(0)\\ \ldots\\ \delta_{target}(p-1)\end{bmatrix}^T K_{\delta t}+x(0)^T K_x\right)^T \qquad (10)$$

and MPC sets: $\Delta\delta(k)=\zeta$ (11)

$$\delta(k)=\delta(k-1)+\Delta\delta(k) \qquad (12)$$

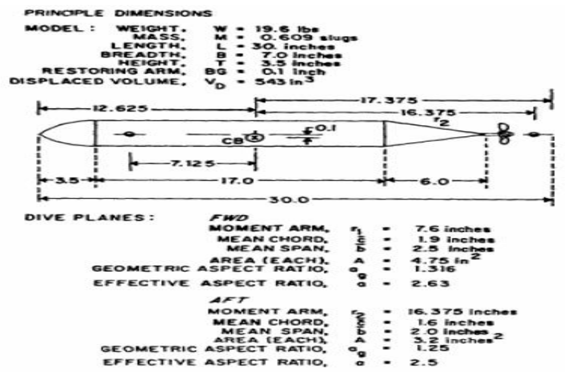

Fig. 3 Physical dimensions of UVM.

3.4. Simulations

We refer to the physical parameter of UVM in [3] (in figure 3), the state space equation is in form:

$$\begin{bmatrix}\dot{\theta}\\ \dot{w}\\ \dot{q}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0.0175 & -1.273 & -3.559 & 0\\ -0.052 & 1.273 & -2.661 & 0\\ -5 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ w\\ q\\ z\end{bmatrix}+\begin{bmatrix}0\\ 0.085\\ 21.79\\ 0\end{bmatrix}\delta \qquad (13)$$

To perform simulation in Matlab [4] with the noise n(t) in figure 5. The simulink diagram of applying MPC to AUV for depth control is in figure 4. Applying the optimal solution is computed in case of the unconstrained MPC to AUV, we get results of simulation in figures below.

The depth z(t) (solid line), and the desired depth r(t) (dashed line) in figure 6. We find that the depth z(t) follow very closely with the desired depth r(t). Figure 7 is the control signal δ(t). Figure 8 is the pitch rate q(t). Figure 9 is the heave velocity w(t). Figure 10 is the pitch angle θ(t).

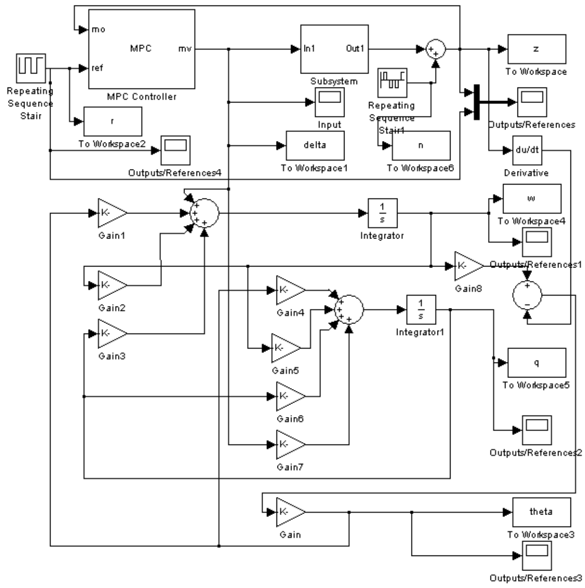

Fig. 4 The simulink diagram of applying MPC to AUV for depth control.

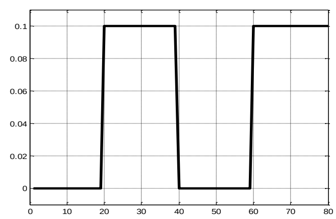

Fig. 5 The noise n(t).

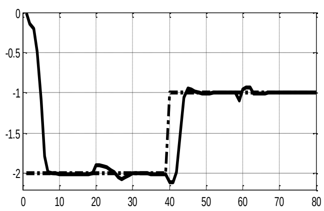

Fig. 6 The depth z(t) (solid line), and the desired depth r(t) (dashed line).

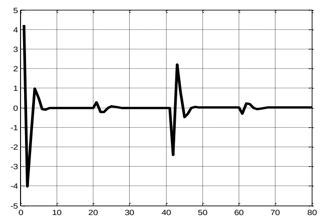

Fig. 7 The control signal δ(t).

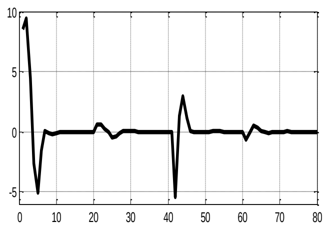

Fig. 8 The pitch rate q(t).

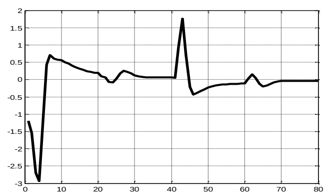

Fig. 9 The heave velocity w(t).

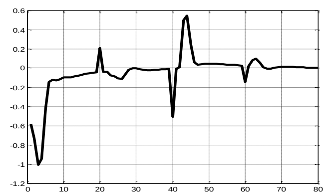

Fig. 10 The pitch angle θ(t).

```
MPC object (created on 18-Apr-2009 09:51:29):
---------------------------------------------
Sampling time:     0.12
Prediction Horizon: 12
Control Horizon:   3
Model:
        Plant: [1x1 ss]
   Disturbance: []
      Noise: []
      Nominal: [1x1 struct]
      Output disturbance model: default method
     (type "getoutdist(mpc55)" for details)
Details on Plant model:
                               --------------
1  manipulated variables   -->|  4 states  |
                              |            |-->  1 measured outputs
0  measured disturbances   -->|  1 inputs  |
                              |            |-->  0 unmeasured outputs
0  unmeasured disturbances -->|  1 outputs |
                               --------------
    1  manipulated variables  -->|  4 states  |
                           |          |--> 1 measured outputs
    0  measured disturbances   -->|  1 inputs  |
                           |          |--> 0 unmeasured outputs
    0  unmeasured disturbances -->|  1 outputs |
Weights: (default)
      ManipulatedVariables: 0
   ManipulatedVariablesRate: 0.1000
         OutputVariables: 1
                ECR: 100000
Constraints:  -25 <= MV1 <= 25,
MV1/rate is unconstrained, MO1 is unconstrained
```

Fig.11 The result of MPC object in Simulink/Matlab.

The result in Simulink/Matlab of MPC which is applied to AUV for depth control is in figure 11.

IV. CONCLUSIONS

Based on the results of simulation by Matlab software when applying MPC to AUV for depth control, it can be shown that this is an effective solution because the depth z(t) follows very closely with the desired depth r(t). Furthermore, these results of this solution also show that MPC is the virtual modern control method. In comparison with other control methods, the advantage of MPC is its easy application. MPC may be applied in controllers for plant types, which are linear or nonlinear MIMO model. As a result, we absolutely conclude that this solution is advantageous and feasible.

REFERENCES

1. N. Syahroni, Y. B. Seo, J. W. Choi; *Depth Control of Autonomous Underwater Vehicle Based on Open Control Platform*, Proceedings of the 17th World Congress IFAC Seoul, Korea, July 6-11, pp. 3707-3712, (2008)**.**

2. R. Cristi, F. A. Papoulias, A. J. Healey; *Adaptive Sliding Mode Control of Autonomous Underwater Vehicles in the Dive Plane*; In: IEEE Journal of Oceanic Engineering, Vol. 15, Issue: 3, pp. 152-160, Piscataway, NJ, USA, (1990).

3. A. J. Healey, P. A. Papoulias, R. Cristi; *Design and Experimental Verification of a Model Based Compensator for Rapid AUV Depth Control;* Proceeding of the Int'l Symposium on Unmanned Untethered Submersible Technology, pp. 458-474, Washington DC, USA, (1989).

4. Alberto Bemporad, Manfred Morari, N. Lawrence Ricker; *Model Predictive Control Toolbox™ 3 User's Guide,* Matlab, MathWorks, October 2008.

Contact: Pham Huu Duc Duc - Tel: (+84)913.238.632; Email: phdduc.uneti@moet.edu.vn
University for Economic and Technical Industries
456, Minh Khai Str., Hanoi, Vietnam