Studying the damping characteristics of the system of building and infrastructure considering the Soil-Structure Interaction

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ABSTRACT

This paper studies the damping characteristics of the soil foundation under a building concerning the Soil-Structure Interaction (SSI). By considering a multi-story building resting on a piled raft foundation as a typical SSI system, subjected to an earthquake (i.e., Chi Chi earthquake, Taiwan 1999), the time-dependent responses of specified locations in the foundation are recorded and analyzed, both in-time domain and frequency domain. Referring to prior knowledge about system identification and damping, this study suggests an overwhelming approach to determining the damping ratio for analyzing more thoroughly a system of SSI systems. Results are a general procedure to establish the damping matrix, namely the Caughey damping matrix which takes several frequencies into account. A different viewpoint that the connection of superstructure and infrastructure in series could help to estimate the damping, and contribute to a wide range of system identification. All these procedures are used to predict more properly the damping characteristics of a system of structures. Suggestions over the findings would be expected to contribute a more rigorous component to the analysis of high-rise buildings in the future.

1. Introduction

During the process of analyzing the structure, especially those which lie on a multi-layered soil foundation subjected to a dynamic effect, and taking the SSI into account, determining the damping ratio is a rather complicated task. People tend to oversee the ratio and accept widely a mere number, for example, 0.05 for reinforced concrete structures and 0.02 for steel works. The problem turns out to be more complicated as many different issues are considered for instance the time-dependent characteristics of the damping or the nonlinearity of the system. During an earthquake, the structure vibrates and propagates the vibration induced by the infrastructure upward; the superstructure in turn participates in motion with the vibration, causing additional internal forces and responses (i.e., internal forces, and displacements, both in the vertical and horizontal direction), and contributes action back to the infrastructure. There would have material and geometrical damping for both elements, and structures, up to the whole system, including structure, footings, and soil foundation. Different structures or structural elements experience different vibrational responses, depending on the stiffness and its restraints or constraints.

For vibration propagating in the far field of which the mechanism of wave propagation strongly depends on the intensity, soil attenuation, or damping, the response is mainly caused by the surface wave (R-wave). The wave propagates only in a definite depth of soil near the soil surface, stirs the receiver footing of the building in its vicinity, and propagates upward to different elements of the structure (Wolf & Song, 2002). The vibration traveling in a distance could attenuate at a far enough distance in the field; some approaches to screening this wave such as deep trench, micro piles, etc. prove to be efficient in mitigating the negative effects of the vibration.

In the near-field, the merge of three types of waves, i.e., prolonged wave (or P-wave), shear wave (or S-wave) and Rayleigh wave (or R-wave) make all the analysis harder. The most efficient way to investigate is at a distance of not less than a 1/5-power of the distance, where the body waves (P- and S-waves) attenuates. As such, in the near field, it is difficult to study the waves separately (Dowding, 1996; McClure, 1995).

The vibrational response of a structure depends on its stiffness and mass. Besides, the damping is vaguely understandable for its dependency on various factors. The questions such as: are the damping of the radiation one? Or does the viscous damping dominate? And how about the hysteretic damping as the soil foundation vibrates? The structure subjected to SSI is the building on a piled-raft foundation (later on, the term "PRF" is used in short), which is laid under consideration in this paper. The general procedure is that the overall system, including the frame of the superstructure and the PRF, could be investigated without separating the structures of the superstructure from the infrastructure.

2. Literature review

About the PRF subjected to vibration, there are at least four damping to be learned (Holeman, 1984, as cited in Dowding, 1996), they are a) Viscous damping; b) Radiation damping (material damping) of which the mechanism of propagation is of the skin friction along the pile shaft; c) Hysteretic damping due to the cyclic shearing stresses; and d) the resistance damping.

As for the superstructure, there are two types of damping, viscous damping, and hysteretic damping. The former is a frequency-independent one, and the latter relates more to non-linearity in the stress-strain relationship, and the frequency-dependent one. One other damping ratio which should be properly understood is Coulomb's damping which tightly correlates to viscous damping. A building on PRF is a multi-degree of freedom system (MdoF). Eigenmodes are complex to determine, providing the damping matrices **C** are not diagonal ones. The damping ratio, in this case, could be formulated by conducting the tests on the small-scaled model to collect data (Clough & Penzien, 1975). The damping ratio for a MdoF system that participates in vibration with the soil medium, i.e., SSI is concerned, is estimated by a linear relationship between the damping ratio and mass and stiffness matrices, or $\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K}$. Constants α and β are computed to the orthogonal conditions between modes of vibration, to have the product $\boldsymbol{\phi}^{T}\mathbf{C}\boldsymbol{\phi}$ a diagonal matrix (Clough & Penzien, 1975). As such, there must have two equations for determining the two constants. Normally, two modes of response are chosen to solve for obtaining α and β.

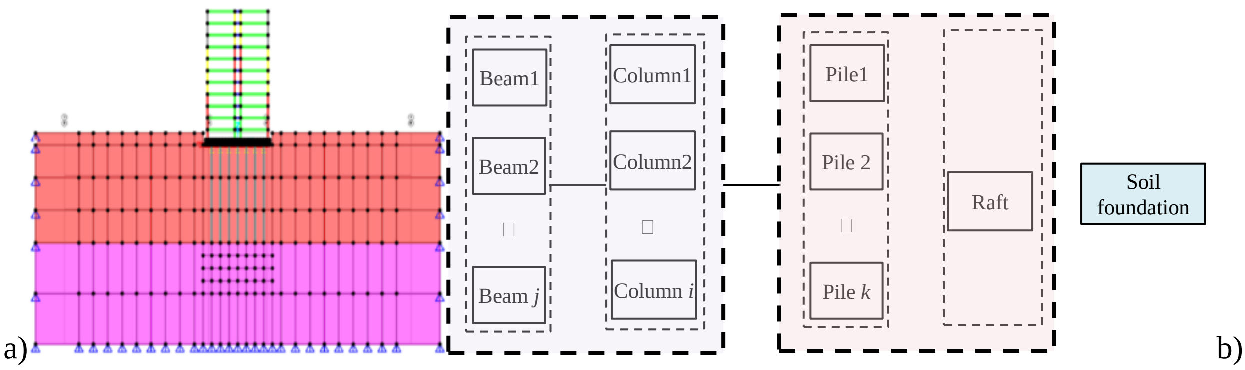


Figure 1. a) System of structures concerning SSI for studying damping characteristics, numerical model; b) Structural elements with connection in series and in parallel

The system is a connection of structure in such a way that beams and columns are connected in series, and the piles and raft are in parallel; the superstructure (frame including beams and columns) is in series with the piled raft. The diagram for this system is in Figure 1. Damping ratios for every element or structure could be combined to obtain the equivalent damping ratio as per the rule of connection as described in Figure 2.

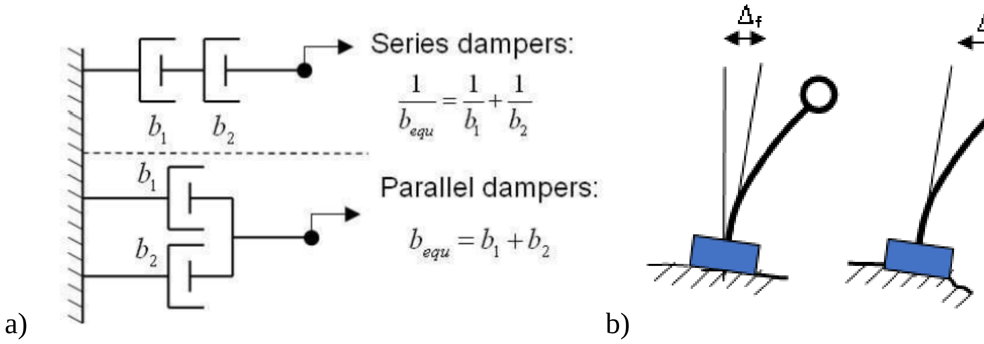


Figure 2. The equivalent damping ratio with dampers in series and parallel connection (Farouk & Farouk, 2014; Manafpour & Moradi, 2012)

Once the damping ratios of various structural elements are determined, the equivalent damping ratios of the system could be computed as below:

$$\xi_{sys} = \frac{\Delta_s \xi_s + \Delta_f \xi_f}{\Delta_d} \quad (1)$$

in which, symbol ξ stands for the damping ratio, of which subscripts s and f refer to super- and infrastructures, respectively; Δ_d is the displacement of the overall system, including superstructure and infrastructure as a whole (i.e., footing and soil foundation). As such the mere procedure is to solve the structure in the earthquake condition as-it, determining the displacements of the system Δ_d, the infrastructure Δ_f, and the superstructure Δ_s. The latter could be determined by studying a fixed base model, and the former displacement could be obtained by solving a soil-as-spring model, providing a preliminary analysis to be conducted to collect the displacement for calculating the spring stiffness (Duong, 2022).

Regarding the system including soil foundation and the footing with piles and rafts, the approach of system identification, developed by Huan, Lin, Wang, and Chen (2010) could be a more efficient one, in which both the stiffness and damping matrices are computed simultaneously, taking into account the SSI of the embedded foundation. The concept of the approach is to solve the two equations of the minimal discrepancy between the predicted and recorded value of the responses.

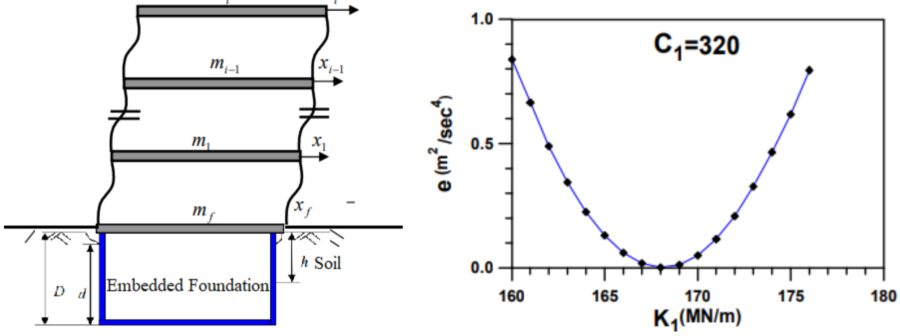


Figure 3. Approach of system identification: a) model with embedded foundation; b) The least square method for determining the damping in the ith-story

By studying the discrepancy e_f between the response of the superstructure and the infrastructure as below:

$$e_f = \sum_{i=1}[\ddot{x}_f + \frac{C_f}{m_f}\dot{x} + \frac{K_f}{m_f}x - \frac{C_1}{m_f}(\dot{x}_1 - \dot{x}_f) - \frac{K_1}{m_f}(x_1 - x_f) + \ddot{x}_g]^2 \qquad (2)$$

Two parameters of damping C_f and stiffness K_f for the foundation system or infrastructure are determined by solving simultaneously the system of equations from the least square method as follows:

$$\frac{\partial e_f}{\partial (C_f/m_f)} = 0 \qquad \frac{\partial e_f}{\partial (K_f/m_f)} = 0 \qquad (3)$$

Data of accelerations and displacements of the different positions such as acceleration $\ddot{x}(t)$ in the i^{th}-story (time-domain then using FFT technique to obtain values in frequency-domain analysis), velocity $\dot{x}(t)$ and displacement x(t) are collected from the responses which are solved by numerical model and viewed as the measured data. Values of error e_f are plotted concerning different values of damping C_i (value of the stiffness K_i is often known by calculation) until the smallest value of ef is attained and nearly equals to zero, then the convergent damping coefficient C_i could be determined (see illustrated Figure 3b). As such, it is necessary to collect well-collected data on masses, stiffness, and damping of every i^{th}-story and the foundation of the building. It is seen as numerical as data collector is essentially necessary.

3. Model

3.1. Multi-mode Caughey damping matrix

The model under consideration is described in Figure 1a. It is truly a complicated non-linear MdoF system in which the SSI is taken into account via PRF and soil participating in motion with structure as a non-linear system. By analyzing the model, taking the first four modes of response into account, the Caughey damping matrix (Caughey & O'Kelly, 1965) is derived by a range of calculations which is described by the flow chart below:

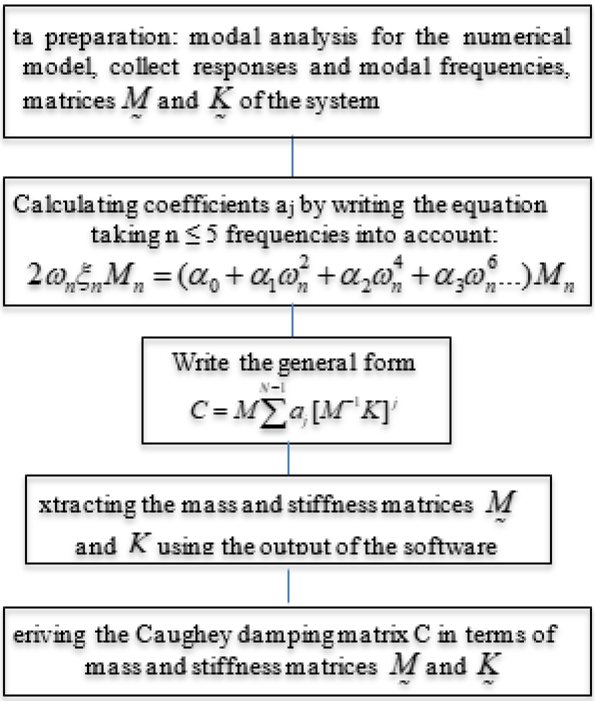
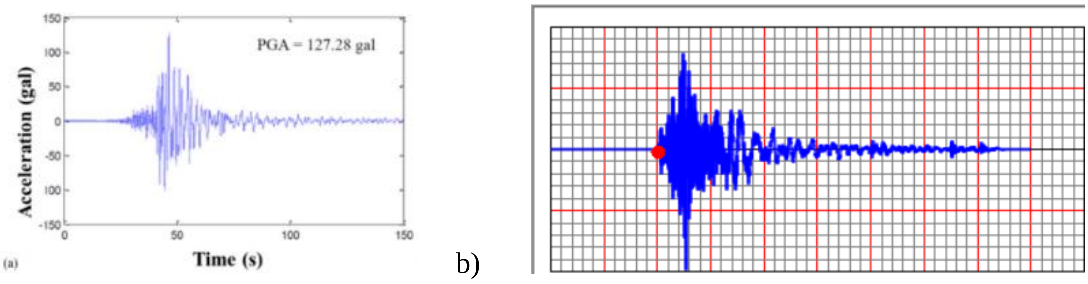


Figure 4. Steps for deriving the Caughey damping matrix

The model to be learned is plotted in the Figure 1a (using SAP2000); the system includes a 10-story frame as a sub-system of the superstructure, the piled raft foundation as a second sub-system of infrastructure, and the far-field medium of wave propagation as the third sub-system (Duong & Nguyen, 2022). That is a more rigorous model for coming closer to the real condition. The model is subject to the 1999 Chi-Chi 6.9 Richter-degree earthquake of which the time history is described in Figure 5 (Data from the PEER Ground Motion Database).

a) 

b)

Figure 5. a) Chi-Chi recordings at a station, east-west direction; b) SAP2000 TH function, from accelerogram No. RSN2752_CHICHI.04_CHY101**N**.AT2 (PEER NGA)

Responses at the different locations indicate the damping is different due to the difference in the mechanical characteristics, and velocity of motion. The stiffer element is, the higher frequency of response. By computing the logarithmic decrement δ, the damping ratio could be obtained by applying the formula:

$$\frac{n\xi}{\sqrt{1-\xi^2}} = \frac{1}{2\pi}\log\left(\frac{u_{i+1}}{u_i}\right) = \delta \tag{4}$$

in which n is the number of successive peaks in the plot of response, ξ being damping ratio.

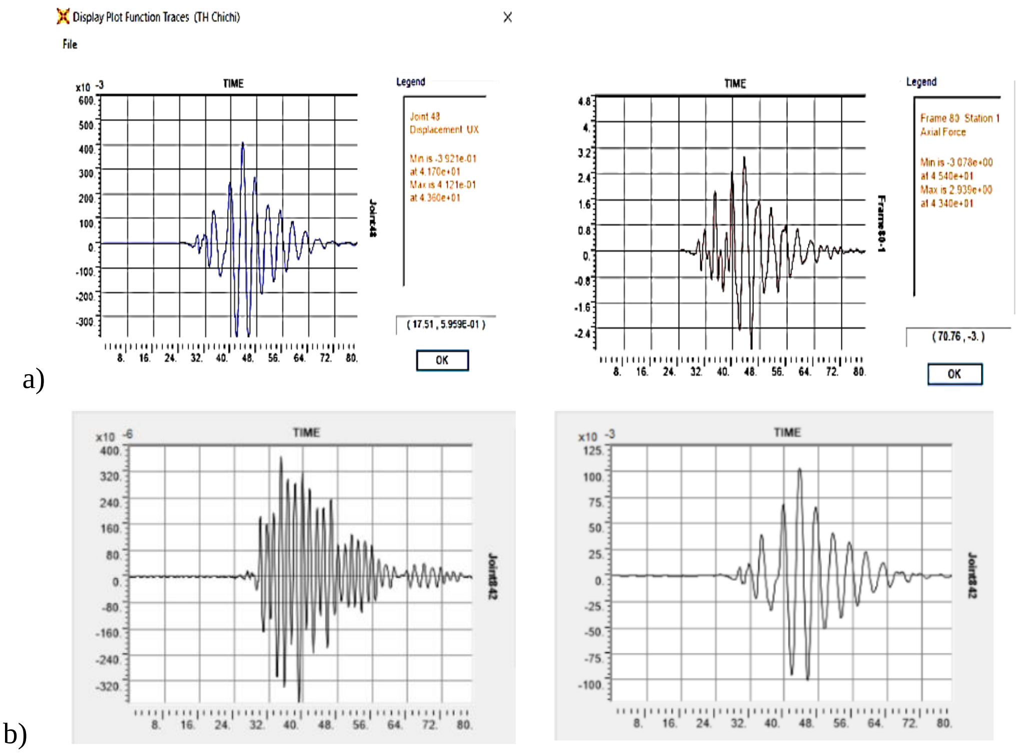


Figure 6. a) The horizontal displacements of the most deflected point (point 40), and axial force in the outer pile (element No 80) as the heaviest loaded pile; b) Responsive displacements of the most sagged beam (left, vertical; right: horizontal displacement, joint 42)

The responses could be used to compute the damping ratios for columns, beam, and piled raft foundation. There are at least 04 frequencies to be studied as tabulated in Table 1.

Table 1

Four modes of response, with respect to two kinds of boundary

Mode	Frequency (Hz)				State of stiffness of piled raft foundation
	1	2	3	4	
Boundary < 5B	0.388	0.534	0.855	26.45	11 square 28cm piles @2m
Boundary >5B	0.268	0.446	0.609	27.83	7 square 35cm piles @3m

High frequency of mode 4th reflects an increase in stiffness of the piled raft footings. For a more reasonable solution, it is necessary to calculate the case of wider boundary, or boundary >5B. As such, the damping of the ith-story C_i and stiffness K_i could be measured (i.e., by numerical model. Results are described in detail (Duong, 2022), where the Caughey's damping matrix of the system considering four frequencies of response as in Table 1 is

$$\underset{\sim}{C} = \frac{1}{0.08}(1.335407\,\underline{M} - 0.05243\,\underline{K} + 0.115198\,\underline{K}\,\underline{M}^{-1}\,\underline{K} - 0.00389.\underline{M}\,\underline{K}\,\underline{M}^{-1}\,\underline{K}) \quad (5)$$

in which $\underline{M}$ and $\underline{K}$ are numerically derived from the output of the software for a specific model. The mass matrix M and global stiffness K of the system could be derived from the output of the model as in Figure 7.

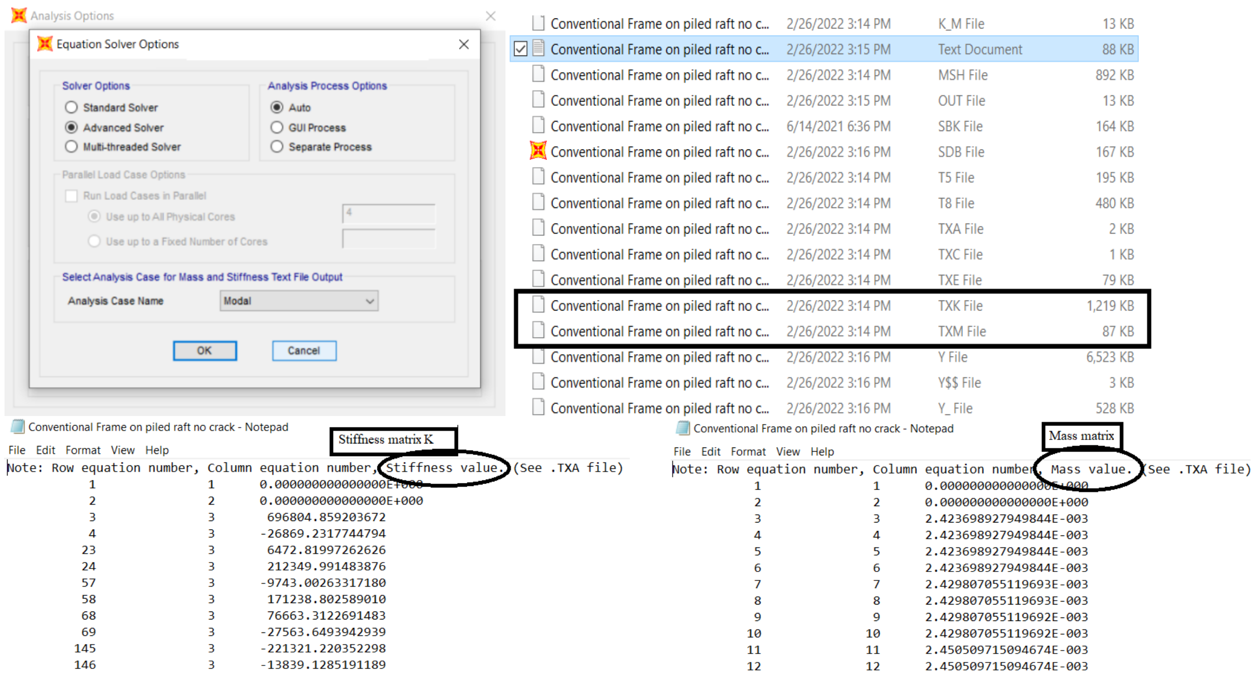


Figure 7. Pull down menu in SAP2000 for extracting the mass and stiffness matrices

Notepad files in Figure 7 are text files having the attribute ".TXK" and ".TXM" for gaining the mass and stiffness matrices. Rows and columns which are notified in the text file could be used to assign respectively into a worksheet for establishing the matrix. In the abovementioned SSI model, the mass matrix is a diagonal 1635x1635 one, and the stiffness matrix is a squared 1635x1635; their size is too large to describe in this paper.

3.2. Equivalent stiffness and damping in system of building and infrastructure

This system identification is based on viewpoint considering the stiffness and damping matrices of the i^{th}-story and the foundation. The concept of equivalent parameters could be used. The story stiffness is the summation of the stiffness of individual columns. Beams have a connection in series with columns and in parallel with each other (see Figure 2b). As such the equivalent parameter could be easily computed by applying the formulas:

Parallel dashpots (dampers) $$c_{eq} = \sum_i c_i \qquad (6a)$$

Series dashpots $$1/c_{eq} = \sum_i 1/c_i \qquad (6b)$$

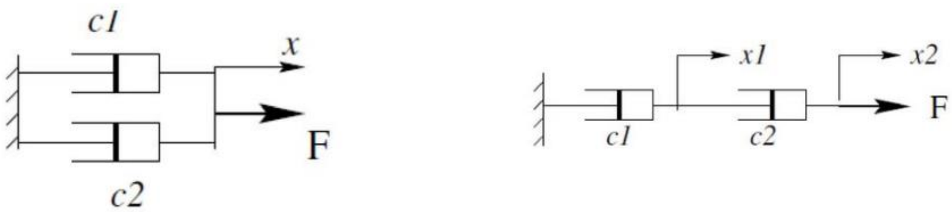


Figure 8. Schematic diagram for computing the equivalent damping

The concept of the story damping is the same as the spring stiffness (see Figure 8).

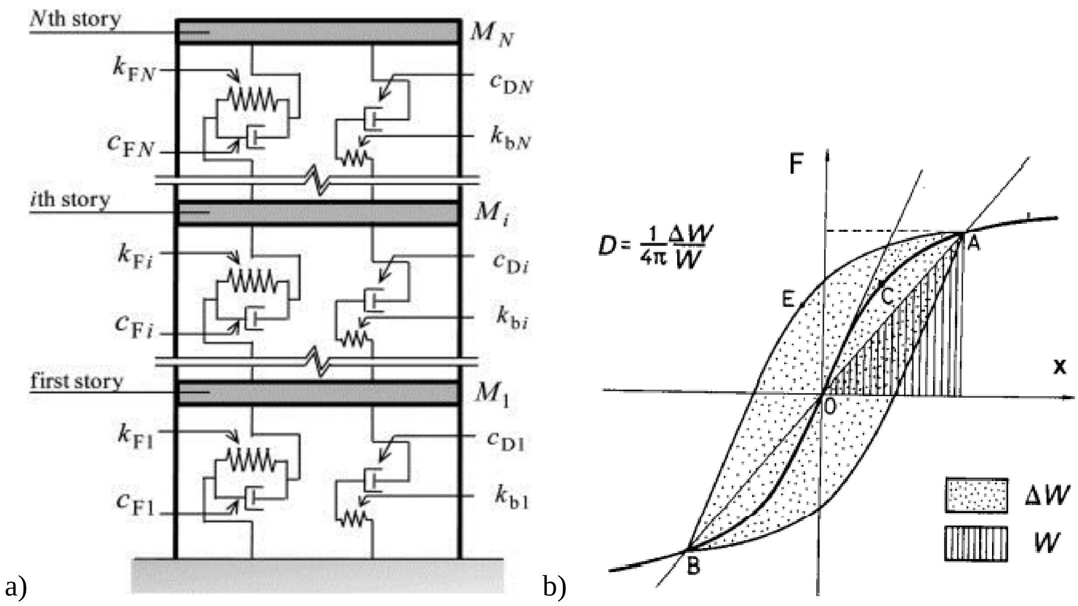


Figure 9. a) *N*-story shear building model with passive dampers and supporting members (Takewaki & Fujita, 2013); b) hysteretic loop from the spectral response to estimate the equivalent damping ratio D

For a structural system having several supporting elements such as outrigger trusses, shear walls or cores, etc., the dampers could be installed in such elements to absorb the energy due to wind with gusts or dynamic effects by earthquakes. These dampers could be tuned suitably to reduce dangerous responses. By considering the responsive internal force-displacement relationship in a specific damper, a hysteretic loop could be plotted and the damping could be estimated as the area of the loop. Within a story, typically as for i^{th} story in Figure 9, the equivalent rheological model could be transferred to one as known as Burger viscoelastic model (Takewaki & Fujita, 2013):

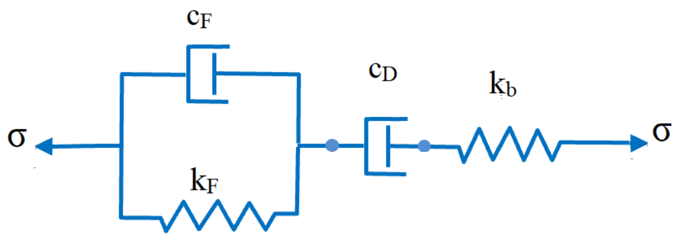


Figure 10. Converted model to compute the equivalent parameters, Burger model

Stiffness K_b is of springs that connects in series to damper having damping coefficient c_D, then, the shear wall or core having the parameters k_F and c_F in parallel, The total strain at constant stress is the sum of the strain in the three elements, or $\varepsilon(t) = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ in which the former for k_F and c_F parallel then combined model could be transformed to the viscouselastic-Burger model (Findley, Lai, & Onaran, 1976). In this study, it is assumed that all the stiffness and damping of the individual structural elements in a specific story is available by both velocity of motion (for viscous damping, c_F) and hysteretic loop (for structural, additional damping, c_D) with a specific dynamic intensity and specific configuration of the structure. In general, at a specific intensity of the earthquake, together with a specific configuration, the n-story building has the dynamic response numerically measured (or recorded by devices in small-scaled models, etc.) at each story, often in the first mode for the lowest responsive frequency. In this study, the stiffness and damping are measurable (Hesam, Irfanoglu, & Hacker, 2019; Mousavi & Ghorbani-Tanha, 2012).

Spring stiffness (providing that in ith story, $k_{eqi}=k_{fi}.k_{bi}/(k_{fi}+k_{bi})$)

$$K = \begin{bmatrix} k_{eq1}+k_{eq2} & -k_{eq1} & \dots & 0 & 0 & 0 \\ -k_{eq1} & k_{eq2}+k_{eq3} & -k_{eq2} & & & \\ \dots & -k_{eq2} & \dots & & \dots & \\ 0 & 0 & & & -k_{eqN-1} & \\ 0 & 0 & 0 & -k_{eqN-1} & k_{eqN-1}+k_{eqN-1} & -k_{eqN} \\ 0 & 0 & 0 & 0 & -k_{eqN} & k_{eqN} \end{bmatrix} \tag{7a}$$

Structural damping (parallel part of the model)

$$C_s = \begin{bmatrix} c_{F1}+c_{F2} & -c_{F1} & \dots & 0 & 0 & 0 \\ -c_{F1} & c_{F2}+c_{F3} & -c_{F2} & & & \\ \dots & -c_{F2} & \dots & & \dots & \\ 0 & 0 & & & -c_{FN-1} & \\ 0 & 0 & 0 & -c_{FN-1} & c_{FN-1}+c_{FN} & -c_{FN} \\ 0 & 0 & 0 & 0 & -c_{FN} & c_{FN} \end{bmatrix} \tag{7b}$$

Viscous damping (series part of the model)

$$C_D = \begin{bmatrix} c_{D1}+c_{D2} & -c_{D1} & \dots & 0 & 0 & 0 \\ -c_{D1} & c_{D2}+c_{D3} & -c_{D2} & & & \\ \dots & -c_{D2} & \dots & & \dots & \\ 0 & 0 & & & -c_{DN-1} & \\ 0 & 0 & 0 & -c_{DN-1} & c_{DN-1}+c_{DN} & -c_{DN} \\ 0 & 0 & 0 & 0 & -c_{DN} & c_{DN} \end{bmatrix} \tag{7c}$$

The dynamic governing equation (in terms of M, K, C mass, stiffness, and damping matrices, respectively) of the system subjected to ground acceleration $\ddot{x}_g$ is

$$M\ddot{x} + (C_s + C_D)\dot{x} + Kx = -M\ddot{x}_g \tag{8}$$

Damping matrices will be of the Rayleigh matrix using two constants α and β for a damping matrix **C**=α**M**+β**K**, or Caughey one which utilizes at least the first four modes (Duong, 2022). To this point, all the parameters in the matrix form of the superstructure are determined.

For not taking SSI into account, parameters of the infrastructure including piled raft foundation and soil medium could be defined by formulas (Mousavi & Ghorbani-Tanha, 2012). The model which includes the piled raft footing, and soil medium in SSI analysis should be the one that the displacement in an arbitrary story drift would alter the damping due to the velocity of motion. As referred to in this point of view, "It can be shown that participation of each story in the seismic input energy depends upon the inter-story drift and the mass index of that story (quoted from the research postulated by Mousavi and Ghorbani-Tanha (2012))".

The connection between the PRF and the subground soil layers is assumed to be in series, both in a vertical and horizontal direction; PRF includes the connection of piles in parallel, and in series between the pile group with the raft. As such the model would be as below:

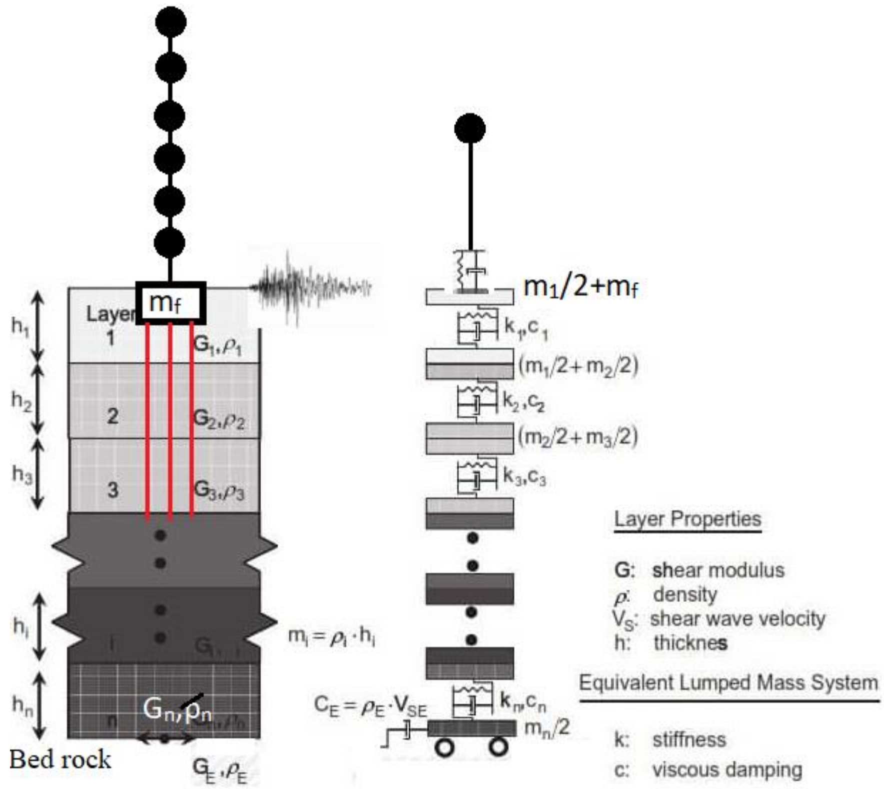


Figure 11. Model of soil column including the mass of PRF, added mass and layers

The model in Figure 11 could be tentatively used to estimate the damping and stiffness of infrastructure using the concept of springs/dampers in series/parallels. The mass matrix is a diagonal one; the damping and stiffness matrices could be determined by the conventional approach, with the indices of the summation should count from 1 to N the number of masses in the vertical direction. This depends on the width of the soil column which participates in motion with the superstructure (a cantilever multi-degree of freedom beam). It is notable, it depends on how far the far field is defined and whether the story drift is considered or not. The damping matrix

c could be a Rayleigh (using two first modes) or a Caughey one (using four first modes). Once matrix ***c*** is known, the damping matrix which takes the drift of stories into account is $\mathbf{C_{db}}$ to be determined as follows:

$$C = \begin{bmatrix} c_1+c_2 & 0 & \dots & 0 \\ 0 & c_2+c_3 & & \\ \dots & & \dots & \\ 0 & & & c_N \end{bmatrix} \quad and \quad C_{db} = \frac{1}{M}\begin{bmatrix} c_1 & 0 & \dots & 0 \\ 0 & c_2 & & \\ \dots & & \dots & \\ 0 & & & c_N \end{bmatrix} \tag{9}$$

where M is the total mass of the building system (Fujita, Ikeda, & Takewaki, 2015; Mousavi & Ghorbani-Tanha, 2012). The main idea for this is that all the masses of the superstructure and infrastructure (i.e., PRF and soil layers) are viewed to be connected in series with each other, and story drift is taken into account. And the damping would be computed by response (i.e., acceleration) measured in each story, taking the smaller values on the safe side.

4. Discussions

This study relates to the two approaches for determining the damping matrix for a system of building and soil columns. Some issues could be studied as follows:

- It is possible to determine _more rigorously_ the damping characteristics in terms of considering the higher modes of response (Caughey damping matrix), system identification using the least square method applied to acceleration of different points in the story (section 2), or equivalent parameters of stiffness and damping matrices (subsection 3.2). As for prior studies, acceleration is preferred to displacement (Hesam et al., 2019; Huan et al., 2010; Takewaki & Fujita, 2013).

- In a numerical model as in Figure 1a, in which the building is modeled as a planar 2D frame model, PRF and the soil as a continuously wave propagating medium, this requires a technique of planar modeling, a wide enough medium of wave propagation, and absorbent boundaries (Duong & Nguyen, 2022).

- Viscous damping in the structural element is in the series part of the viscoelastic Burger model, and the structural damping or hysteretic damping is in the parallel part of the model. In the case without stiffness kb of the additional structures (core, outrigger structures, shear wall, etc.), the model turns out to be the Voigt-Kelvin model, which is useful for assessing the creep (time-dependent deformation) and stress relaxation (time-dependent decreasing stress). In some conventional models, only the parallel part is assigned to the story (Fujita et al., 2015). The combination of all structural elements results in this simple model.

- The overall stiffness K and damping C are determined by formulas (7a, 7b, and 7c) and (9), in which each story, the viscoelastic-Burger model is applicable (see Figure 10); all the structural components in each story could have the computable stiffness K_D for additional elements (i.e., core, shear wall), and K_F for walls or some other structures; as for damping C_D and C_F, the best and practicable way for determining them is to conduct in a numerical model or small scaled model to collect data of responses (Hesam et al., 2019).

- Two procedures for determining the damping matrices as postulated in this study, i.e., a) using a numerical model (for instance, SAP2000 model) and extracting the mass and stiffness matrices, then computing the Caughey damping matrix or Rayleigh one; b) considering all the elements to be connected in series and calculating the overall matrices of mass, stiffness, and damping, might lead to a difference in damping ratios. The selected value would be the smaller one instead of, the larger one because the less damping effect would not underestimate the responses; it is on the safe side for practical purposes. By using data of response obtained in the already solved numerical model, the damping matrix could be able to determine with the abovementioned approaches. It is preferable to use responsive acceleration to displacement to avoid errors due to the twice numerical integration of the acceleration, and its frequency-dependent characteristics.

- The lumped mass concept is applicable to both superstructure and infrastructure, in which the width of soil mass participating in motion with the structure is satisfactorily defined. All the lumped masses are connected vertically in series, as in Figure 11.

5. Conclusion

Damping is important for structural elements when subjected to high-energy effects like earthquakes. It refers to the extent to which the energy is dissipated and in some specific cases, to the level of damage within structures. Properly determining the damping, the response of the structural elements in the system could be exactly assessed. As such, some more reasonable solutions for controlling and tuning the negative effects could be applied at a reasonable cost. The contribution of this study is: a) a set of Caughey damping matrices, in which the contribution of responses of the higher frequency modes to the overall motion could be considered; b) a conceptual model of super- and infra-structure connected in series in vertical direction subjected to ground acceleration, from which, damping and stiffness matrices could be computed; and c) a more rigorous investigation on the PRF system, concerning the strongly non-linear SSI. Due to the giant mass of calculations regarding the 1635 x 1635 mass and stiffness matrices, this study could not clarify quantitatively computed results; the essential issue of the procedure is described instead. Nevertheless, some theoretical key points could be practicable, that as the application of the viscoelastic-Burger model to each story of the building for replacing the structural elements such as a shear wall, core, outrigger belt-truss, etc. The efficiency of this model is confirmed by a variety of prior research (Chen, Sun, Yuan, & Zhang, 2008; Zubair & Shilpa, 2016), and strongly recommended, in the hope of providing more understanding of SSI dynamics to postgraduates.

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