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How Trading the S&P 500 ETFs “Beats” the Market by an Average of 30% in Annual Returns

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Abstract

The market refers to the S&P 500 index, which is an important benchmark of U.S. stock performances, and “beating” the market means earning a return greater than the market. The historical average annual market return is approximately 10% since its inception in 1928. The purpose of this paper is to showcase a trading strategy that earns an average annual return of about 13%, which is 30% higher than the historical average annual market return. The strategy is contained in the website WaveletTrader.com, which is quite user-friendly, and no special skills or prior practice is needed. It has been generally believed that coming up with a scientific system to “beat” the market is impossible. But the results of this paper strongly suggest otherwise.

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1 INTRODUCTION

We often hear different people, like friends or television financial reporters and see books, magazines or websites talking about “beating” the market with their investment strategy. It is talked about in such high esteem that you might think it is something akin to finding the holy grail for an investor. In this paper, we are going to explain a strategy for “beating the market” that anyone can implement. Before we get into the strategy though we’ll review the basics of stock market investing and explain everything you need to know and define all the fancy terms so that you can execute the plan without anyone’s help, even if you have never bought a single share of stock in your life.

1.1 What Is “the Market”?

When people save their money for retirement or other long term goals, they often invest some of their savings in the stock market. The stock market in the most general sense means the place where people can buy and sell shares of stock in public companies. The buying and selling of stocks is also often called trading. The physical location or marketplace where the stocks are traded are called exchanges. The two biggest exchanges in the United States are the New York Stock Exchange (NYSE) and the NASDAQ.

A stock means a certificate issued by a company that represents a share of the ownership interest in the company. A stock is one type of investment security, which is a word professionals use to describe a financial product that can

be traded on an exchange and involves risk. A public company is a company that has issued stock certificates to the general public that can be legally traded on an exchange. The process of becoming a public company is time consuming and expensive because a company must go through a registration process with the United States Government through an agency called the Securities and Exchange Commission (SEC) and comply with requirements to report its financial results at regular intervals using established accounting rules. So the term “stock market” can also be used in a general sense to describe this complicated system for people to trade ownership interests in companies and share in their success or failure.

1.2 What does “Beating the Market” Mean?

Sometimes when people say “the market”, they aren’t literally talking about the physical exchange or the marketplace for stock that we just reviewed. Instead they are referring to the average **performance** of all stocks generally in the stock market. This average performance is measured by a stock market index, like the Dow Jones Industrial Average or the Standard and Poor’s 500. An index is a value calculated using the prices of a grouping of stocks. This is the sense of the term we are referring to when we say “beating the market.” We mean getting a better return from our investments (also called “outperforming” or “beating”) compared to the average return of stocks in general.

There are a number of stock market indices that are calculated on a

real-time basis throughout the day during trading hours. A particular index represents a grouping of certain stocks which share some common feature that investors use to get a sense of the benchmark performance of a sector of stocks. For example the Dow Jones Industrial Average, sometimes called just the “Dow” is a grouping of 30 of the largest U.S. public companies.

One of the most commonly quoted major (meaning overall) indices in newspapers and television reports is the Standard and Poor’s 500 index, often referred to as the S&P 500. The S&P 500 is a very broad index that represents a grouping of the 500 largest U.S. public companies that have a stock listing on either the NYSE or NASDAQ (the two largest exchanges). This grouping of companies is a good representative benchmark of the performance of the entire stock market because it represents about 80% of the U.S. economy compared to only about 25% for the 30 stocks in the Dow.

The S&P 500 is a market capitalization weighted index which means that mathematically, the index value is calculated not by taking a simple average of the prices of all of the stocks in the group, but by using a **weighted** average of the prices. The weighting value applied to each stock price is the total value of all of the shares of the company, called the market capitalization or market value. So a particular company that has a market value twice as high as another company, will have its share price counted twice as much in the average. This weighting approach allows the value of the S&P 500 to be

a better representation of the overall value of all stocks and the total wealth change of the stock market as a whole as prices fluctuate. We use the term benchmark to mean a base to compare other things to. Since the S&P 500 is a such a broad index and represents overall wealth changes, it is a good base to compare other portfolio’s performances to, so we say it represents a benchmark portfolio.

Now we understand that when we use the term “beating the market” in this paper we mean using an investment strategy that generates a higher average investment performance than the S&P 500 over a period of time.

1.3 Objective

“Beating the market” is a problem of much interest to stock traders, academicians and people with interest in business finance and economics. There is a large body of literature on this topic. Googling “beat the market” yields more than 10 million results. The reader is referred to the references [13] and [2] listed at the end of the paper for an account of this information. It has been always believed that trying to come up with a scientific strategy to “beat” the market is a useless waste of time.

Currently, there are many S&P 500 ETFs (Exchanged Traded Funds) that closely track the S&P 500. These ETFs have relatively low volatility and are well diversified since they contain all 500 stocks in the S&P 500. The biggest S&P 500 ETF is the SPY index stock issued by State Street Global Advisors. In this paper, my objective is to show how a

trader can “beat” the market by trading these ETFs to attain an average annual return of about 13%, which is 30% higher than the 10% historical market average annual return. An increase of 30% is a very significant amount.

The strategy is implemented in a website (WaveletTrader.com), which is quite easy to use and no special skills, licenses or prior practice is needed as detailed instructions on how to trade and how to use the website will be given later. A trader using this website and the website will be referred to as WT for brevity. The term wavelet is used to indicate that WT makes decisions to buy, hold or sell depending on past movements of small waves caused by fluctuations of market prices.

All that is required of you is a computer or a smart phone with internet to get to the website. On the first day of trading, you enter on the screen the date and price of the S&P 500 ETF you trade, and then activate the website. WT is programmed to ask you to buy 1,000 shares on the first day. On each day of trading, WT needs the dates and prices of all previous days of trading from the first day up to and including today’s date and price to make calculations. This data containing dates and prices has to be saved as a cumulative data file. WT processes the information, then recommends one of the following three actions: buy, hold or sell. The most common action is to hold.

You are advised to buy or sell only occasionally when prices have changed by a significant amount. Barber and Odean [1] show that traders who buy and sell frequently usually end up los-

ing more money than those who trade less often. You are advised to buy or sell only about thirty to forty times in a calendar year. The number of shares traded is also given in case it is a buy or sell. You carry out the strategy as follows:

Start trading at any trading day by buying, say, 1,000 shares of any S&P 500 ETF. Trade back and forth according to the guidance of WT for a year of 365 days. Sell all shares on the last day of trading. For example, if the first day of trading is September 6, 2017 then the last day of trading is September 5, 2018. Your returns are then compared with the returns of the market on the last trading day. The website will tell you what the last day of trading is.

What if you don’t want to start with buying 1,000 shares. Then you can start by buying any amount that you wish but you will need to modify the number of shares traded by WT. More detailed instructions on how to modify your trades, activate and use the website will be provided later on. An example with numerical data will be illustrated in Section 6

After completing one year of trading, you can quit or start another year at any trading day of your choice. WT’s strategy is data-dependent in the sense that on each trading day, WT has to analyse the stock prices of all previous trading days of the trading year plus today to make decisions as to hold, buy or sell. The maximum time-length that WT can handle is one year due to the heavy computations involved.

WT is an annualized strategy with a time frame of one year. Your portfolio

contains a S&P 500 ETF and you trade this ETF occasionally at about once every two weeks. Fifteen minutes a day would be sufficient to manage your trading. The website carries out all the necessary computations and no skill is required of you.

To achieve a higher average annual return than the market requires you to take some additional risk, but you are well compensated for the risk taken, resulting in a higher risk adjusted return than the market. More discussion on risk adjusted returns will come later.

“Beating” the market is a problem of much interest to stock traders, academicians, economists, people with interest in business finance and proponents of the random walk hypothesis (RWH) and efficient market hypothesis (EMH). There is a large body of literature on the EMH and RMH. For a bibliography, see [3], [4], Fama and French [5], Malkiel [9], and the references therein.

1.4 The Link

Many big sets of data are referred to in this paper. They are displayed at <https://iu.box.com/s/huilt6haxsdbh346miu71zvgm2wk9rrf> which will be referred to as “the link”. Many tables containing outputs from the website are contained at the link. A more detailed version of this paper is stored at the folder Research Paper at the link.

Before using the website for trading, you should test the website on past historical and simulated data that will be provided to you. There is only a limited amount of past historical data.

However, the website can generate

an unlimited amount of simulated market data. Use the website for trading only if you are persuaded that WT does indeed “beat” the market. The theory behind the operations of the website will be briefly described.

2 THE S&P 500 EXCHANGED TRADED FUNDS

Since the S&P 500 represents the market capitalization weighted average price of 500 different stocks, it is impractical for the average investor to trade the benchmark directly by buying and selling individual stocks.

Fortunately, there are special types of funds, called the S&P 500 ETFs for short, that closely track the performance of the S&P 500 using complex computer models and automated trading techniques to keep the funds portfolio of underlying stocks in line.

Investors can buy shares in an ETF and trade it just like any other stock, but the ETF’s performance is designed to track the performance of the grouping of a portfolio of stocks and the investor owns shares in the fund that owns the stocks instead of the stocks directly. *If an investor wants to invest in the S&P 500, a practical way to do that is to invest in a S&P 500 ETF.*

2.1 A list of S&P 500 Exchange Traded Funds (ETFs)

The S&P 500 ETFs provide full diversification of the market. They slightly underperformed the S&P 500 due to expense ratios. Below is a list of 6 of the biggest ones in terms of their Assets Under Management (AUM).

Ticker	Fund Name	Issuer	AUM	Expense Ratio
SPY	SPDR S&P 500 ETF Trust	State Street Global Advisors	\$255.16B	0.09%
IVV	iShares Core S&P 500 ETF	BlackRock	\$158.40B	0.04%
VOO	Vanguard S&P 500 ETF	Vanguard	\$99.20B	0.04%
SSO	ProShares Ultra S&P 500	ProShares	\$2.30B	0.90%
SH	ProShares Short S&P 500	ProShares	\$1.69B	0.89%
UPRO	ProShares UltraPro S&P 500	ProShares	\$1.26B	0.92%

There are about 13 S&P 500 ETFs. See [12] for a complete list of these ETFs.

2.2 SPY

The biggest ETF that tracks the S&P 500 is the SPDR S&P 500 ETF SPY issued by the State Street Global Advisors. The AUM of SPY is \$255.16B and the expense ratio is 0.09%. The inception data is 1/22/1993.

The SPY historical data set displayed at the link lists the dates and corresponding adjusted closing prices of SPY from January 29th, 1993 to September 7, 2018. The data was downloaded from the Yahoo Finance website online. This set of data plays an important role in the paper. It will be used in many examples and also for back testing to show that WT “beats” the market based on historical data. A date of the year will often be displayed in the same style of the SPY data set. For example, January 27th, 2018 is written as 1/27/2018 or 1/27/18.



Let us look at the graph of the historical data of the SPY displayed below. Note that SPY increases from \$27.234995 on 1/29/93 to \$287.600006 on 9/7/18, which amounts to over 10.00% a year on average. From the graph above, it is clear that SPY has an upward trend. However, investing in SPY is still a risky business. A buy and hold investor does not have a positive return every year. The price of SPY can drop dramatically. The cost of a share dropped from \$106.674629 on 3/11/08 to \$56.779598 on 3/6/09.

A model (Geometric Brownian Motion) will be fitted to the SPY set of historical data in Section 9. The model can then be used to generate an unlimited amount of simulated data of the SPY. Before you use the strategy for actual trading, you should play with the website using historical data and simulated data to persuade yourself that the strategy works as claimed above. Please be reminded that WT is only designed to trade the S&P 500 ETFs and should not be used to trade any individual stocks.

2.3 Performance of the S&P 500 ETFs

Since the S&P 500 ETFs began, they have increased annually at the rate slightly less than the rate of increase of the S&P 500 which is about 10% each year, also called an annual return of 10.00%. This is expected due to the ex-

pense ratio, which is between 0.04% and 0.90%. One common benchmark investment strategy is called “buy and hold” which indicates a strategy which literally means for an investor to buy the security one time at the beginning and hold it through the duration of time being measured. Since the “buy and hold” strategy is simple and easy and guarantees you a return equal to the market, many investors consider it to be the best investment strategy.

Academicians have studied various investment strategies and have assembled extensive actual data to suggest that a “buy and hold” strategy outperforms or “beats” the average actively managed account (meaning an account which is traded). This data shows that because of the cost of trading more often and the fees charged by investment managers, an average portfolio traded by brokers and fund managers does not perform as well as a “buy and hold” strategy on average. In other words, the average fund manager does not “beat the market.”

3 PREPARING TO TRADE

3.1 Opening a Brokerage Account

In order to purchase stock on an exchange from a seller, you must open a special type of financial account with a stock broker. A brokerage account is similar to a bank account in that you can deposit and withdraw money, but the difference is that you can also buy and sell stocks as well. A stock broker is a licensed individual who works for

a licensed company that is trained to help you buy and sell stocks and give you investment advice.

3.2 Applying for a Margin Account

You need to apply for a margin account. Margin is a form of borrowing using your stocks as collateral for a loan. It is similar to borrowing money for a mortgage when buying a home or a car loan for a vehicle. A bank is willing to lend you money because it knows that you can pay it back if you have to by selling your home or car. Since the bank has collateral in your personal property (called an “asset”), it feels that its investment in your loan is safe. For the same reasons, a brokerage firm is willing to lend you money using your stock assets as collateral because it knows you can sell them to repay the loan. The other reason that your financial assets are good collateral to a brokerage is because they have a readily available price and can be sold quickly, which is called “liquidity” by financial professionals.

Usually when you are opening your brokerage account, the stock broker will ask you if you want a margin account, which means an account that has margin lending authorized. If you are opening the account online, you may need to search for how to activate margin on your account or call a helpline. Enabling margin can be done quickly and easily on a new or existing account quite easily but it usually requires your specific approval so it probably was not done for you automatically.

3.3 How Much Can I Borrow?

Different brokers lend different percentage amounts to investors based on the value of their assets in the brokerage and their overall financial relationship with the account owner. A good rule of thumb though is 50% of the purchase amount of a particular purchase. The total amount you can buy at any point is also called your “purchasing power.” It is important to note that the rules for brokers on how much margin they can extend are set by the U.S. Federal Reserve Board through its Regulation T.

Regulation T is the Federal Reserve Board rule that governs how much money you can borrow from the brokerage for the purchase price of a security. The initial margin is 50% of the cost.

3.4 Margin Calls

One of the dangers of using margin when you are investing is that if the price declines too much, the value of the stock in your portfolio might no longer meet the margin ratio set by your broker and you could have to deposit more money to make up the difference.

This situation is called a “margin call” since it is a call for you to deposit additional money in your account. This can scare many people who might not want to have to put more money in their account on short notice. There is an alternative to depositing additional funds to your account though, because you can sell some of your shares to get your account back in compliance with the margin leverage requirement.

3.5 Diversification

Normally, a trader’s stock portfolio contains at least several stocks in different sectors of the economy. If one stock decreases in price, then another stock might increase to make up for it. Diversification is very important in controlling the risk associated with trading.

WT is designed to trade any one of the S&P 500 ETFs, which contain stocks from 500 companies. Thus WT’s portfolio is fully diversified. To limit the scope of the paper, assume that the stock is the SPY. There is more data on the SPY than on other S&P 500 ETFs. SPY is a very popular stock and is currently traded about 5 times as often as Apple (APPL).

4 GETTING STARTED

After you open your brokerage account and authorize margin, you need to make your initial investment deposit to the account. In my examples, I assume that you start out by buying 1,000 shares, however this number is chosen arbitrarily and you can buy any amount that you wish. You need to be sure to cover the total cost of the shares, plus the cost of commission that you must pay the broker to purchase the shares. If you use a number different than 1,000 shares as your initial purchase, then you can make modifications to the tools I’ve provided as follows:

WT is programmed to always start by buying 1,000 shares. Suppose you can only afford to start with 500 shares, then just buy or sell half as much as WT does. Rounding off the number of shares

traded to an integer won't change your returns much. If WT sells 71 shares, then you can sell 35 or 36 shares. Suppose you start with 2,000 shares. Then you can buy or sell twice as many shares as commended by the website.

4.1 Costs: Margin Interest and Commissions

Just like any loan, if you have a margin loan outstanding, you will be charged interest on the balance of the loan. Margin interest is calculated on a daily basis and subtracted from the cash balance in your brokerage account automatically. The brokerage firm that has your account and buys and sells shares for you will also charge you a commission every time that you buy or sell a stock. You should therefore look for the amount that the brokerage charges you for both margin interest rate and commissions.

To give you an idea of the amount of the costs, at the time of this writing, with an account balance of \$25K, Interactive Brokers charges an interest rate of 3.63% per year on margin loans and commissions of \$1.00 per share if the number of shares per trade is 200 or less or \$0.005 per share if a trade in-

volves more than 200 shares. Below is a table listing the margin interest rates of various brokerages.

It is important to use a brokerage with low margin interest rates. Note that the margin rates of interest at Interactive Brokers are lowest among the brokerages. For a listing of margin interest rates of various brokerages, see [10]. The margin rates of interest vary with the prime rate of interest set by the Federal Reserve.

4.2 Assumptions

Trading by WT is done through a brokerage which must satisfy the following assumptions:

Assumption 1. The brokerage charges an interest rate equal to or less than 4% per year and collects interest daily (compounded once a day). The trader gets 2% interest for surplus cash in her account.

Assumption 2. The brokerage allows WT a margin leverage $a:1$ where a is a number between 2 and 2.25, and checks WT's margin every day (usually around 4:00 PM) before closing time.

Assumption 3. The brokerage charges a commission equal to or less than .04% of the cost of a transaction.

	\$25K	\$300K	\$1.5M	\$3.5M
Interactive Brokers	3.63%	3.29%	2.99%	2.75%
E-Trade	10.25%	8.75%	7.25%	7.25%
Fidelity	9.07%	7.82%	5.25%	5.25%
TD Ameritrade	10.00%	8.50%	7.75%	7.75%
Vanguard	9.25%	7.75%	6.00%	6.00%

Currently the margin rate of interest at Interactive Brokers (IB) is 3.63%. However, IB does not pay interest for surplus cash in the trader’s account. WT seldom holds surplus cash, so the interest she gets does not affect her return much. A trader with a lot of surplus cash can transfer her money to a sweep account to collect some interest. The result of the paper would not change much if WT gets no interest for surplus cash. The commission charged by the brokerage in Assumption 3 is much higher than that charged by IB.

The margin ratio is assumed to be 2 unless stated otherwise.

4.3 Brokerages

Among the major brokerages, Interactive Brokers (IB) satisfies our assumptions with respect to the commissions charged and the margin rates of interest. IB has the largest electronic trading platform in the United States. IB is 2019 Barron’s best online brokers and the commissions and margin rates of interest charged by IB are much cheaper in comparison with other brokerages, for example, E-Trade, Fidelity and most others. The rates can be lower for high volume traders. For a review of IB, see [7].

5 THE IDEAS BEHIND WT’S STRATEGY

This chapter is devoted to explain intuitively to the reader why and how WT manages to “beat” the market. For brevity, only the trading of SPY will be discussed in detail. You start out by buying 1,000 shares of SPY and this is

the only stock in your portfolio. Below are the 3 most important features that WT utilizes to “beat” the market.

- (i) *The low volatility of the SPY.*
- (ii) *The increasing trend at about 10% of the SPY .*
- (iii) *The low margin interest rate at 3.63% a year.*

5.1 WT Uses a Moderate Amount of Margin

Using margin to buy additional shares brings additional risk. If the stock rises, you will make more money and if the stock drops, you will lose more. The effects of losses or gains are magnified. The idea of using margin is to use the brokerage’s (someone else) money to help you get rich faster.

Recall that SPY increases at a rate approximately equal to 10% a year while margin rate of interest is 3.63% a year. The large difference between the two rates provides a good opportunity for you to increase your return by borrowing margin to buy some additional shares. The sum of the interests and extra commissions you pay to hold additional shares increases more slowly than your gain due to the increases in prices of these shares. Borrowing at a low rate of interest to buy shares that increase at a higher rate is one of the keys to winning here.

The SPY has relatively low volatility in the sense that its share price does not fluctuate wildly. The days that SPY price moves up or down more than 5% are rare. Due to the low volatility of the SPY, a substantial amount of additional shares can be bought without much increase in additional risk. The

number of shares in each trade is carefully calculated by WT. You never run out of money you can borrow to buy. Hence no margin calls can occur.

Individual stocks generally have high volatility as compared with the volatility of the SPY. I would not use margin to buy any additional individual stocks.

5.2 Buy Low and Sell High

WT buys low and sell high to increase her return. WT always buys at pits when price drops. Since SPY has an increasing trend, WT can generally sell the additional shares at prices higher than the buying prices after waiting for some time. Occasionally, WT has to sell low to avoid margin calls.

5.3 WT Buys More than Sells

WT’s cash in her account is more often negative than positive. The strategy is to keep on accumulating more and more shares when possible, by borrowing moderately using margin and also by buying low and selling high. Holding shares is more profitable than holding cash.

The computer program in the website does all the computations to advise you as to whether you should hold, buy or sell. In case of a buy or a sell, the number of shares is also given. Understanding how the website does the computations is not necessary to operate the website.

6 USING THE WEBSITE FOR TRADING

6.1 Instructions

Click on www.WaveletTrader.com and the screen shows: **Welcome to WaveletTrader.com and Run Annualized Wavelet Trading Strategy.** Set your margin ratio equal to 2. As an example, enter the following data and then click on “Generate Table”.

Date	Price
10/10/2018	276.01
10/11/2018	273.34
10/12/2018	271.56
10/15/2018	276.78
10/16/2018	284.45
10/17/2018	289.12

“Results Table” will show up at the bottom of the screen as follows:

Date	Price	Shares	Comm	CumComm	Cost	CumCost	CumShares
10/10/2018	276.01	1000	110.40	110.40	276010.00	276010.00	1000
10/11/2018	273.34	0	0.00	110.40	0.00	276010.00	1000
10/12/2018	271.56	229	24.87	135.27	62187.24	338197.24	1229
10/15/2018	276.78	0	0.00	135.27	0.00	338197.24	1229
10/16/2018	284.45	0	0.00	135.27	0.00	338197.24	1229
10/17/2018	289.12	-113	13.07	148.34	-32670.56	305526.68	1116

MV	Cash	BuyingPower	Interest	Balance	TradingReturn	MarketReturn
276010.00	0.00	276010.00	0.00	276010.00	-110.40	-110.40
273340.00	0.00	273340.00	0.00	273340.00	-2780.40	-2780.40
333747.24	-62212.11	209323.02	0.00	271535.13	-4585.27	-4560.40
340162.62	-62232.85	215696.92	-20.74	277929.77	1809.37	659.60
349589.05	-62239.76	225109.53	-6.91	287349.29	11228.89	8329.60
322657.92	-29589.19	263479.54	-6.92	293068.73	16948.33	12999.60

"Shares" is the number of shares you buy or sell. "Comm" and "CumComm" are, respectively, the commissions and cumulative commissions you have to pay. "MV" is your market value, which is just the total number of your shares times the share price. "Cash" is usually a negative number, which is the money you owe the brokerage. "Balance" is the amount you have in your account. "Trading return" is your return if you trade following the guidance of the website and "Market Return" is your return if you buy and hold.

7 PERFORMANCE OF WT ON SPY HISTORICAL DATA FOR 24 CONSECUTIVE YEARS

This chapter investigates the performance of WT in comparison to the market by back testing using the set of SPY historical data at the link. The data of WT's trades for 24 consecutive years is contained in the folder 24-year at the link. This set of data is too big to be included here.

1st year

You started trading on 1/29/93 and sold all shares one year later on 1/28/94. The initial price of a share of SPY is \$27.23 dollars a share, so you have to pay \$27,230.00 for the cost of the 1,000 shares and also \$10.89 in commission. The sum of the two expenses is \$27,240.89, which is your starting cost (abbreviated as Start Cost). Your return is \$4,399.17 while the market return is \$3,179.11.

Your rate of return is $100 \times 4399.17/27,240.89 = 16.15\%$ while

the market's rate of return is $100 \times 3,179.11/27,240.89 = 11.67\%$.

For brevity, denote your rate of return and market's rate of return, respectively, by TR% and MR%. The difference between the two rates of return is called the excess return, which is abbreviated as ExRet on the table.

The excess return for the year starting on 1/29/93 is 4.48%.

2nd year

You started trading again on 2/8/94 and sold all shares one year later on 2/7/95. The starting cost is \$30,012, which includes \$30,000 for the cost of the 1,000 shares and \$12.00 in commission. Your return is \$2,244.31 while the market return is \$1,498.00. TR% and MR% are, respectively, equal to 7.48% and 4.99% and the excess return for the year is 2.49%.

Calculations for the remaining years are carried out similarly. The results for all 24 years are summarized in the table below. Note that ExRet is positive in 19 of the 24 years and that you don't beat the market every year, but only on approximately 79% of the years.

WT began her trading on 1/29/93 and ended her trading for the year on 1/29/94. She took a break for about ten days and then started another trading year on 2/8/94, which ended on 2/7/95. She continued this pattern of trading for 24 consecutive years. The last year of trading began on 10/26/16 and ended on 10/25/17. The length of her break is chosen rather arbitrarily. The results of her trades would only change slightly if the length of the break is 9 days or less. The average return of WT is 13.03%

as compared to 9.58% for the market. Thus WT outperforms the market by 35.97% by trading.

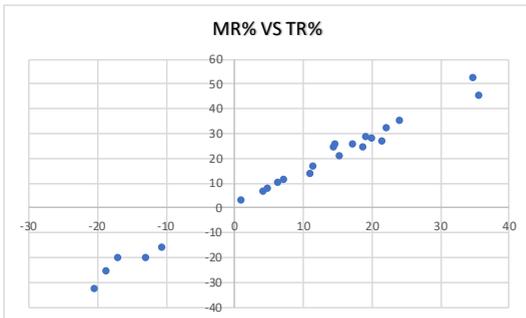
“Reset Inputs” to clear the memory of the computer at the website before you enter new data on the screen.

Reset Inputs. *You should press*

Summary Results

Start Date	End Date	Start Cost	TR %	MR%	ExRet
1/29/93	1/28/94	27240.89	16.15	11.67	4.48
2/8/94	2/7/95	30012.00	7.48	4.99	2.49
2/21/95	2/20/96	31612.64	44.59	35.74	8.85
3/1/96	2/28/97	43327.32	34.91	24.36	10.56
3/12/97	3/11/98	54841.93	51.77	34.95	16.82
3/24/98	3/23/99	76670.66	20.49	15.52	4.97
4/6/99	4/4/00	92757.09	25.37	14.84	10.54
4/13/00	4/12/01	102390.94	-20.93	-16.76	-4.17
4/26/01	4/25/02	88755.49	-16.36	-10.41	-5.96
5/14/02	5/13/03	80092.02	-20.60	-12.64	-7.96
5/27/03	5/25/04	70508.19	23.97	19.05	4.92
6/7/04	6/6/05	86104.43	9.63	6.66	2.97
6/17/05	6/16/06	93267.29	6.04	4.53	1.51
6/29/06	6/28/07	99579.82	27.51	20.28	7.23
7/13/07	7/11/08	123379.33	-26.26	-18.45	-7.81
7/24/08	7/23/09	102020.79	-32.95	-20.12	-12.82
8/5/09	8/4/10	83823.52	23.79	14.61	9.18
8/17/10	8/16/11	93237.28	13.45	11.26	2.18
8/26/11	8/24/12	102380.94	31.48	22.45	9.03
9/7/12	9/6/13	127911.14	25.42	17.57	7.85
9/23/13	9/22/14	154721.86	28.08	19.43	8.65
10/3/14	10/2/15	182412.94	2.46	1.19	1.27
10/15/15	10/13/16	191616.62	10.71	7.49	3.22
10/26/16	10/25/17	206762.67	26.55	21.81	4.74

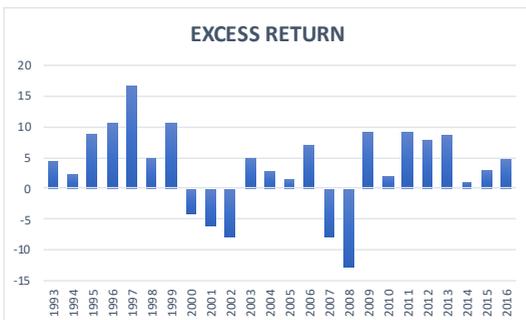
7.1 Scatter Diagram of MR% vs TR%



The graph of the scatter diagram shows the relationship between MR% and the TR%. The horizontal axis represents MR% and the vertical axis represents TR%. The two variables MR% and TR% are strongly correlated. In bull years, the trader outperforms the market whereas the market outperforms the trader in bear years.

7.2 Excess Return

WT's excess returns per year is shown in the graph below.



The graph above yields a lot of information in comparing the performances of WT and the market. The market “beats” WT three years in a row in the years 2001-2003. During the 24 years, the market never “beats” WT for a longer period. The trader WT “beats”

the market 7 years in a row from 1993-1999, and then continuously for another 8 years from 2009-2016.

7.3 Sharpe Ratios

The Sharpe ratio (SR) helps you to evaluate your excess returns compared to the additional risk you take. Risk measures the uncertainty associated with the strategy's return. The most common measure of risk is the standard deviation, which measures the dispersion or variability around the mean. The standard deviation of the excess returns is the risk associated with the strategy. It can be easily computed using the function STDEV in Excel.

From the table, the mean excess return for the 24 years is found to be 3.47%. In general, to achieve a higher mean excess return, a trader has to take higher risk. If two strategies, say, A and B, generate the same amount of excess returns, then the one with less risk is considered a better one.

The excess returns above have a mean of 3.47% and standard deviation of 6.98%. The ex-post Sharpe ratio (SR) is defined in [14] as the mean excess return divided by its standard deviation. It indicates how well you are compensated for the risk to attain excess returns. A positive SR shows that you have higher risk-adjusted return than the market. A division using 3.47% for the mean and 6.98% for the standard deviation establishes that SR is 0.49. Thus WT does quite well in comparison to the market.

Sharpe ratios assume that returns are normally distributed. Daily returns are well known to have heavy-tailed dis-

tributions. Here the returns are yearly returns which are sums of daily returns and the assumption that they are normally distributed is quite reasonable.

Sharpe ratios play an important role in the paper. They vary with the amount of margin loan borrowed and also with the margin rates of interest. Borrowing too much from the brokerage to buy additional shares can decrease Sharpe ratios. An increase in margin interest rate will reduce excess returns, resulting in a decrease of Sharpe ratios.

8 DETAILED INVESTIGATION OF WT'S PERFORMANCE USING SPY HISTORICAL DATA

The historical data set displayed in the folder SPY-Historical-Data at the link contains 6,450 dates and prices of SPY starting on 1/29/93 and ending on 9/7/18. The folder Histo-Data-Perfom at the link contains 6,198 files in the "Inputs" created as follows:

random-0000.csv lists the dates and prices of SPY from 1/29/93 to 1/28/94,

random-0001.csv lists the dates and prices of SPY from 2/1/93 to 1/31/94,

...

random-6197.csv lists the dates and prices of SPY from 9/7/17 to 9/6/18,

The folder "ouputs" contains trades made by WT using the files in the "inputs" folder. The output files are labeled consecutively, as result-0000.csv to result-6197.csv.

Upload File. *This button is created to check quickly that the output files correspond to the input files. The files in "Inputs" may not be in the right order.*

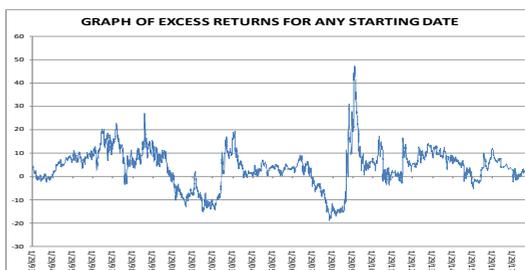
Click on "Name" to get the files ordered. You may just want to download one file if you wish. After you upload the file, click on "Generate Data" to obtain the output file.

Note that the buying powers in the output files are always positive, indicating that you never run out of money to buy or get a margin call. The results from the output files are summarized in stats.csv in the folder Stats at the link. Some interesting statistics are:

You "beat" the market in 4,715 out of 6,198 cases, which amounts to 76.7%. That means if you choose a date randomly to start your trading, then after a year you have 76.8 percent chance of beating the market; your probability of "beating" the market is about .77. Your average trading return is 14.76% as compared with the market's return of 10.93%, which amounts to an increase of 35% in return. You earn an average excess return of 3.83%.

8.1 Excess Returns

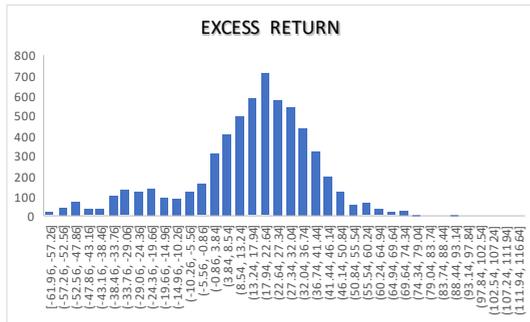
The graph of the excess returns for all 6198 years is given below. The dates on the horizontal axis are the first trading day of the year. Recall that excess returns are measured using percentages. By inspecting the graph, it is clear that WT "beats" the market more often than otherwise.



The market “beats” you if you start on almost all dates between 1/29/00 and 1/29/03 and between 1/29/07 and 1/29/09. But the market never “beats” the trader consecutively for 3 years. You “beat” the market if you start on practically all other remaining dates.

8.2 Distribution of Excess Returns

The frequency graph shows the number of occurrences of excess returns in each interval. The graph looks like a bell shaped curve.



The mean and standard deviation of excess returns, are respectively, equal to 3.83% and the 8.18% and the Sharpe Ratio is .47, which is obtained by dividing the mean of excess returns by its standard deviation.

9 How the Website Performs with Simulated Data

9.1 Fitting a Geometric Brownian Motion to Historical Data

Geometric Brownian motion (GBM) is used to model stock prices in the Black Scholes model and is about the most popular model for stock market prices. It is quite easy to fit a GBM to SPY historical data. The equation

for a GBM is given by:

$$S_t = S_0 \exp \left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right),$$

where W_t is standard Brownian motion. Here, S_t is the value of GBM at time t and S_0 is the initial value. The parameters $-\infty < \mu < \infty$ and $\sigma > 0$ are constants. GBM serves as an important example of a stochastic process satisfying a stochastic differential equation.

Taking logarithm yields,

$$\log S_t = \log S_0 + \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W_t.$$

Replacing t by $t - 1$, the equation above becomes,

$$\log S_{t-1} = \log S_0 + \left(\mu - \frac{\sigma^2}{2} \right) (t - 1) + \sigma W_{t-1}.$$

Let $X_t = \log S_t - \log S_{t-1}$. Then

$$X_t = \mu - \frac{\sigma^2}{2} + \sigma(W_t - W_{t-1}).$$

Note that $W_t - W_{t-1}$ is an increment of a standard Brownian motion over a time period equal to 1. The distribution of the increment is normal with mean zero and variance 1 (standard normal). Hence X_t is normally distributed with mean $\mu - (\sigma^2/2)$ and standard deviation σ .

The data SPY historical data set at the link contains 6,450 historical values of S_t . A total of 6,449 values of X_t is obtained as follows:

$$X_1 = \log S_1 - \log S_0,$$

$$X_2 = \log S_2 - \log S_1,$$

...

$$X_{6,448} = \log S_{6,448} - S_{6,447},$$

$$X_{6,449} = \log S_{6,449} - S_{6,448},$$

Note that there are 6,450 values of listed historical values of S_t but only 6,449 values of X_t .

Brownian motion is a stochastic process with independent increments so $X_1, X_2, \dots, X_{6449}$ are independently and identically distributed random variables. Using these values, estimates of the drift parameter $\mu - (\sigma^2/2)$ and standard deviation σ are, respectively, 0.000365494 and 0.011508575. These estimates are, respectively, the sample mean and sample standard deviation of 6,449 observations of X_t 's. The fitting of the GBM is carried out in the file GBM-fit.xlsx at the link.

If $\mu - (\sigma^2/2) = 0.000365494$, then $\mu = 0.000365494 + \frac{(0.011508575)^2}{2} = 0.0004317$.

The GBM that fits the historical data is:

$$S_t = 287.60 \exp\left(0.000365494t + 0.011508575W_t\right)$$

9.2 Random Data

The GBM found above is used to generate simulated SPY prices for the trading year starting on 9/7/2018. The first day of trading starts with $S_0 = 287.60$ and $W_0 = 0$ at $t = 0$.

The next trading day is 9/10/2018 since 9/8/2018 and 9/9/2018 are weekend days. The simulated price of SPY on 9/10/2018 is S_1 . A year has 365 days but only 261 days of trading since there are 104 weekend days when the market is closed. The website generates S_0, S_1, \dots, S_{260} for the year starting on 9/7/2018 using the fitted GBM above.

The probability density function of S_t is log-normal given by:

$$f_{S_t}(s) = \frac{1}{\sqrt{2\pi}} \frac{1}{s\sigma\sqrt{t}} \exp\left(-\frac{(\ln s - \ln S_0 - (\mu - \frac{1}{2}\sigma^2)t)^2}{2\sigma^2t}\right)$$

where s is a positive number (see [6]). On the last day of trading, the time parameter t is 260. A simple computation shows that,

$$\sigma\sqrt{260} = 0.011508575 \times \sqrt{260} = 0.1855702$$

$$\sqrt{2\pi}\sigma\sqrt{260} = \sqrt{2\pi} \times 0.011508575 \times \sqrt{260} = 0.4651555,$$

$$\frac{1}{\sqrt{2\pi}} \frac{1}{\sigma\sqrt{260}} = \frac{1}{0.4651555} = 2.149819,$$

$$-\ln S_0 - \left(\mu - \frac{1}{2}\sigma^2\right) \times 260 = -5.661571 - 0.09502844 = -5.756599,$$

and

$$2\sigma^2 \times 260 = 2 \times (0.011508575)^2 \times 260 = 0.0688726.$$

The probability density function of S_{260} is:

$$f_{S_{260}}(s) = \frac{2.149819}{s} \exp\left(-\frac{(\ln s - 5.756599)^2}{0.0688726}\right).$$

The mean and variance of S_t are:

$$E(S_t) = S_0 \exp(\mu t), \text{Var}(S_t) = (S_0)^2 \exp(2\mu t) (\exp(\sigma^2 t) - 1).$$

Then

$$E(S_{260}) = \$287.60 \times \exp(0.000431717 \times 260) = \$321.76.$$

A simple computation shows,

$$(S_0)^2 = (287.60)^2 = 82713.76,$$

$$\begin{aligned} \exp(2\mu \times 260) &= \exp(2 \times 0.000431717 \times 260) = 1.251688, \\ \exp(\sigma^2 \times 260) - 1 &= 1.03503609 - 1 = 0.03503609. \end{aligned}$$

Therefore, the variance and standard deviation of S_{260} are ,

$$\begin{aligned} \text{Var}(S_{260}) &= 82,713.76 \times 1.251688 \\ &\times 0.03503609 = 3627.35, \\ \sigma(S_{260}) &= \sqrt{3627.35} = 60.22. \end{aligned}$$

On the last day of trading, the simulated SPY share price has a mean of \$321.76 and standard deviation of \$60.22.

The cumulative distribution function (see [8]) of S_{260} is:

$$\begin{aligned} F(s) &= P(S_{260} \leq s) = \\ &= \frac{1}{2} \left[1 + \text{erf} \left(\frac{\ln s - 5.756599}{0.1855702\sqrt{2}} \right) \right], \end{aligned}$$

where the erf function is defined as:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dx.$$

It is easy to show that $\text{erf}(x) = 2\Phi(x\sqrt{2}) - 1$, for any $x \geq 0$, where $\Phi(x\sqrt{2})$ is the area under a standard normal curve up to $x\sqrt{2}$. Thus erf(s) can be computed using a normal table.

9.3 Testing WT with Simulated Data

Go to the webpage and click on “Run Annualized Wavelet Trading Strategy” and then enter the margin that you hold. Click on “Random Data” and then on “Generate Data”. The website generates an Excel.csv file containing simulated values of SPY for one year. You can continue by clicking on “Generate Table” to see how WT trades with simulated data. If you play with “Random Data” long enough, you will notice that

WT is more likely to “beat” the market in a “bull” year when prices increase, and the market is more likely to “beat” WT in a “bear” year when prices decrease.

According to [11] , over 88 years, from 1928 to 2015, the S&P 500 went up 64 years and went down 24 years. which means the S&P 500 went down about once every 4 years on average. Since there are more “bull” years than “bear” years, WT “beats” the market in the long run.

9.4 Multi File Simulator

The Multi-File Simulator page is created to replicate “Random data” up to 1,000 times. Go to the webpage WaveletTrader.com and click on “Multi-File Simulator” then enter the margin ratio allowed by your brokerage and the number of times you want to replicate. As an illustration, set the number of replications to be 1,000 and set the margin ratio equal to 2. Click on “Run Simulation” then wait a couple of minutes. The website generates three items: stats.csv, inputFiles.csv and outputFiles.csv. The inputFiles.csv and outputFiles.csv are folders containing 1,000 files in each. A sample of these three items is stored at the folder 1000-Simulated-Sample at the link.

The file stats.csv contains relevant statistics obtained from an analysis of the the output files. A total of 1,000 output files is listed in the first column of stats.csv. They are labeled as result-0000.csv to result-0999.csv. The second column and the third column in stats.csv list, respectively, the starting

date 9/7/18 and the last date 9/6/19 of the trading year. The fourth column in stats.csv records the stock price at the last day of the trading year. The fifth and sixth columns in stats.csv show, respectively, the starting cost \$28,7715.04 and the ending balance of the trader. The seventh and eighth column in stats.csv list the trading returns and market returns.

Recall that market return is your return if you buy and hold for the entire year. The ninth column in stats.csv lists the values of alpha, which is the trading return subtracted by the market return. Note that market returns, trading returns and alpha are values in dollars. The Sharpe ratio for each year is in the 10th column. They can be negative or positive for a year but the average over 1,000 simulated years is 0.023.

The variables in file stats.csv are defined as follows: WT Ave is the average return of WT for the 1,000 years

of simulation. This average is equal to \$42,012.91. Market Ave is the average return of the market for the 1,000 years of simulation. Market Ave is \$33,320.28. Alpha Ave is the difference between WT Ave and Market Ave. Alpha Ave is \$8,692.63. SR Ave is the average Sharpe ratio for all the years.

The results displayed in stats.csv show that WT outperforms the market by about 26% a year as compared with 35% using real data. This discrepancy is not surprising. The GBM is a markov process whose future moves are dependent only on the current state. Buying at pits helps to boost WT's returns with real data but not with data generated by a GBM. Buying low means that you think the market is more likely to rise at pits. The GBM does not account for this type of human behaviour. Buy low and sell high boosts the average annual return of WT by about 9%, which is the difference between 35% and 26%.

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