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MATHEMATICAL MODELLING FROM THE COGNITIVE POINT OF VIEW, THE TRANSITION FROM IMPLICIT MODEL TO EXPLICIT MODEL

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Abstract. Thecognitive aspect of mathematical modeling competencehas been of interest in the mathematics education community. From the cognitive point of view, the individual's behavior and problem-solving abilities have a strong connection with the inner cognitive activities. We will present BorromeoFerri’s diagram of cognitive mathematical modeling, including the transformation from an implicit model to an explicit model. We then describe one group of 10th graders work on building a bridge crossing Huong River project to simulate students' mathematical modeling process to illustrate how the model is used.

Keywords. Mathematical modeling, cognitive, implicit model, explicit model.

1. Introduction

Mathematics comes from reality and is closely related toreal life. Reality is the origin of mathematical concepts to emerge and develop, and mathematics has a wide range of applications in many different fields of science and technology as well as in production and social life. The NationalCouncil of Teachers of Mathematics (NCTM, 2000, p.2) suggests:

In a coherent curriculum, mathematical ideas are linked to and build on one another so that students’ understanding, and knowledge deepen and their ability to apply mathematics expands. An effective mathematics curriculum focuses on important mathematics that will prepare students for continued study and for solving problems in a variety of school, home, and work settings.

The Theory of Realistic Mathematics Education developed in the Netherlands brings out two principles (Tran Vui, 2014):

(1) Mathematics must be linked to the real world, and
(2) Mathematics should be viewed as a human activity.

Therefore, besides providing students with mathematical knowledge and skills such as concepts, theorems, formulas, rules, developing students' ability to use the knowledge and skills in solving problems in learning real-life situations is a must (Tran Vui, 2014, p.78). When solving problems in life, the mathematical model and mathematical modeling process are necessary.

Mathematical modeling gets its emphasison mathematics education in the recent few decades (Blum et al., 2007). Its importance is reflected in current curricula, such as the Common Core State Standards for Mathematics (CCSSM, 2012).

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At the 3rd International Conference on Mathematical Education (ICME-3) in 1976, Henry Pollak proposed to integrate applications and mathematical modelinginto teaching. The Program for International Student Assessment (PISA) program also emphasizes that the purpose of mathematics education is to develop the capacity for students to use mathematics in their lives. The biennial international conference on teaching modeling and application (ICMA) also aims at promoting application and modeling in all areas of mathematics education.

Many different mathematics educators around the world have studies competence relating to mathematical modeling. Maaß (2006) classified mathematical modeling competence into three distinct areas: cognitive, affective, and metacognitive. The cognitive aspect includes conscious activities that students participate in modeling. The affective aspect is related to students' beliefs, emotional orientations of mathematics, the nature of problems, as well as the role of mathematics in solving real problems. Finally, the metacognitive aspectisthe factor that supports cognition, the thought about one's thinking and controlling one'sthought processes. Allthree aspects belong to the implicit model (BorromeoFerri, 2006). However, to be able to observe students' mathematical modeling competence, a transition from an implicit model to an explicit mathematical model is necessary to be monitored and analyzed.

It is critical for researchers to provide an account for the cognitive process students go through when they engage in a modeling problem. In particular, how such process can be described and measured empirically can inform future actions such as how to scaffold such process efficiently and derive recommendations for teachers to guide students during the modeling process. Studies have been examining teachers and students in context-bounded mathematics lessons from a cognitive perspective (BorromeoFerri, 2006, 2013) mostly in Western countries. Therefore, we integrated a true-modelingtask (Tran & Dougherty, 2014) into the activities of Vietnamese students in a mathematics class to address the scarcity of research as well as to highlight the complication of student cognitive model in the highest level of authenticity of modeling task (Tran et al., 2019). This study addresses the following research question: How is students' mathematical modeling competence expressed through the transition from an implicit cognitive model to an explicit model?

2. Content

2.1. Mathematical modeling from a cognitive view

BorromeoFerri (2007) focused on studying mathematical modeling from the cognitive view. By using the mathematical didactical and cognitive psychology approach of mathematical thinking styles, the researcher analyzed teachers and students' process of participating in modeling problems in the classroom. From a cognitive point of view, BorromeoFerri emphasized (see BorromeoFerri, 2006):

• Individual students' modeling processes need to be rebuilt at the level of small processes with a cognitive psychology approach of mathematical thinking styles.

• The teachers'handling during the pupils'modeling process and their classroom discussion afterward will be reconstructed.

BorromeoFerri focused on the analysis of individual students' modeling processes based on the analysis of individual modeling routes (BorromeoFerri, 2006). Treilibs and colleagues (1979) (Treilibs, Burkhardt&Low, 1980) also examined how learners build models in their process of modeling. Therefore, Treilibs did not examine the process of making a complete model but focusing on the "construction phase" when the model is formed.

Matos' and Carreira' research (1995, 1997) put a particularemphasis on 10^{th} learners' cognitive processes. The authorsreconstructed the performances of students when theyparticipate insolving practical problems. In so doing, various explanations used by the students were exposed in their process of participating in modeling.

Modeling cycle from a cognitive view

Many different modeling cycles have been developed. Fromthe cognitive aspect, we used the cycle proposed by Blum and BorromeoFerri (2009) to examine student mathematical modeling. The modeling cycleincludes real situation, mental representation of the situation, real model, mathematical model, mathematical results, and real results. Reusser (1997) assumed that a situation model would exist when an individual illustrates the situation described in the task through an internal mental representation. The term mental representation of the situation was used instead of the situation model as it better describes the kind of internal processes of an individual after reading the given modeling task (BorromeoFerri, 2006). A mathematical modeling cycle consists of six steps: (a) understanding a situation and building a model for that situation; (b) simplifying the situation and using appropriate variablesto build a real model of the situation; (c) transfroming from real model to mathematical model; (d) working in a math environment to achieve math results; (e) interpreting the results into the reali context; and (f) validating the suitability of the result or making the second cycle (see Fig. 1).

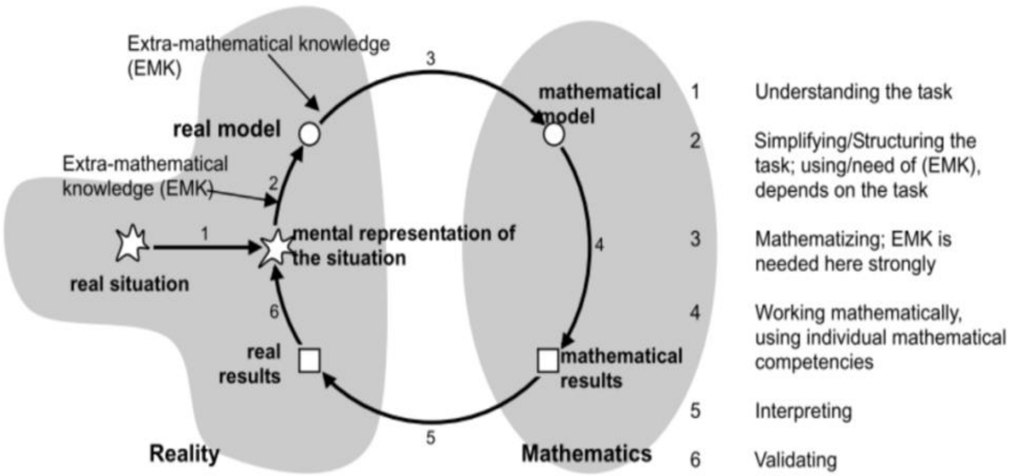

Figure 1. Modeling cycle from a cognitive view, Blum and BorromeoFerri (2009, p.266)

2.2. From an implicit model to an explicit model

The term mathematical modeling is discussed in ways that are significantly different around the world (Kaiser &Sriraman, 2006). Views on the concept of "model" are not often easy to agree. Nevertheless, the term *situation model* used by Blum and Leiß (2007) in modeling cycles has gained more attention in mathematics education. In particular, this term has been used in combination with modeling problems - precisely with verbal problems (Kintsch & Greeno, 1985; Nesher 1982; Verschaffel et al., 2000). A situation model could be described as a mental representation of the situation in a verbal problem, and BorromeoFerri termed *mental representation* in the modeling cycle (BorromeoFerri, 2006). In addition, BorromeoFerri held that this term would be better to describe internal processes. The mental representation is a special way of thinking about a particular situation and influenced by personal experiences, and so, it is somewhat difficult to share with others. While mathematical models are often expressed explicitly in a textual or verbal form, it is better to convey to others. Yet, it usually includes implicit model building stages (Fig. 2). Therefore, some aspects of a situation model can also act as an implicit model.

From a cognitive-psychological view, we cannot know how the internalized system and model of an individual's brain. That underlying system and model include cognitive aspects and beyond-cognitive aspects. However, when developing a real modelor a mathematical model, an individual is often conscious of his actions. Therefore, transitioning from an implicit model to an explicit model could be very helpful for research (see Fig. 2). Implicit models are on an unconscious level, and that explicit models are more conscious and can be better communicated to others.

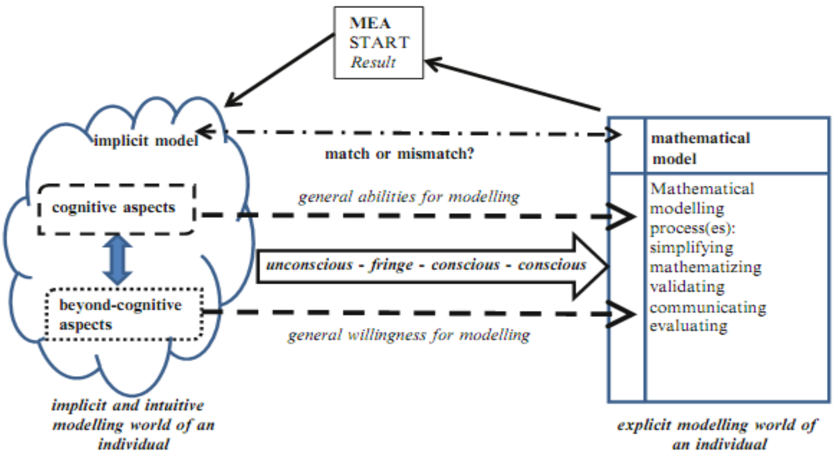

Fig. 4.3 Implicit and explicit modelling world of an individual

Figure 2. Implicit and explicit modeling world of an individual, Borromeo Ferri (2006, p. 64).

Model-eliciting activities (MEAs) (Borromeo Ferri & Lesh, 2013) is a diagram emphasizes parallel and interacting developments between the "explicit modelling world of an individual" (right side of the figure) and a fuzzy "implicit and intuitive modeling world of an individual" (left side of the figure). The first stage when working on an MEA or another modeling problem is this fuzzy "implicit modeling world" including cognitive aspects and beyond-cognitive attributes which influence each other in several (unconscious) ways. Cognitive aspects contain general abilities for modeling, which means mathematical abilities and modeling competencies and thus are necessary for developing an adequate mathematical model (Borromeo Ferri& Lesh, 2013). At the same time, these cognitive aspects are influenced by beyond-cognitive attributes such as beliefs and feelings, which in turn build a basis for general willingness dealing with the MEA at all. It is impossible to reconstruct these mental actions of an individual. However, these interpretation systems within this fuzzy unconscious world are the bricks for the upcoming mathematical model in the "explicit modeling world" through the fringe-consciousness and finally consciousness. The match or the mismatch of the interpretation systems developed in the implicit world as an implicit model with the mathematical model in the explicit world cannot be investigated, because it disappeared in the mystery of the unconsciousness (Borromeo Ferri & Lesh, 2013).

The beyond-cognitive aspects of mathematical models are essential. When students solve modeling problems, they participate in the system of mathematical concepts and express emotions, attitudes, and beliefs. Many of the beyond-cognitive attributes are conscious and often present when the logical-mathematical aspects of the model are active. Kaiser and Maaß (2007) demonstrated in their one-year experimental study that many seventh graders changed their beliefs in mathematics. In particular, the usefulness of mathematics was essential that the students had not realized before. Such a realization increases math motivation for the students.

An upcoming reform initiative of Vietnam school curriculum and textbooks will follow a competency-based model (Vietnam Department of Education, 2018). This initiative calls for preparing students to develop mathematical competencies which involve mathematical thinking and reasoning, mathematical modelling, problem-solving, mathematical communication, and using mathematical tools. Mathematical modeling is emphasized as a way for students to develop an appreciation of mathematics and its role in the real world; students should be able to use the mathematics they learn in school in everyday life (Tran et al., 2019).However, current secondary mathematics textbooks still focus on pure mathematics and ignore contextualized problems. When such contextualized tasks appear in textbooks, they tend to be contrived and superficial (Tran et al., 2019). Moreoverresearch related to mathematical modeling competence from a cognitive view is still a new field and has not received much attention from mathematics education researchers in Vietnam. The purpose of this study is to illustrate how students' mathematical modeling processes are exhibited from a cognitive view.

2.3. Methodology

We used a teaching experiment approach in this study. The investigation was conducted in three 10th grade classes from Thuan Hoa and Hai Ba Trung high school (Hue city, Viet Nam). The sample comprised of 128 pupils and two teachers. We chose 10^{th}-grade students because they are in the transition from middle school to high school, and this transition provides the potential todevelop innovative perspective about mathematics for students.

A total of 128 students were introduced mathematical modeling with class-size task. For the last task, the students were assigned to work in a project, and they had three weeks to accomplish. In the third week, students presented their work in class.

The project description is as follows:

To support traffic flow between two parts of Hue city, the govement develops a new bridge that goes across Huong river. The bridge is located to connect between Nguyen Hoang street and Bui ThiXuan street (Phuong Duc Ward). Can you propose a bridge model and explain your proposal with supporting documents?

This task can be considered as true modeling (Tran & Dougherty, 2014). True-modeling problems involve the full modeling cycle: Start with a question, then formulate a model, and finally solve, interpret, and validate betweena mathematical and contextual situation.Students experience the role of a designer, planning, conducting, and explaining their ideas. Also, students are allowed to use any resources, such as the internet, books, newspapers that they find helpful. The group was chosen because they had added more extra-mathematical knowledge than othersin their modeling process.We chose this context for the project as their was a campaign in Hue cityabout the design of a bridge that goes across Huong riverat the time when the study was conducted. This realistic context is one of the criteria recommended in Tran et al. (2019) in designing modeling tasks.

Before solving this task, students engaged in three class-size modeling tasks, which prepare them familiarize with the modeling cycle and the transition from traditional mathematical tasks to realistic tasks. For the scope of this paper, we only discuss a true-modeling task carried out at the end of the project. The task presented in this study is thefourth and final task, which was a 3-week project. The choice of this analysis is to help exemplify the cognitive aspect of the modeling cycle.This study is exploratory; therefore we will use one group to illustrate how such framework can be used in analyzing students' process when they solve a project-based task.We describe the modeling routes of one group concerning the task Across Huong river task using the cycle (Fig. 1) that is to mapping student actions to the six steps specified in the cycle.

2.4. Results

This group changed from the situation described in the project into the mathematical model: “Looking for the number of bridge abutments so that the building cost is minimum” (*Real situation* → *Mathematical model 1*)

Doing this, they started thinking about another mathematical model: “what is an equation to model the bridge” (*Mathematical model 1* → *Mathematical model 2*)

The bridge has 6 piers, the distance between two piers is 48,5 m, the height of the bridge is 4 m.

The students used variables to create a mathematical model and calculated a, b, c of $y = ax^2 + bx + c(a \neq 0)$ (P) using the information: (P) goes through the points A(-170, 0); $B(170,0)$ and $I(4,0)$. Although they did not show the process explicitly in their report, the symbols on the graph show this (see Fig. 3).In addition, this process was confirmed by the students during the interview: *"We put O (0,0) at the center of the two ends, attached the Oxy system, with Ox coinciding with the straight line connecting the two ends, Oy and Ox are perpendicular. Call the general equation (P) and looking for a, b, c by replace A(-170, 0); B(170, 0) and I(0, 4)"*

They found a parabolic equation: $f\left(x\right) = \frac{-1}{7225}x^2 + 4$.

After they had found a parabolic function, the students returned to the first mathematical model and tried to look for the answer.

They worked in the mathematical environment for Mathematical model 1 by adding the variables

Each bridge meter costs a billion dongs $(a > 0)$

Each bridge abutment costs b billion dongs $(b > 0)$

The length of the bridge is l (m) $(l > 0)$

The number of bridge’s abutmentsis n $(n > 0)$

So, the cost after completion (not including decoration and pavement) is $al + bn$ (billion dongs)

In order to solve this problem, they used the Cauchy Schwarz inequality (*Mathematical model* → *Extra-mathematical knowledge*).

$$al + bn \geq \frac{(\sqrt{al} + \sqrt{bn})^2}{2}$$

Students argued that: The "=" sign occurs, or the bridge has the minimum construction cost if and only if $al = bn$ or $n = \frac{a}{b}l$

To find n, they had to calculate the length of the bridge. It means they accepted a and b as constants, which can befound from real data. Although this model was not explicitly reported, the way this group drew graphs and calculated what? reveals a hidden model in their cognition (see Figure 3) (*Extra-mathematical knowledge* → *Mathematical results*)

$f\left(x\right)=\dfrac{-1}{7225}x^{2}$, A(-170, 0) và B(170,0)

The length of the bridge is equal to the length of the arc AB

$$l=\int_{x_A}^{x_B}\sqrt{1+\left(f'(x)\right)^2}\,dx = \int_{-170}^{170}\sqrt{1+\left(2.\frac{-1}{7225}x\right)^2}\,dx \approx 340{,}1254485\left(m\right)$$

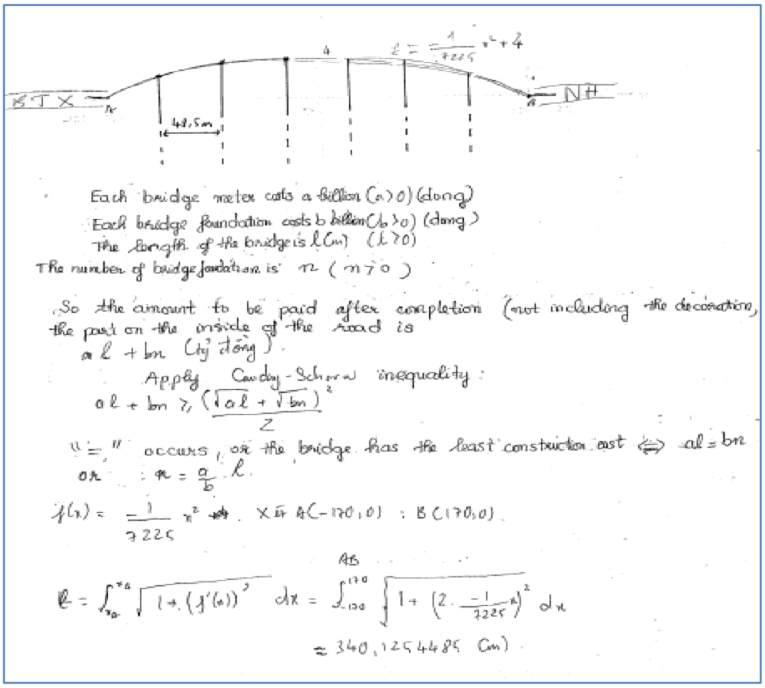

Figure 3. Report of group 3

(The student report was translated from Vietnamese)

As for decoration, this group chose lotus to represent Hue. They thought that Hue has many temples and isa land of spirituality. Also, lotus also creates features of Hue, serving festivals, and combining tradition and modernity (*Mathematical result→Real factors*).

Student's data and calculations were acceptable when compared to real data. In addition, the mathematical tools that were quite divers, such as Cauchy - Schwarz inequality, integrals, and derivatives. As seen, the mathematical knowledge is not easy for 10th graders. In interviews related to this issue, students said that: *"The mathematical knowledge is used because we are interested and curious, so we have learned some knowledge such as inequality, integrals, and derivatives and its application. In addition, we also refer several books such as Diamonds in math inequality, document for Advanced math 10th, websites: diendantoanhoc, toanmath and teacher supports".*

3. Conclusion

We analyzed the students' mathematical modeling steps using the explicit mathematical modeling cycle. From these explicit models, we can infer implicit models that have been existed in the students' cognition (e.g., looking for an equation to model the bridge). It helps illustrate the process the students went through when dealing with a true-modeling problem. With a project limited to 3 weeks, students were allowed unlimited support. In such an open project, the internet was extremely valuable, helping students become familiar with and connect with knowledge, science and technology. It motivates students to discover new knowledge, including mathematical, as described in the results of the project implementation above (such as integrals, derivatives). Student work showed that the models that students used are quite rich and varied, although some cases have not been optimal. This might inform teachers how to scaffold student learning when they solve such tasks. One might argue that the mathematics students used in this study was beyond what is required by the curriculum for Year 10 students. However, first, the selection of data here was for an illustrative purpose, and we did not argue this was typical. Second, when involving in modeling, students have freedom to choose the mathematics they are familiar and comfortable with to use. This contrasts with mathematical application when researchers/teachers have a mathematical topic in mind and look for real-life problems for students to apply the mathematical knowledge.

The analysis focuses on the cognitive aspect of modeling. However, we also observed other aspects related to the modeling process. To the best of the researchers' knowledge, this is the first time these students have access to realistic problems in math class. At the beginning, they were quite nervousand hesitated to participate in class modeling problems, but then they gradually became more active and flexible. Many of them came to be interested in the project. Moreover, presenting a project report was also new for them. Their ideas and arrangement of ideas, hence, were not expressed coherently and clearly. However, their presentation revealed hidden thoughts in their cognitive system. Also, the students gradually realize the value of learning and the applications of mathematics in practical life. Future research can continue to investigate other aspects, such as emotion and attitudes (non-cognitive) (See Fig. 2, left).

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