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THE DIDACTIC TRANSPOSITION COMPETENCE OF MATHEMATICS PRESERVICE TEACHERS

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Abstract. In the context of the implementation of the Vietnamese General Education Curriculum in 2018, teachers must be able to use knowledge from a variety of sources due to the demand to develop students' competence. Teachers must transform it into knowledge to be taught and convert knowledge to be taught into taught knowledge. This process is related to the term called didactic transposition. To teach students, teachers need to be adept at didactic transposition. The study aims to propose a set of criteria for the internal didactic transposition competence, surveying the internal didactic transposition competence of mathematics preservice teachers presently training at Hanoi National University of Education, and suggesting some measures to enhance that competence.

Keywords: didactic transposition, didactic transposition competence, internal didactic transposition, mathematics preservice teachers.

1. Introduction

Stemming from the development of anthropology, the Anthropological Theory of Didactics (ATD) was founded in the 1970s in France and became a new section of education science. This field of research quickly spread around the globe, especially in French-speaking countries.

One of the highlights of didactic theory is didactic transposition. According to the research of Nguyen Thi Thanh Van [1], the term "didactic transposition" was established by Verret in 1975. In 1985, didactic transposition began to sprout by Chevallard's definition through his investigation into the didactics of Mathematics. Didactic transposition was later seen in other subjects. Johsua & Dupin introduced the teaching of science and mathematics under the aspect of didactic as a radical alternative that should be progressively distinguished from other approaches related to science teaching in 1993 [2]. In 2001, Koliopoulos & Ravanis presented the didactical implications of designing Physics curriculums' content [3]. In 2014, Pascal Terrien studied the know-how of artists using the theoretical framework of didactics, the process that allows teachers to help students develop their musical skills, and the forms of knowledge [4]. These works focused on the consideration of the teaching and learning process of different subjects in the field of didactic transposition and the application of ATD in teaching. In Vietnam, the theory of didactic transposition was transmitted later, and the research was mostly conducted by didacticians, influenced by the French theory of didactics, as another approach to the relation between knowledge and learners. In 2001, Tien Le Van questioned the conditions and constraints

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concerning the study of equations using functions in the teaching of mathematics in France and Vietnam [5]. In 2005, Nguyen Ba Kim presented basic notions in the Theory of Didactical Situations in Mathematics and gave some pedagogical conclusions for mathematics teachers [6]. In 2009, Annie Bessot, Claude Comiti, Le Thi Hoai Chau, and Le Van Tien acknowledged the fundamental elements of didactic in Mathematics including didactic transposition, didactic contract, and the application of anthropology in teaching mathematics [7]. In 2020, Tra Huong Do and Van Nguyen Thi Thanh proposed a structure of didactic transposition capability and analyzed an example of didactic transposition in training Physics preservice teachers [8].

While the Vietnamese General Education Curriculum of 2018 is widely and thoroughly propagated and applied in every school in the nation, it raises an important problem for teachers to manage the method of transforming the knowledge in the curriculum to the one taught in class. That obstacle is not only an issue for teachers but also for students who are preservice teachers. Angeliki Vellopoulou and Konstantinos Ravanis (2010) stated that the curriculum, the syllabus, and the manuals intended for the pupil and the teacher are of great importance since they are the result of the initial process of didactic transposition [9]. Without the help of official documents, there is no "bridge" object connecting students and knowledge; hence, this role is fully occupied by teachers. Then, teachers must combine textbooks and the standard outcomes and choose from them a section of knowledge that later becomes teaching content, and the difficulty of this task escalates considerably due to the application of the new curriculum.

This research will briefly illustrate the theory of didactic transposition in teaching mathematics and mainly focus on some measures to develop the didactic transposition competence of mathematics preservice teachers. Although the theory of didactic transposition was raised from mathematics education and there has been much research on didactic transposition and didactic transposition competence, most of those works are in fields other than Mathematics. In addition, the objects of those research are teachers, while in this work, the owners of didactic transposition competence are mathematics preservice teachers, who are professionally trained in pedagogical institutions but lack practical teaching experience.

2. Content

2.1. The didactic transposition

As mentioned above, the term “didactic transposition” (originally in French: transposition didactique) was first introduced in 1975 by a French sociologist named Michel Verret. Verret studied the process of teaching and learning as an aspect of sociology to illustrate the social meaning of pedagogy and education.

In Verret’s research, he assumed education was restrained by the environment consisting of the processes of teaching and learning. Therefore, to propagate knowledge, people must choose and reform certain knowledge to make it teachable and understandable (Nguyen Thi Thanh Van, 2017 cited in Philippe Perrenoud, 1998) [1].

Chevallard and Marianna Bosch (2014) asserted that the concept of didactic transposition was developed by Chevallard in 1985. Those authors also suggested that “The process of didactic transposition refers to the transformations an object or a body of knowledge undergoes from the moment it is produced, put into use, selected, and designed to be taught until it is taught in a given educational institution” [10].

Marianna Bosch and Josep Gascón (2006), in their work evaluating the theory of didactic transposition after 25 years, suggested a detailed description as a complement to the definition of Chevallard: “… what is being taught at school (“contents” or “knowledge”) is ... something generated outside school that is moved “transposed” to school transpositive work needs to be

carried out so that something that was not made for school changes into something that may be reconstructed inside school" [11].

In 2006, Emil Paun also published a definition of didactic transposition. According to Emil Paun, didactic transposition is a complex process influenced by many factors that have as a starting point the whole of scientific knowledge and as an endpoint all the knowledge acquired by the students [12, in French].

From the above definitions, this research suggests that the didactic transposition may be understood as *the process of transforming scholarly knowledge into knowledge to be taught and transforming knowledge to be taught into taught knowledge that is compatible with the physical and psychological development of certain groups of students.*

2.2. Didactic transposition competence

In mathematics education, teachers play an essential role in the process of didactic transposition. The key is to refine the teaching content, which means modifying the amount of knowledge as well as its complexity to be in line with students' level of comprehension. To implement the didactic transposition, before working professionally in educational institutions, mathematics preservice teachers must be capable of performing its manifestations.

2.2.1. Didactic transposition competence in teaching Mathematics

According to Adrienne Kozan Naumescu (2008), the competence of didactic transposition is defined in the fusion with linguistic ability. The research mentioned "the competence in didactical transposition and use of language" [13] as one of the competencies that teachers, especially those teaching natural science, must manage. According to Tra Huong Do and Van Thi Thanh Nguyen (2020), didactic transposition competence is defined as "the ability to analyze the transformation of scholarly knowledge into the knowledge that needs to be taught in textbooks and the transformation of the knowledge in textbooks into the one that can be taught in classrooms with a sense of responsibility, effectiveness under specific circumstances of the curriculum, which is based on the awareness and requirements of didactic transposition in accordance with the appropriate learners and school conditions" [8].

Based on the previous conceptions, this research proposes that didactic transposition competence in mathematics education is the competence of comprehending the process of transforming mathematics knowledge and managing to transform mathematics knowledge of academically higher levels into mathematics knowledge in the general curriculum and to transform mathematics knowledge in the general curriculum into the mathematics knowledge that can be taught and will be taught in the micro-scale of classrooms, conducted by pedagogical specialists, educational management authorities, mathematics educators, and mathematics teachers, depends on the practical conditions for teaching and learning Mathematics of the educational institutions as well as the physical and psychological development of certain groups of students.

2.2.2. The manifestation of didactic transposition competence

For teachers to perform didactic transposition competence, it is necessary to establish a set listing the components, the elements, and some criteria of the didactic transposition competence. It is shown that there are two main stages of the didactic transposition procedure: external transposition, determined by the process of transforming scholarly knowledge, and internal transposition, determined by the process of transforming the knowledge in the curriculum to the one taught in class (Lurdes de Fátima Polidoro and Robson Stigar, 2010 cited in Chevallard, 1991) [14]. The external didactic transposition is usually done on a macro-scale by curriculum planners, pedagogical specialists, educational administrators, and sometimes teachers, while the internal

didactic transposition is mainly performed by teachers in class, represented by the act of choosing the knowledge formation paths that are suitable for students [6].

Tra Huong Do and Van Thi Thanh Nguyen (2020) proposed a list of criteria to identify the level of accomplishment of preservice Physics teachers in didactic transposition competence and listed the manifestation of the external transposition competence, including (i) analyzing the knowledge of different stages in the transposition process, (ii) analyzing the objectives, knowledge contents in textbooks, (iii) analyzing the diagram of the scientific process of developing and applying knowledge, (iv) pointing out the applications of knowledge in practice, and (v) restructuring the knowledge content in textbooks [8].

The group of authors also investigated the capability of internal didactic transposition and provided some criteria in Table 1 [8].

Table 1. Elements and criteria of the internal transposition capability

No.	**Capability of internal transposition**		
	Symbol	**Elements**	**Criteria**
1	CIT1	Determine appropriate targets of capacity development for learners.	- Establish a goal in a certain topic/module at different levels of awareness. - Determine the skills that need to be obtained in a lesson. - Identify the attitudes needed in a lesson.
2	CIT2	Utilize knowledge that ensures the instructional objectives.	- List the knowledge related to the objective. - Simplify and shorten teaching content horizontally (range of the learners). - Simplify and shorten teaching content vertically (levels of difficulty for learners).
3	CIT3	Design lesson plans that meet the instructional objectives.	- Identify the logic that shapes the knowledge content. - Identify learning situations that correspond to the logic of knowledge content. - Select suitable instructional forms and methods with the knowledge content. - Select suitable forms of revising, summarizing, testing, and evaluating after class.
4	CIT4	Conduct the lesson plans.	- Conduct the instruction in class step by step as scheduled. - Be flexible in organizing learning activities. - Assess the students' knowledge, skills, and abilities in class.

Source: Do Huong Tra and Nguyen Thi Thanh Van (2020) [8]

This research focuses on studying the internal didactic transposition of mathematics preservice teachers. Based on the work of Do Huong Tra and Nguyen Thi Thanh Van (2020), this paper promotes a set of criteria as an adaptation for mathematics education.

Table 2. Elements and criteria of the internal didactic transposition competence in mathematics education

No.	**The internal didactic transposition competence in mathematics education**	
	Elements	***Criteria***
1	Determine appropriate competence-developing objectives for students of different levels.	- Construct objectives for a certain mathematics lesson or topic for different levels of students' awareness, including (i) low-performing, (ii) average-performing, and (iii) high-performing.
2	Determine knowledge that ensures the instructional objectives.	- Determine the mathematics knowledge related to the objectives of the lesson. - Simplify and shorten content in the textbook to be suitable for each level of students, including (i) low-performing, (ii) average-performing, and (iii) high-performing. - Simplify and shorten some activities in the textbook to be suitable for the standard outcomes of the Mathematics Curriculum.
3	Construct lesson plans that meet the instructional objectives.	- Identify typical situations in teaching a mathematics lesson or topic. - Identify the path of mathematics concepts, theorems, and methods formations in the prior determined typical situations. - Select suitable teaching methods and teaching aids with the knowledge content. - Select suitable forms of revising, summarizing, testing, and evaluating at the end of the lesson.
4	Implement the lesson plans flexibly.	- Conduct the lesson plans suiting certain classes. - Be flexible in organizing learning activities. - Assess the students' knowledge, skills, and abilities in class.

It is essential to explicate the eleven criteria so that mathematics teachers can understand and develop didactic transposition competence. In this article, the criteria will be presented explicitly and exemplified in teaching the lesson "Combination" in *Mathematics 10*, Volume 2, written by Do Duc Thai (Chief Editor), Pham Xuan Chung, Nguyen Son Ha, Nguyen Thi Phuong Loan, Pham Sy Nam, Pham Minh Phuong, Pham Hoang Quan (2019) [15].

- Construct objectives of a certain mathematics lesson or topic for different levels of student awareness, including (i) low-performing, (ii) average-performing, and (iii) high-performing: In planning a lesson, the first step that a teacher must be able to perform is to determine the instructional objectives. The teacher must rely on two sources of information to construct appropriate objectives. First, the standard outcomes regulated in the curriculum restrict the minimum requirements for students to attain in a lesson. Second, the awareness levels of students in the class, which is categorized into 3 groups of low-performing, average-performing, and high-performing, stipulate the objectives to be constructed into corresponding levels. At each level, students are required to perform activities and express their apprehension of knowledge differently. Mathematics preservice teachers need to determine the competencies that students are required to achieve, then denote a set of specific expected behaviors or actions performed by students that is relevant to both the standard outcomes and their levels of cognition. The objectives need to be written clearly and understandably, and based on that, the teacher can evaluate students' competence effectively and thoroughly. Constructing objectives requires teachers to practice and improve themselves through years of professional work, and teachers also have to grasp the curriculum as well as the standard outcomes of a specific lesson or topic in order to construct appropriate objectives.

Example of constructing objectives in teaching the lesson "Combination":

+ For low-performing students:

Compute the number of combinations;

Compute the number of combinations using a hand-held calculator.

+ For average-performing students:

Compute the number of combinations;

Compute the number of combinations using a hand-held calculator;

Solve complex problems related to combination.

+ For high-performing students:

Compute the number of combinations;

Compute the number of combinations using a hand-held calculator;

Solve complex problems related to combination;

Prove the formula of combination;

Understand the properties of combination.

- Determine the mathematics knowledge related to the objectives of the lesson: In teaching mathematics, teachers need to decide which section of knowledge is necessary for the lesson. That knowledge must serve the instructional objectives and circulate the core teaching content of that lesson. It includes knowledge presented in the textbook and additional knowledge, such as knowledge learned in lower grades or previous lessons, knowledge in other subjects that are related to the theme of the lesson, or real-life knowledge as an application. The teacher investigates the teaching content in the textbook, determines which part of the lesson does not match the objectives, and addresses any knowledge, task, or question by replacing it with more suitable content. Besides, the teacher should update the teaching content by integrating the additional knowledge into mathematics lessons.

Example of determining the mathematics knowledge related to the objectives in teaching the lesson "Combination":

+ Knowledge in the lesson of "*Combination*":

The concept of combination;

The formula of combination;

+ Knowledge learned in previous lessons:

Addition and multiplication rules;

Permutation and partial permutation.

- Simplify and shorten content in the textbook to be suitable for each level of students, including (i) low-performing, (ii) average-performing, and (iii) high-performing: The teaching content is not fixed. It should be adjusted based on the students' comprehension. The teacher should consider the general awareness level of students in a class, then determine which teaching contents need to be executed in a mathematics period. If the average perception level of students is low, then the teacher can remove some complicated teaching contents, but if students have a high level of intelligence, the teacher must enhance the quantity and the quality of the contents as well as the study activities in class.

Example of simplifying and shortening content in the textbook to be suitable for each level of students, including (i) low-performing, (ii) average-performing, and (iii) high-performing, in teaching the lesson "Combination": Provided the cognition level of students is the following:

+ High-performing: maintain the whole lesson to teach students;

+ Average-performing: remove the proof of the combination formula and the properties of the combination;

+ Low-performing: maintain only the concept, the formula of combination, and the method to compute using a hand-held calculator;

- Simplify and shorten some activities in the textbook to be suitable for the standard outcomes of the Mathematics Curriculum: While teaching mathematics, the lesson or topic presented in textbooks may include sections of knowledge and their corresponding requirements that do not appear in the curriculum. In the case of contents in textbooks surpassing the standard outcomes, the teacher can simplify the teaching content, but it is not allowed for the teacher to simplify the knowledge attached to the standard outcomes.

Example of simplifying and shortening some activities in the textbook to be suitable for the standard outcomes of the Mathematics Curriculum in teaching the lesson of "*Combination*":

+ The Teacher classifies the knowledge and the activities according to the level of difficulty.

Easy: activity 1, the definition of combination, example 1, the notation of combination; the formula of combination, activity 3, example 3 (part a and part b).

Moderate: activity 2, example 2, example 3 (part c).

Hard: the properties of combination.

+ The teacher chooses which part of moderate or hard activities to discard depending on the difficulty for learners.

- Identify typical situations in teaching a mathematics lesson or topic: The teacher needs to correctly determine which of the four typical situations in teaching mathematics resembles the mathematics knowledge presented in the curriculum or the textbook. Each lesson contains different sections of knowledge, and each section includes a certain situation for teachers to operate the class in.

Example of identifying typical situations in teaching the lesson of "*Combination*":

+ The definition of combination: teaching concept.

+ The combination formula: teaching theorem.

+ The method to compute combination using a hand-held calculator: teaching method.

+ Solving exercises: teaching solving mathematics problems.

- Identify the path of mathematics concepts, theorems, and methods formations in the prior determined typical situations: Based on the typical situations in teaching mathematics identified

earlier, the teacher needs to choose an appropriate path for forming mathematics knowledge. Each section of mathematics knowledge requires the teacher to conduct learning in different ways; therefore, teachers should consult the concept, theorem, and method formation path presented in textbooks or select another path and construct suitable activities to form new knowledge for students.

Example of identifying the path of mathematics concepts, theorems, and methods formations in the prior determined typical situations in teaching the lesson of "*Combination*":

+ Teaching concept: inductive way;

+ Teaching theorem: deductive way;

+ Teaching method: teaching directly.

- Select suitable teaching methods and teaching aids for the knowledge content: Teachers should use teaching methods that boost some or all of the specific competencies of students by having them complete a task or achieve certain goals. Also, the teaching aids greatly contribute to the teaching and learning process; therefore, they should be designed to be student-friendly, attractive, and not require any complicated actions for students to interact. To accomplish this criterion, teachers, and mathematics preservice teachers in particular, need to consult different sources about modern teaching methods and teaching aids, and then practice applying them in class.

Example of selecting suitable teaching methods and teaching aids with the knowledge content:

+ Teaching methods:

Collaborative learning: Let students complete a worksheet about combinations in groups.

Problem-solving: Place students into a problematic situation and require them to figure out the solution. This can be performed in various ways, such as the teacher giving a problem and asking students to find out the answer, or the teacher asking multiple questions successively to keep students thinking and giving answers continuously.

+ Teaching techniques: Think - Pair - Share: The teacher organizes the class using this technique for students to solve exercise 3.

+ Teaching aids: Projector, slides; Worksheets: For students to accomplish in groups; Bulletin board: For students to attach their product onto for them to review later.

Computer algebra systems, in the form of software and websites, such as GeoGebra, Wolfram Alpha, etc.: For the teacher to illustrate the process of choosing counting, and calculating the number of combinations.

- Select suitable forms of revising, summarizing, testing, and evaluating at the end of the lesson: The teacher should detail the activities to revise and summarize knowledge at the end of the lesson or in an independent review session and apply a variety of methods of assessment without causing a lack of interest in students. Besides using traditional methods, such as question and answer or doing tests on paper, teachers may use games and quizzes as an appealing yet effective way to revise and assess students' capability of understanding mathematics knowledge.

Examples of selecting suitable forms of revising, summarizing, testing, and evaluating at the end of the lesson are as follows:

+ Revising and summarizing: Let students practice by completing worksheets or giving answers in interactive games;

+ Assessment: Combine various assessment forms and let students perform self-assessment and peer assessment.

Using the Minute paper: After finishing the lesson, let students answer briefly two questions: "What is the most important knowledge you have learned?" and "Which question remains unanswered?".

Using the Memory matrix: After finishing the lesson, assess students by letting them fill in some crucial information (the keywords in the definition, the notations, and the formula) from the previous lessons on permutation, partial permutation, and combination.

- Conduct the lesson plans suiting certain classes: Teachers are required to perform the teaching session following the lesson plans. A specific lesson must contain four main activities (motivating, introducing new knowledge, practicing, and applying) as regulated in the Official Dispatch 5512/BGDDT-GDTrH dated December 18, 2020 [16]. Sometimes, the lesson plan might be changed by teachers themselves according to the situations happening in class, but the replacement should be prepared as part of the scenario that teachers have forecasted.

- Be flexible in organizing learning activities: Based on the teaching methods, teaching aids, and lesson plans that have been constructed, teachers should be able to (i) be flexible in choosing and planning the way to conduct a learning activity and (ii) be flexible in arranging and implementing the chosen one. The order or the content of the learning activities can be adjusted while the teacher is planning or conducting them with the class, depending on the circumstances of the in-class environment.

- Assess the students' knowledge, skills, and abilities in class: Teachers need to promptly gather information from students using suitable forms of evaluation, which have been mentioned earlier and briefly process that information to determine the capability of students in attaining the instructional objectives.

Example of assessing the student's knowledge, skills, and abilities in class: The teacher let students perform self-assessment and peer assessment by allowing them to grade and comment on the products of their classmates and themselves, assess students' knowledge and skills using various techniques such as the Minute paper, the Memory matrix, etc., and assess the general competencies of students developed through learning activities (for instance, the competence of collaboration and communication) using some assessment tools, namely rubric tables, checklists, student portfolios, etc. Teachers should not heavily rely on summative assessment and traditional assessment tools. Instead, it is better to combine with formative assessment, which provides opportunities for teachers to collect and process the students' feedback in time. Based on the results of the assessment, the teacher can modify the teaching content on the cognition levels of the students.

2.3. Case study on the didactic transposition competence of mathematics preservice teachers

In this work, the research focuses on studying the internal didactic transposition competence of mathematics preservice teachers. The case studies were conducted with a group of 51 mathematics preservice teachers in their third year of professional training at Hanoi National University of Education. The time of the case studies is in April 2023. During the time of the study, the participants had already finished studying the modules on *Theory and methods of teaching mathematics, Planning lessons of Mathematics,* and *Microteaching practices*.

2.3.1. The methodology

The methodology of studying the practice of mathematics preservice teachers for didactic transposition competence includes 3 steps:

Step 1: Select a lesson or topic in the General Mathematics Curriculum 2018 for the research participants to practice creating and conducting a lesson plan based on the criteria given in Table 2.

The participants were required to demonstrate didactic transposition competence when teaching the lesson "Functions and Graphs" in *Mathematics 10, Volume 1*, written by Do Duc Thai (Chief Editor), Pham Xuan Chung, Nguyen Son Ha, Nguyen Thi Phuong Loan, Pham Sy Nam, Pham Minh Phuong, Pham Hoang Quan (2019) [17]. In this research, the participants only practiced the first three elements that are equivalent to eight criteria.

Step 2: Create a questionnaire with the requirements written in Vietnamese. The survey is conducted online, using Google Forms for participants to respond by submitting their answers or products, which are also in Vietnamese.

Step 3: Analyze the participants' results using a rubric.

2.3.2. The results

Overall, the participants cannot give desirable answers to all the requirements in the questionnaire. The majority of the preservice teachers engaging in the study perform quite well in determining the mathematics knowledge related to the objectives of the lesson, selecting suitable teaching methods and teaching aids with the knowledge content, and selecting suitable forms of revising, summarizing, testing, and evaluating at the end of the class. These sections of knowledge are introduced in the teaching method modules; therefore, the participants have the chance to interact with those sections of knowledge and practice them in the micro-scale of classrooms.

Besides those advantages, the results show some problems in the internal didactic transposition competence of mathematics preservice teachers.

The opening question: "Before doing this survey, have you ever heard/read about the term *didactic transposition*?". The results showed 74.51% of the respondents chose "I have never heard/read about this term before".

Conducting the requirement "Construct objectives for different levels of student awareness, including (i) low-performing, (ii) average-performing, and (iii) high-performing", 76.47% of the participants underachieved the task, while 19.61% achieved and only 3.92% overachieved. The majority of the respondents can construct the general objectives of a lesson, but most of them cannot differentiate the objectives for dissimilar levels of students' cognition. In most products, the objectives were vague, ambiguous, and uncorrelated to the standard outcomes of the lesson.

Conducting the requirement "Simplify, shorten or substitute some content in the textbook to be suitable for each level of students, including (i) low-performing, (ii) average-performing, and (iii) high-performing", the number of underachieving responses was 43, which was equivalent to 84.31%. On the contrary, the data for achieving and overachieving responses was relatively small, only 9,80% and 5,88% respectively. Some preservice teachers had a first-hand idea of how to simplify and shorten the content in the textbook based on the students' levels of awareness, while the rest of them either did not comprehend the need to simplify and shorten or simplified and shortened the content without considering the cognition levels of the students.

Conducting the requirement "Simplify and shorten some activities in the textbook to be suitable for the standard outcomes of the Mathematics Curriculum 2018", 88.24% of the participants underachieved the criterion. Their products shared the same problem which is the lack of comparison between the standard outcomes and the activities in the textbooks, therefore the underachieving participants usually discard the activities connected to the standard outcomes. The portion of the achieved and the overachieved were 7.84% and 3.92% respectively.

Conducting the requirement "Identify typical situations in teaching the given lesson", the research obtained that 72.55% of the participants could not identify correctly the typical situations in teaching the lesson "Combination", some of them did not know what typical situations in teaching mathematics were.

Conducting the requirement "Identify the path of mathematics concepts, theorems, and methods formations in the prior typical situations", although 13.73% of the participants performed this criterion as overachieving, there was still a portion of 78.43% underachieved. The underachieving respondents used the path of mathematics knowledge formation that was

uncorrelated to the prior typical situations in teaching mathematics. Some of them cannot name any path to form mathematics knowledge.

Conducting the requirement "Select suitable teaching methods and teaching aids with the knowledge content", a portion of 21.57% of participants answered incorrectly because they could not determine what teaching methods and teaching aids were. Most of the remaining ones were chosen appropriately, but the answers were not varied, mainly presentation, questions and answers, revision, intuition, and visual teaching. Consider the promulgation of the Vietnamese Mathematics Curriculum of 2018. This teacher-centered teaching style cannot satisfy the need for developing students' qualities and competencies.

From the survey, it is obvious that there are some problems in performing didactic transposition competence. Therefore, some measures are required to improve the didactic transposition competence of mathematics preservice teachers.

2.4. Measures to improve didactic transposition competence of mathematics preservice teachers

This research will display the curricula of the mathematics teacher education major of Ho Chi Minh City University of Education [18] and Can Tho University [19], and compare them to the one of Hanoi National University of Education [20] under the aspect of training and developing didactic transposition competence for mathematics preservice teachers.

In the curriculum of Ho Chi Minh City University of Education, the theory of Didactic appears in an elective module, code Math 1439, called *Situation Theory*. The content of this module covers some sections of knowledge: the theory of Mathematics Didactic, the theory of Didactical Situations in Mathematics, and the practice of analyzing some situations in teaching mathematics.

In the curriculum of Can Tho University of Education, mathematics preservice teachers have the chance to experience didactic transposition in the module *Trends of Teaching Mathematics*, code SG244. The module illustrates the theory of Mathematics Didactic, the theory of Didactical Situations in Mathematics, and some models of teaching mathematics.

From the aforementioned examples, it is clear that ATD is now one of the trends in mathematics education. It should be noted that the didactic transposition competence has been taught implicitly in the curriculum of Hanoi National University of Education, but the teaching is not explicit and systematic. This paper suggests some measures to develop the didactic transposition competence more intentionally. Those measures could be performed by adding some knowledge, skills, and competencies into the mathematics teaching method modules including Theory and methods of teaching mathematics, Planning lessons of Mathematics, Implementing lessons of Mathematics, and Microteaching practices [20].

Based on the theoretical foundation of didactic transposition competence and the result of the previously conducted experiment, this work proposes some measures mentioned below.

2.4.1. Include didactic transposition and other knowledge related to didactic transposition competence in teaching method modules

It is necessary to teach preservice teachers the ATD and the theory of Didactical Situations in Mathematics, the definition of didactic transposition and didactic transposition competence, especially the elements and the criteria for didactic transposition competence in teaching mathematics.

The module *Theory and methods of teaching mathematics* offers various opportunities to yield knowledge, skills, and didactic transposition competence of preservice teachers. Based on the elements of didactic transposition competence and their specific manifestations, lecturer of pedagogical institutions needs to determine which content or topic of the module can be combined with the knowledge, skills, and didactic transposition competence to deliver proper knowledge to preservice teachers.

Example 1: In the content of "Objectives in teaching Mathematics," preservice teachers need to be provided with sufficient knowledge related to the element "Determine appropriate competence-developing objectives for students of different levels" in preservice teachers' didactic transposition competence. There, the theories of determining appropriate competence-developing objectives for different levels of students are to be explicated as a preparation for practicing in the following modules.

Example 2: In the topic of "Mathematics teaching content," preservice teachers must be prepared with knowledge and skills related to the element "Determine knowledge that ensures the instructional objectives." The connection between the standard outcomes and the competence-developing objectives in the mathematics curriculum should be clarified. By comparing the standard outcomes of the curriculum and the knowledge presented in the Mathematics textbooks, preservice teachers acquaint themselves with the act of determining which section of knowledge in the textbooks ensures the standard outcomes regulated in the curriculum and which knowledge and skills surpass the standard outcomes, so they can schedule lesson plans that are suitable for different cognition levels of students.

Example 3: In the chapter "Non-traditional methods of teaching mathematics," the lecturer provides knowledge about didactic transposition and didactic transposition competence in the theory of Didactical Situations in Mathematics. This theory will be taught particularly in some topics in the graduate level course of Theory and methods of teaching mathematics. Nonetheless, spending a suitable amount of teaching periods in the syllabus to present the theory of Didactical Situations in Mathematics for mathematics preservice teachers is necessary.

2.4.2. Practice the skills of didactic transposition in the modules of *Planning lessons of Mathematics* and *Implementing lessons of Mathematics*

It should be noted that besides attending theoretical classes about didactic transposition, mathematics preservice teachers need to practice their skills in the modules of *Planning lessons of Mathematics* and *Implementing lessons of Mathematics*. These modules contribute to the process of orientating teaching styles of mathematics preservice teachers after finishing their undergraduate-level courses and offer various opportunities for preservice teachers to practice the skills of didactic transposition. The lecturer conducts the lessons for preservice teachers to practice essential skills to form and develop didactic transposition competence based on the elements of didactic transposition competence and their manifestations.

Example 1: While investigating the content of "Analyze the Mathematics Curriculum of 2018", the lecturer asks preservice teachers to give illustrative examples about the manifestation of elements of mathematics competence and determine the instructional objectives to develop certain competencies based on the standard outcomes. These are critical skills for preservice teachers to practice didactic transposition.

Example 2: While investigating the content of "Planning lessons of Mathematics in a specific lesson or topic," the lecturer asks preservice teachers to (i) determine the objectives of a lesson to be suitable for each cognition level of students, including low-performing, average-performing, and high-performing students, as a manifestation of "Determine appropriate competence-developing objectives for students of different levels," (ii) determine the appropriate path of forming a concept, a theorem or a method, (iii) selecting suitable teaching methods and teaching aids, (iv) constructing a sequence of learning activities suited the factors of controlling the teaching procedure such as revising, summarizing, and evaluating, as a manifestation of "Construct lesson plans that meet the instructional objectives."

2.4.3. Developing didactic transposition competence in teaching modules of *Implementing lessons of Mathematics* and *Microteaching practices*

To form and develop didactic transposition competence for preservice teachers, they need to experience and practice teaching based on their knowledge and skills about didactic transposition. The modules *Implementing Lessons of Mathematics* and *Microteaching Practices* provide valuable opportunities for preservice teachers to develop didactic transposition competence in Mathematics education. The lecturer needs to let preservice teachers participate in some activities: Conduct the lesson plans suiting certain classes; Be flexible in organizing learning activities; Assess the students' knowledge, skills, and abilities in class (the criteria to evaluate the element "Implement the lesson plans flexibly"). After preservice teachers finish their experience and practice, the lecturer organizes for preservice teachers to discuss, comment, and perform assessments and self-assessments. Under the guidance of the lecturer, preservice teachers can gradually improve their lesson plans.

Example 1: When teaching the modules *Implementing lessons of Mathematics* and *Microteaching practices*, the lecturer can execute a procedure of lesson study by conducting the following steps:

Step 1: Each preservice teacher constructs a lesson plan by themselves, based on the knowledge and skills about didactic transposition received from the previous modules.

Step 2: Preservice teachers discuss in groups of 4 or 5 members to complete their lesson plans.

Step 3: Every preservice teacher must practice teaching a section of the lesson plan, while others play their roles as students. This format is used in the course of Microteaching practices. The lecturer divided the class of preservice teachers into some groups corresponding to different levels of students' cognition. Members of those groups play their roles as low-performing, average-performing, and high-performing students and interact with the preservice teachers currently teaching.

Step 4: When a preservice teacher finishes teaching, others comment on the lesson plan and the microteaching performance. The lecturer comments and assesses the lesson plan and the performance in class.

Step 5: Preservice teachers make changes in case needed to their lesson plans, then upload the products on Google Drive (or other platforms) to create a digital archive for the class.

This article has presented some pedagogical measures to develop didactic transposition competence for mathematics preservice teachers. In the reality of teaching at Hanoi National University of Education, by utilizing the modules of teaching methods, preservice teachers – to some extent - form and develop didactic transposition competence. However, this development is still spontaneous and unsystematic. The aforementioned measures assist the formation and the development of becoming more initiative and organized.

3. Conclusions

In the ideology of didactic transposition, didactic transposition competence is one of the most crucial aspects that teachers need to manage and constantly develop. Mathematics teachers, especially the preservice ones, may encounter significant challenges during the procedure of transposing knowledge from the curriculum into the one taught in class, which needs to be interpretable for students of different levels of comprehension. This article has investigated the didactic transposition competence of mathematics preservice teachers, given an example illustrating the application of internal didactic transposition in a specific lesson, and proposed some measures to improve their knowledge, skills, and attitudes about practicing didactic

transposition in reality, which include building a firm foundation of knowledge for mathematics preservice teachers about didactic transposition and didactic transposition competence which is not officially comprised in the training program of Hanoi National University of Education and organizing suitable forms of activities for mathematics preservice teachers to experience the learned theory in practical situations. Those measures can enlarge the quantity of mathematics preservice teachers engaging in the process of learning and practicing didactic transposition competence by turning the knowledge into a part of the university curriculum, increase the interest of mathematics preservice teachers in practicing didactic transposition competence by producing various experiential learning methods for them to approach, and be conducted inside pedagogical institutions without disrupting the training program. Using the presented measures, mathematics preservice teachers may attain didactic transposition competence in Mathematics education, hence surmounting the serious difficulties laid down by the new curriculum.

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