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PRODUCTION OF SAXION AT e^+e^- COLLIDER VIA VECTOR UNPARTICLE EXCHANGE IN THE SUPERSYMMETRY AXION MODEL

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Abstract. The dynamics of vector unparticle behavior within a supersymmetric axion model have been thoroughly examined, with a focus on their impact on saxion production. Our investigation involved a comprehensive analysis, encompassing the computation of vector unparticle exchange contributions to the cross-section (CS) and evaluating the dependence of the differential cross-section (DCS) on the scattering angle (θ). These calculations, in turn, allowed us to determine the production rates of both saxions and vector unparticles in the e^+e^- collider, both in s- and t-channels such as the distribution of missing energy. The results have been plotted in the energy ranges currently used in accelerator designs and for the near-future energy upgrades of the International Linear Collider (ILC) and the Compact Linear Collider (CLIC).

Keywords: saxion, vector unparticle, DCS, CS.

1. Introduction

Supersymmetric axion models offer an intriguing synthesis of two fundamental concepts in physics: supersymmetry (SUSY) and the Peccei-Quinn (PQ) mechanism [1]. These models are primarily designed to resolve two pivotal issues in theoretical physics: the hierarchy problem and the enigmatic strong CP violation. From a cosmological perspective, these models take on particular importance due to the presence of relatively light particles, specifically the saxion (with spin 0) and axino (with spin ½), which serve as the supersymmetric counterparts to the axion [2]. The scalar potential, which incorporates SUSY-breaking effects, can be explicitly formulated as follows [2]:

$$V = m_\phi^2|\phi|^2 + m_{\bar\phi}^2|\bar\phi|^2 + \lambda^2\left(\left|\phi\bar\phi - f^2\right|^2 + |X|^2(|\phi|^2 + |\bar\phi|^2)\right) + 2\lambda m_{3/2} f^2 (X + X^\dagger) \quad (1.1)$$

where ϕ and $\bar\phi$ represent PQ superfields, each carrying PQ charges +1 and -1 respectively, and X stands as a singlet superfield of PQ (with the distinctive property: it has a charge +2 under the R-symmetry, ensuring this type of superpotential). In this context, m_ϕ and $m_{\bar\phi}$ denote soft SUSY breaking masses, and $m_{3/2} = W_0 / M_P^2$ signifies the gravitino

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mass, with M_P is Planck scale. It is worth noting that we consider the constants λ, f and W_0 to be positive and real without loss of generality. In the SUSY, with limiting $m_\phi, m_{\bar{\phi}} \to 0,$ there are two massless modes, called axion and saxion. These modes correspond to the flat direction $\phi\bar{\phi} = f^2$. The saxion acquires a mass of the order of the soft SUSY breaking, but it remains greatly lighter than the PQ scale $\sim f$ [2].

Based on the effective field theory of low energies [3] and the intriguing concept of "unparticles" introduced by Georgi in the context of unusual virtual effects at high energy processes [4], a scale-invariant sector has been established between a Standard Model operator Θ_{SM} and an unparticle operator Θ_{UV}, which combine to form $\Theta_{UV}\Theta_{SM}$. In the unparticle physics, the Lagrangian is formulated as follows [5]:

$$L = \frac{c_n}{M^{d_{UV}+n-4}} \Theta_{UV}\Theta_{SM} , \tag{1.1}$$

where M represents the energy scale of the new physics, Θ_{UV} and Θ_{SM} are operators with dimensions d_{UV} and *n* respectively, and c_n is a dimensionless constant. In the low-energy effective theory, the form of the operator can be expressed as:

$$L = c_n \frac{\Lambda_U^{d_{UV}-d_U}}{M^{d_{UV}+n-4}} \Theta_U\Theta_{SM} , \tag{1.2}$$

In this expression, the unparticle operator Θ_U is characterized as a dimension d_U.

Recently, there have been studies showing that the unparticle contribution is significant in the $e^{+}e^{-}$ collision process in the Beyon standard model [6-9]. In this paper, we calculate the productions of saxion and vector unparticle in the $e^{+}e^{-}$ collider in s- and t-channels such as the missing energy distribution.

2. Content

2.1. The collision processes $e^{+}e^{-} \to sU^{\mu}$ in the supersymmetry axion model

The common effective interaction that satisfies the supersymmetry axion model for the vector unparticle operator is given by the Lagrangian:

$$L_{U^\mu - s - U^\mu} = \frac{\lambda_1}{\Lambda_U^{d_U-1}} C_U s(\partial^\mu O_U^\nu - \partial^\nu O_U^\mu)(\partial_\mu O_{U\nu} - \partial_\nu O_{U\mu}) . \tag{2.1}$$

Where λ_1 represents the coupling constant, C_U is a coefficient function, and s denotes the saxion and the O_U^μ is the vector unparticle operator. The Feyman diagram illustrating the production of saxions and vector unparticles at an electron-position colloider via the vector unparticle exchange is shown in Figure 1.

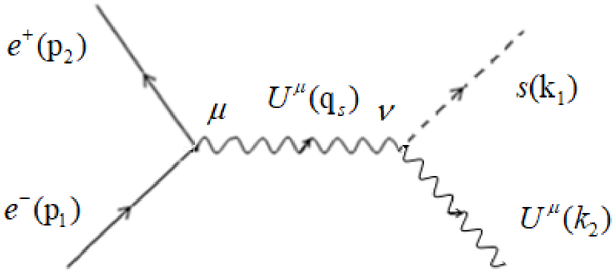


Figure 1. The Feynman diagram for the saxion- vector unparticle productions on the positron-electron via the vector unparticle exchange

Using Feynman's rule, we obtain the scattering amplitude expression:

$$M = -\left(\frac{\lambda_1}{\Lambda_U^{d_U-1}}\right)^2 C_U[2(q_s k_2) g_\alpha^\beta - q_s^\beta k_{2\alpha} - q_{s\alpha} k_2^\beta]\left(\frac{A_{d_U}}{2\sin(d_U \pi)}\right)(-q_s^2)^{d_U-2}\left(-g^{\alpha\mu}+\frac{q_s^\alpha q_s^\mu}{q_s^2}\right)$$
$$\times \varepsilon_\beta(k_2)\bar{v}(p_2)\gamma_\mu(1+\gamma_5)u(p_1) \qquad (2.2)$$

Taking the Hermitian conjugate of expression (2.1), we have

$$M^+ = -e\left(\frac{\lambda_1}{\Lambda_U^{d_U-1}}\right)^2 C_U \frac{1}{q_s^2}[2(q_s k_2) g_{\alpha'}^{\beta'} - q_s^{\beta'} k_{2\alpha'} - q_{s\alpha'} k_2^{\beta'}]\left(\frac{A_{d_U}}{2\sin(d_U \pi)}\right)(-q_s^2)^{d_U-2}$$
$$\times\left(-g^{\alpha'\mu'}+\frac{q_s^{\alpha'} q_s^{\mu'}}{q_s^2}\right)\varepsilon^*_{\beta'}(k_2)\bar{u}(p_1)(1-\gamma_5)\gamma_{\mu'}v(p_2). \qquad (2.3)$$

From expressions (2.1) and (2.2), we obtain the squared expression of scattering amplitude

$$|M|^2 = -8\left(\frac{\lambda_1}{\Lambda_U^{d_U-1}}\right)^4 |C_U|^2\left(\frac{A_{d_U}}{2\sin(d_U \pi)}\right)^2(-q_s^2)^{2(d_U-2)}$$
$$\times(2\{4(q_s k_2)^2(p_1 p_2) - 3(p_1 k_2)(q_s k_2)(q_s p_2)) - 3(q_s p_1)(q_s k_2)(k_2 p_2)$$
$$+q_s^2(k_2 p_2)(p_1 k_2) + (q_s p_1)k_2^2(q_s p_2)$$
$$-\frac{(q_s p_1)}{q_s^2}[(q_s k_2)^2(q_s p_2)) - 2q_s^2(q_s k_2)(k_2 p_2) + q_s^2 k_2^2(q_s p_2)]$$
$$-\frac{(q_s p_2)}{q_s^2}[(q_s k_2)^2(p_1 q_s) - 2(p_1 k_2)(q_s k_2)q_s^2 + (q_s p_1)k_2^2 q_s^2]$$
$$+\frac{(q_s p_1)(q_s p_2)}{q_s^4}[-(q_s k_2)^2 q_s^2 + q_s^4 k_2^2]\}$$
$$-(p_1 p_2)[10(q_s k_2)^2 + 2q_s^2 k_2^2] + (p_1 p_2)\frac{1}{q_s^2}[-(q_s k_2)^2 q_s^2 + q_s^4 k_2^2]). \qquad (2.4)$$

From that, we estimate the DCS as a function of $\cos\theta$ and the CS as a function of $\sqrt{s}$ and the parameter d_U in the next section.

2.2. Numerical results and discussions

In the center of the mass system, we can estimate the numerical values and evaluate the DCS $\left(\frac{d\sigma}{|C_U|^2\, d\cos\theta}\right)$ and the CS $\left(\frac{\sigma}{|C_U|^2}\right)$ from the following formula.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{k}_1|}{|\vec{p}_1|}|M|^2,\ d\Omega = d(\cos\theta)d\varphi. \tag{3.1}$$

To study this work, we choose $m_e = 0.00051GeV$; the range of mass evaluation for saxion ($10^{-2}GeV \le m_s \le 10^5 GeV$) [10]; $\lambda_1 = 1$, $\Lambda_U = 1TeV$, $1.1 \le d_U \le 1.9$ [5], $\sqrt{s} = 3000GeV$ (CLIC) to evaluate the CS of the processe $e^+e^- \to sU^\mu$ depending on d_U (Figure 2) and m_s (Figure 3) and the center of mass energy $\sqrt{s}$ (Figure 4), and the DCS as a function of $\cos\theta$ (Figure 5).

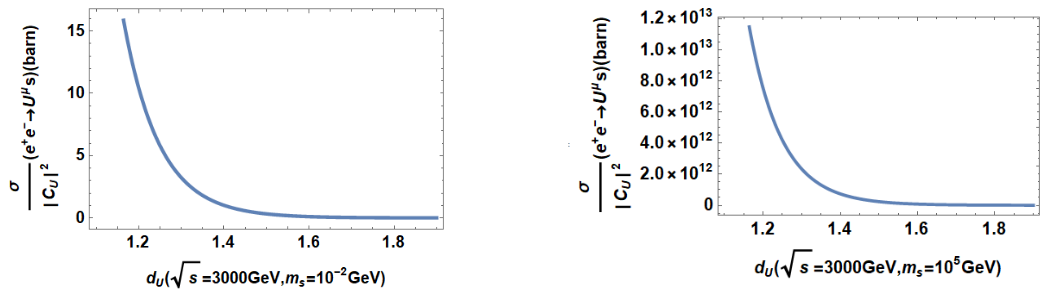


Figure 2. The $\frac{\sigma}{|C_U|^2}$ ***of process*** $e^+e^- \to sU^\mu$ ***as a function of*** d_U

In Figure 2, we have evaluated the $\frac{\sigma}{|C_U|^2}$ according to the parameter d_U for the cases $m_s = 10^{-2}GeV$ and $m_s = 10^5 GeV$. The results show that when d_U increases from 1.1 to 1.9, the $\frac{\sigma}{|C_U|^2}$ decreases, and decreases rapidly when $d_U < 1.3$.

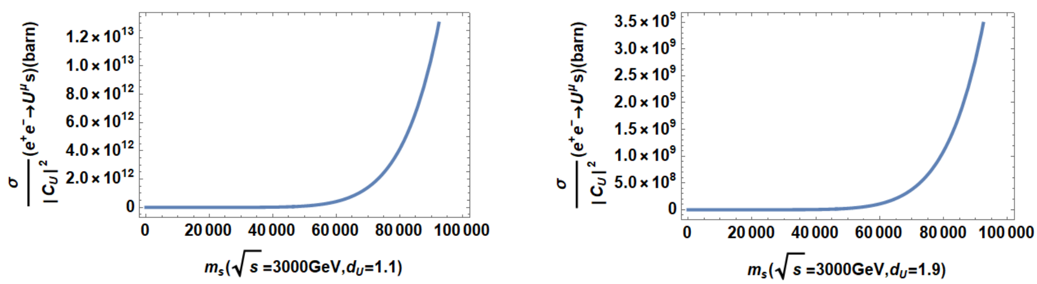
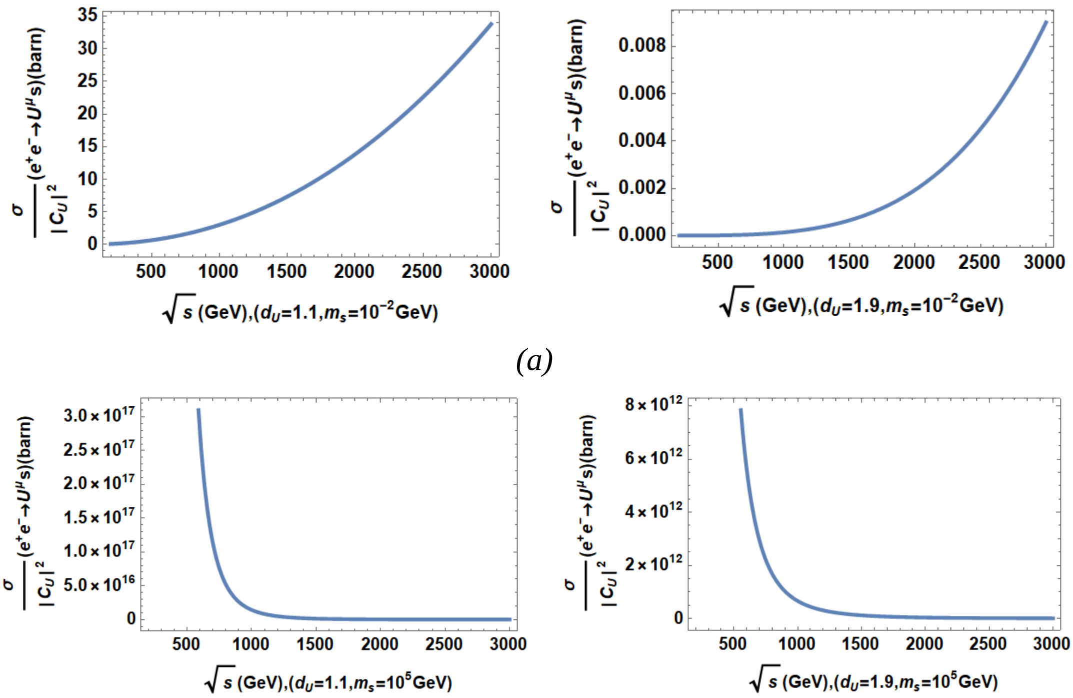


Figure 3. The $\frac{\sigma}{|C_U|^2}$ ***of process*** $e^+e^- \to sU^\mu$ ***as a function of*** m_s

In Figure 3, we investigate the dependence $\frac{\sigma}{|C_U|^2}$ on the m_s at two limiting values of d_U, which are $d_U = 1.1$ and $d_U = 1.9$. From that, we observe that $\frac{\sigma}{|C_U|^2}$ increases and increases rapidly in the region of large m_s. Furthermore, with the $d_U = 1.1$, the $\frac{\sigma}{|C_U|^2}$ has a significantly larger value compared to that at $d_U = 1.9$, approximately 10^5 times larger.



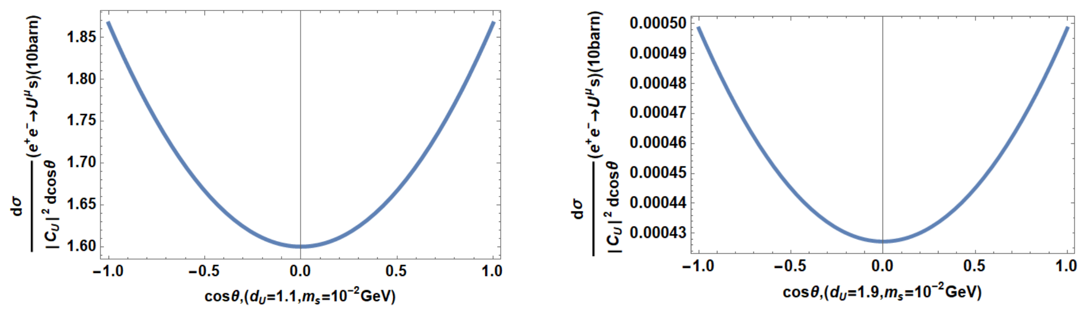
(b)

Figure 4. The $\frac{\sigma}{|C_U|^2}$ ***of process*** $e^+e^- \to sU^\mu$ ***as a function of*** $\sqrt{s}$

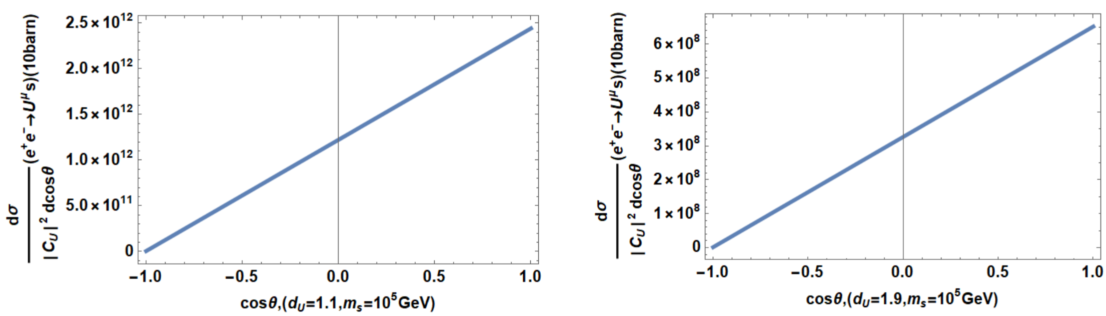
Investigating the dependence on the center of mass energy $\sqrt{s}$ across different values of d_U and within the range of mass evaluation for saxion (m_s), we see that, for the lower bounds of m_s at $d_U = 1.1$ and $d_U = 1.9$, the $\frac{\sigma}{|C_U|^2}$ increases as $\sqrt{s}$ increases, meaning that the probability of finding particles in the high-energy region is greater than in the low-energy region (Figure 4a). Conversely, for the upper bounds of m_s, the

$\dfrac{\sigma}{|C_U|^2}$ decreases as $\sqrt{s}$ increases, and almost decreases to the saturation value from a value of 1500 GeV. In other words, when the mass of saxion is high, the probability of finding particles in the low-energy region is higher than in the high-energy region (Figure 4b).

In the following Figure 5, we consider the dependence of $\dfrac{d\sigma}{|C_U|^2\, d\cos\theta}$ on $\cos\theta$ across different values of d_U and within the range of the m_s: For the lower bounds of m_s, at $d_U = 1.1$ and $d_U = 1.9$, $\dfrac{d\sigma}{|C_U|^2\, d\cos\theta}$ reaches its maximum value when $\cos\theta$ is either +1 or equal to -1 (Figure 5a). Additionally, the upper bounds the m_s, $\dfrac{d\sigma}{|C_U|^2\, d\cos\theta}$ reach their maximum value when $\cos\theta = 1$ and minimum when $\cos\theta = -1$ (Figure 5b).



(a)



(b)

Figure 5. The $\dfrac{d\sigma}{|C_U|^2\, d\cos\theta}$ ***of process*** $e^+e^- \to sU^\mu$ ***as a function of*** $\cos\theta$

3. Conclusions

From a detailed study of the saxion production during the collision process of e^+e^- involving the participation of the vector unparticle within the energy range available in the current designs of accelerators and anticipated energy upgrades of ILC and CLIC, the obtained results are as follows: the cross-section decreases rapidly when the parameter $d_U < 1.3$ and increases in the region of high saxion mass values. Meanwhile, in the lower vicinity of the saxion mass evaluation region, the cross-section shows an increase as $\sqrt{s}$ increases, and the optimal observation direction for particle generation is either in the same direction or in the opposite direction to the incoming particle beams. Additionally, in the upper vicinity of the saxion mass evaluation region, the cross-section decreases as $\sqrt{s}$ increases, and the best observation direction for particle generation is only when it aligns with the incoming particle beams. Furthermore, the cross-section decreases significantly at low energy values, reaching saturation when $\sqrt{s} > 1500 GeV$.

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