HNUE JOURNAL OF SCIENCE DOI: 10.18173/2354-1059.2023-0054
Natural Science, 2023, Volume 68, Issue 3, pp. 3-12
This paper is available online at http://stdb.hnue.edu.vn

ROBUST STABILIZATION OF UNCERTAIN DISCRETE-TIME STOCHASTIC BILINEAR SYSTEMS WITH MARKOVIAN SWITCHINGS

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Abstract. In this paper, the problem of robust stabilization is addressed for a class of uncertain discrete-time stochastic bilinear systems with Markovian switchings. The parameter uncertainties are assumed to be time-varying and norm-bounded. The jumping process is driven by a discrete-time Markov chain with finite states and its transition probability matrix is partially unknown. Sufficient conditions are established in terms of tractable linear matrix inequalities to ensure the closed-loop system is robustly stochastically stable.
Keywords: stabilization, stochastic bilinear systems, stochastic stability, linear matrix inequalities.

1. Introduction

Stochastic models have been extensively studied due to their flexibility in modeling real-world phenomena in biology, economics, engineering applications, and many other areas, see [1-3] and the references therein. One of the most important classes of systems that has been focused on in stochastic modeling is stochastic bilinear systems. This kind of system is referred to as a linear system with noises. Many results dealing with stability analysis and control of stochastic bilinear systems have been reported. We refer the reader to [4-9] and the references therein.

On the other hand, Markov jump systems (MJSs) form an important class of stochastic systems. They are widely used to model practical and physical processes subject to random abrupt changes in their state variables, external inputs, and structure parameters caused by sudden component failures, environmental noises, or random loss of digital packages in interconnections [10-13]. A number of results on stability analysis, H_∞ control, dynamic output feedback control, and state bounding for MJSs can be found in [14-18]. Besides that, stochastic bilinear systems with Markovian switchings have been investigated by many researchers, see [19-22] and the references therein.

Received October 8, 2023. Revised: October 20, 2023. Accepted October 27, 2023.
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In [19], sufficient conditions in terms of tractable linear matrix inequalities to design a mode-dependent stabilizing state-feedback controller were formulated and the problem of robust H_∞ control was studied in [23]. However, most of the obtained results are concerned with stochastic bilinear systems with Markovian switchings and the transition probability matrix is assumed to be completely known. Such an assumption is not reasonable in practical applications. To the best of the author's knowledge, the problem of robust stabilization of uncertain discrete-time stochastic bilinear systems with Markovian switchings and partially unknown transition probabilities has not been fully investigated in the literature.

In this paper, we deal with the problem of robust stabilization of uncertain discrete-time stochastic systems with Markovian switchings. The parameter uncertainties are assumed to be time-varying and norm-bounded. The transition probability matrix of the jump switching is assumed to be partially unknown. Sufficient conditions are established in terms of tractable linear matrix inequalities to ensure the closed-loop system is robustly stochastically stable.

2. Preliminaries

2.1. Notation

Throughout this paper, $\mathbb{Z}$ and $\mathbb{Z}^+$ denote the set of integers and positive integers, respectively. For an $a \in \mathbb{Z}$, $\mathbb{Z}^a = \{k \in \mathbb{Z} : k \geq a\}$. $\mathbb{E}[.]$ is the expectation operator with respect to some probability measure Pr, $\mathbb{R}^n$ is the n-dimensional Euclidean space with vector norm $\|.\|$, $\mathbb{R}^{n\times p}$ is the set of $n \times p$ real matrices and $\mathbb{S}_n^+$ is the set of symmetric positive definite matrices. For matrices $A, B \in \mathbb{R}^{n\times p}$, $\mathrm{col}\{A, B\}$ and $\mathrm{diag}\{A, B\}$ denote the block matrix $\begin{bmatrix} A \\ B \end{bmatrix}$ and $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$, respectively.

2.2. Problem formulation

Let $(\Omega, \mathcal{F}, \mathrm{Pr})$ be a complete probability space. Consider a class of discrete-time stochastic systems with Markovian switchings of the form

$$x(k+1) = A_1(r_k)x(k) + B_1(r_k)u(k) + [A_2(r_k)x(k) + B_2(r_k)u(k)]w(k),\ k \in \mathbb{Z}^0, \tag{2.1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^p$ is the control input. The system matrices $A_1(r_k)$, $B_1(r_k)$, $A_2(r_k)$, $B_2(r_k)$ belong to $\{A_{1i}, B_{1i}, A_{2i}, B_{2i}, i \in \mathcal{M}\}$, where $A_{1i}, B_{1i}, A_{2i}, B_{2i}, i \in \mathcal{M}$, are known constant matrices.

For notational simplicity, in the sequel, whenever $r_k = i \in \mathcal{M}$, matrices $A_1(r_k)$, $B_1(r_k)$, $A_2(r_k)$, $B_2(r_k)$ will be denoted by A_{1i}, B_{1i}, A_{2i}, and B_{2i}, respectively.

The sequence $\{w(k), k \in \mathbb{Z}^0\}$ consists of scalar standard random variables satisfying

$$\mathbb{E}[w(k)] = 0, \mathbb{E}[w(k)]^2 = 1, \tag{2.2}$$

and $w(0), w(1), \ldots,$ are independent. The jump switching parameters $\{r_k, k \in \mathbb{Z}^0\}$ is driven by a discrete-time Markovian jump process specifying the system mode which takes value in a finite set $\mathcal{M} = \{1, 2, \ldots, m\}$ with transition probabilities (TPs) given by

$$\Pr\{r_{k+1} = j | r_k = i\} = \pi_{ij}, \quad i, j \in \mathcal{M},$$

where $p_{ij} \geq 0$, $i, j \in \mathcal{M}$, and $\sum_{j=1}^{m} p_{ij} = 1$ for all $i \in \mathcal{M}$. We denote by $\Pi = (\pi_{ij})$ the transition probability matrix and $p = (p_1, p_2, \ldots, p_m)$ the initial probability distribution, where $p_i = \Pr\{r_0 = i\}$, $i \in \mathcal{M}$. We also assume that the Markov chain $\{r_k\}$ and the stochastic noise $\{w(k)\}$ are independent.

In this paper, we assume in general that the transition probability matrix Π is only partially accessible (i.e., some entries of Π are completely unknown). In the sequel, we denote by $\hat{\pi}_{ij}$ the unknown entry $\pi_{ij} \in \Pi$, $\mathcal{M}_a^{(i)}$ and $\mathcal{M}_{na}^{(i)}$ the sets of indices of known and unknown TPs in row $\Pi_i = \begin{bmatrix} \pi_{i1} & \pi_{i2} & \ldots & \pi_{im} \end{bmatrix}$ of Π, respectively,

$$\mathcal{M}_a^{(i)} = \{j \in \mathcal{M} : \ \pi_{ij} \text{ is known}\}, \ \mathcal{M}_{na}^{(i)} = \{j \in \mathcal{M} : \ \pi_{ij} \text{ is unknown}\}. \tag{2.3}$$

Moreover, if $\mathcal{M}_a^{(i)} \neq \emptyset$, we denote $\mathcal{M}_a^{(i)} = (\mu_1^i, \mu_2^i, \ldots, \mu_l^i), 1 \leq l \leq m$. That is, in the ith row of Π, entries $\pi_{i\mu_1^i}, \pi_{i\mu_2^i}, \ldots, \pi_{i\mu_l^i}$ are known.

Corresponding to the system (2.1), we consider the system with uncertainties as follows

$$\begin{aligned} x(k+1) = & A_1(k, r_k)x(k) + B_1(k, r_k)u(k) + [A_2(k, r_k)x(k) \\ & + B_2(k, r_k)u(k)]w(k), \ k \in \mathbb{Z}^0, \end{aligned} \tag{2.4}$$

with

$$\begin{aligned} A_1(k, r_k) = & A_1(r_k) + \Delta A_1(k, r_k), \\ A_2(k, r_k) = & A_2(r_k) + \Delta A_2(k, r_k), \\ B_1(k, r_k) = & B_1(r_k) + \Delta B_1(k, r_k), \\ B_2(k, r_k) = & B_2(r_k) + \Delta B_2(k, r_k), \end{aligned} \tag{2.5}$$

where $\Delta A_1(k, r_k)$, $\Delta A_2(k, r_k)$, $\Delta B_1(k, r_k)$, and $\Delta B_2(k, r_k)$ are uncertain matrices which have the following forms

$$\begin{aligned} \Delta A_1(k, r_k) = & D(r_k)\Delta(r_k)E_{A_1}(r_k), \\ \Delta A_2(k, r_k) = & D(r_k)\Delta(r_k)E_{A_2}(r_k), \\ \Delta B_1(k, r_k) = & D(r_k)\Delta(r_k)E_{B_1}(r_k), \\ \Delta B_2(k, r_k) = & D(r_k)\Delta(r_k)E_{B_2}(r_k), \end{aligned} \tag{2.6}$$

$D(r_k)$, $E_{A_1}(r_k)$, $E_{A_2}(r_k)$, $E_{B_1}(r_k)$, and $E_{B_2}(r_k)$ are known matrices of appropriate dimensions. The term $\Delta(r_k)$ satisfies the following condition

$$\Delta^T(r_k)\Delta(r_k) \leq \mathbb{I}, \tag{2.7}$$

where $\mathbb{I}$ is the identity matrix of the appropriate dimension. The uncertain matrices $\Delta A_1(k, r_k)$, $\Delta A_2(k, r_k)$, $\Delta B_1(k, r_k)$, and $\Delta B_2(k, r_k)$ are said to be admissible if both (2.6) and (2.7) hold.

In this paper, a memoryless state feedback controller is designed in the form

$$u(k) = K(r_k)x(k), \tag{2.8}$$

where $K(r_k) \in \{K_i, i \in \mathcal{M}\}$, and $K_i, i \in \mathcal{M}$ are the controller gains which will be designed. With the state feedback controller (2.8), the closed-loop systems of (2.1) are given by

$$x(k+1) = A_{1c}(r_k)x(k) + A_{2c}(r_k)x(k)w(k), \ k \in \mathbb{Z}^0, \tag{2.9}$$

where $A_{1c}(r_k) = A_1(r_k) + B_1(r_k)K(r_k)$ and $A_{2c}(r_k) = A_2(r_k) + B_2(r_k)K(r_k)$.

In addition, the closed-loop systems of (2.4) is given by

$$x(k+1) = A_{1c}(k, r_k)x(k) + A_{2c}(k, r_k)x(k)w(k), \ k \in \mathbb{Z}^0, \tag{2.10}$$

where $A_{1c}(k, r_k) = A_1(k, r_k) + B_1(k, r_k)K(r_k)$ and $A_{2c}(k, r_k) = A_2(k, r_k) + B_2(k, r_k)K(r_k)$.

First, we introduce the following definitions, see [24].

Definition 2.1. *The open-loop system of (2.1) (i.e.,$u(k) = 0$) is said to be stochastically stable if there exists a constant $T(r_0, x_0)$ such that*

$$\mathbb{E}\Big[\sum_{k=0}^{\infty} x^T(k)x(k)|r_0, x_0\Big] \leq T(r_0, x_0).$$

Definition 2.2. *The open-loop system of (2.4) is said to be robustly stochastically stable if it is stochastically stable for all admissible uncertainties. Moreover, system (2.4) is said to be robustly stochastically stabilizable if there exists a state feedback controller in the form of (2.8) such that closed- loop system (2.10) is stochastically stable for all admissible uncertainties.*

The main objective of this paper is to establish conditions to design a state feedback controller in the form of (2.8) which guarantees system (2.4) with partially unknown transition probabilities robustly stochastically stabilizable.

2.3. Auxiliary lemmas

We introduce some useful technical lemmas to proof our main results as follows.

Lemma 2.1 (Schur complements)**.** *Given constant matrices M, L, Q of appropriate dimensions, where M and Q are symmetric and $Q > 0$, then $M + L^TQL < 0$ if and only if*

$$\begin{bmatrix} M & L^T \\ L & -Q^{-1} \end{bmatrix} < 0, \tag{2.11}$$

or equivalently

$$\begin{bmatrix} -Q^{-1} & L \\ L^T & M \end{bmatrix} < 0. \tag{2.12}$$

Next, in order to establish conditions for robust stability, we employ the inequalities in the following lemma (see, e.g., [25]).

Lemma 2.2. *Let A, D, Δ, E be real matrices of appropriate dimensions with $\|\Delta\| \leq 1$. Then,*

(i) *for any matrix $P > 0$ and scalar $\epsilon > 0$ satisfying $\epsilon\mathbb{I} - EPE^T > 0$,*

$$(A + D\Delta E)P(A + D\Delta E)^T \leq APA^T + APE^T(\epsilon\mathbb{I} - EPE^T)^{-1}EPA^T + \epsilon DD^T \tag{2.13}$$

(ii) *for any matrix $P > 0$ and scalar $\epsilon > 0$ satisfying $P - \epsilon DD^T > 0$,*

$$(A + D\Delta E)P^{-1}(A + D\Delta E)^T \leq AT(P - \epsilon DD^T)^{-1}A + \frac{1}{\epsilon}E^T E. \tag{2.14}$$

The following lemma gives necessary and sufficient conditions for the stochastic stability of system (2.1) with $u(k) = 0$, see [24].

Lemma 2.3. *System (2.1) with $u(k) = 0$ is stochastically stable if and only if there exist matrices $Q_i \in \mathbb{S}_n^+, i \in \mathcal{M}$, such that one of the two following conditions holds:*

(i) *For all $i \in \mathcal{M}$, the algebraic Riccati inequality (ARI) holds*

$$A_{1i}^T G_i A_{1i} + A_{2i}^T G_i A_{2i} - Q_i < 0, \tag{2.15}$$

where $G_i = \sum_{j=1}^m \pi_{ij} Q_j$;

(ii) *The following LMI holds*

$$\begin{bmatrix} -Q_i & J_{1i}^T & J_{2i}^T \\ J_{1i} & -Q & 0 \\ J_{2i} & 0 & -Q \end{bmatrix} < 0, \forall i \in \mathcal{M}, \tag{2.16}$$

where

$$J_{1i}^T = \left[\sqrt{\pi_{i1}} A_{1i}^T Q_1, \ldots, \sqrt{\pi_{im}} A_{1i}^T Q_m\right],$$
$$J_{2i}^T = \left[\sqrt{\pi_{i1}} A_{2i}^T Q_1, \ldots, \sqrt{\pi_{im}} A_{2i}^T Q_m\right].$$

3. Main results

In this section, sufficient conditions for robust stability and stabilization of system (2.4) will be derived. First, the conditions for robust stability of system (2.4) with $u(k) = 0$ are given in the following theorem.

Theorem 3.1. *The open-loop system of (2.4) with deficient TPs (2.3) is robustly stochastically stable if there exist matrices $Q_i \in \mathbb{S}_n^+$, scalars $\epsilon_{1i} > 0$, $\epsilon_{2i} > 0$, $\epsilon_{3i} > 0$, and $\epsilon_{4i} > 0$, $i \in \mathcal{M}$ such that*

$$\begin{bmatrix} -\pi_a^i Q_i & A_{1i}^T \Omega_i & A_{2i}^T \Omega_i & E_{A_{1i}}^T & E_{A_{2i}}^T \\ * & \epsilon_{1i}\Omega_i^T D_i D_i^T - \Gamma & 0 & 0 & 0 \\ * & * & \epsilon_{2i}\Omega_i^T D_i D_i^T - \Gamma & 0 & 0 \\ * & * & * & -\epsilon_{1i}\mathbb{I} & 0 \\ * & * & * & * & -\epsilon_{2i}\mathbb{I} \end{bmatrix} < 0, \tag{3.1}$$

and

$$\begin{bmatrix} -Q_i & A_{1i}^T & A_{2i}^T & E_{A_{1i}}^T & E_{A_{2i}}^T \\ * & \epsilon_{3i} D_i D_i^T - Q_j^{-1} & 0 & 0 & 0 \\ * & * & \epsilon_{4i} D_i D_i^T - Q_j^{-1} & 0 & 0 \\ * & * & * & -\epsilon_{3i}\mathbb{I} & 0 \\ * & * & * & * & -\epsilon_{4i}\mathbb{I} \end{bmatrix} < 0, \forall j \in \mathcal{M}_{na}^{(i)}, \tag{3.2}$$

where $\Gamma = diag(Q_{\mu_1^i}^{-1}, Q_{\mu_2^i}^{-1}, \ldots, Q_{\mu_l^i}^{-1})$, $\Omega_i = [\sqrt{\pi_{i\mu_1^i}}\mathbb{I},\ \sqrt{\pi_{i\mu_2^i}}\mathbb{I},\ \ldots,\ \sqrt{\pi_{i\mu_l^i}}\mathbb{I}]$.

Proof. According to Lemma 2.3, the open-loop system (2.4) is robustly stochastically stable if there exist matrices $Q_i \in \mathbb{S}_n^+$, $i \in \mathcal{M}$, such that the following inequality holds for all admissible uncertainties

$$A_1(k,i)^T G_i A_1(k,i) + A_2(k,i)^T G_i A_2(k,i) - Q_i < 0, \tag{3.3}$$

where $G_i = \sum_{j=1}^m \pi_{ij} Q_j$, $A_1(k,i) = A_{1i} + D_i \Delta_i E_{A_{1i}}$, and $A_2(k,i) = A_{2i} + D_i \Delta_i E_{A_{2i}}$.

From the fact $\sum_{j=1}^m \pi_{ij} = 1$ for all $i \in \mathcal{M}$, condition (3.3) is equivalent to the two following conditions

$$\sum_{j=1}^m \pi_{ij}\big[A_1(k,i)^T Q_j A_1(k,i) + A_2(k,i)^T Q_j A_2(k,i)\big] - \sum_{j=1}^m \pi_{ij} Q_i < 0, \tag{3.4}$$

or

$$\sum_{j\in\mathcal{M}_a^{(i)}} \pi_{ij}\big[A_1(k,i)^T Q_j A_1(k,i) + A_2(k,i)^T Q_j A_2(k,i) - Q_i\big] + \sum_{j\in\mathcal{M}_{na}^{(i)}} \pi_{ij}\big[A_1(k,i)^T Q_j A_1(k,i) + A_2(k,i)^T Q_j A_2(k,i) - Q_i\big] < 0. \tag{3.5}$$

Denote $\tilde{G}_i = \sum_{j \in \mathcal{M}_a^{(i)}} \pi_{ij} Q_j$ and $\pi_a^i = \sum_{j \in \mathcal{M}_a^{(i)}} \pi_{ij}$. Then, with the notice that $\pi_{ij} \geq 0$, $\forall i, j \in \mathcal{M}$, condition (3.5) holds if the two following conditions hold

$$A_1(k,i)^T \tilde{G}_i A_1(k,i) + A_2(k,i)^T \tilde{G}_i A_2(k,i) - \pi_a^i Q_i < 0 \tag{3.6}$$

and

$$A_1(k,i)^T Q_j A_1(k,i) + A_2(k,i)^T Q_j A_2(k,i) - Q_i < 0, \text{ for all } j \in \mathcal{M}_{na}^{(i)}. \tag{3.7}$$

On the other hand,

$$A_1(k,i)^T \tilde{G}_i A_1(k,i) = \left[\Omega_i^T A_{1i} + \Omega_i^T D_i \Delta_i E_{A_{1i}}\right]^T \Gamma^{-1} \left[\Omega_i^T A_{1i} + \Omega_i^T D_i \Delta_i E_{A_{1i}}\right], \tag{3.8}$$
$$A_2(k,i)^T \tilde{G}_i A_2(k,i) = \left[\Omega_i^T A_{2i} + \Omega_i^T D_i \Delta_i E_{A_{2i}}\right]^T \Gamma^{-1} \left[\Omega_i^T A_{2i} + \Omega_i^T D_i \Delta_i E_{A_{2i}}\right]. \tag{3.9}$$

For given $\epsilon_{1i} > 0$, $\epsilon_{2i} > 0$, $\Gamma - \epsilon_{1i} \Omega_i^T D_i D_i^T \Omega_i > 0$, and $\Gamma - \epsilon_{2i} \Omega_i^T D_i D_i^T \Omega_i > 0$, by utilizing Lemma 2.2, we get

$$A_1(k,i)^T \tilde{G}_i A_1(k,i) \leq A_{1i}^T \Omega_i (\Gamma - \epsilon_{1i} \Omega_i^T D_i D_i^T \Omega_i)^{-1} \Omega_i^T A_{1i} + \epsilon_{1i}^{-1} E_{A_{1i}}^T E_{A_{1i}}, \tag{3.10}$$
$$A_2(k,i)^T \tilde{G}_i A_2(k,i) \leq A_{2i}^T \Omega_i (\Gamma - \epsilon_{2i} \Omega_i^T D_i D_i^T \Omega_i)^{-1} \Omega_i^T A_{2i} + \epsilon_{2i}^{-1} E_{A_{2i}}^T E_{A_{2i}}. \tag{3.11}$$

Thus, condition (3.6) holds if

$$\begin{aligned} &A_{1i}^T \Omega_i (\Gamma - \epsilon_{1i} \Omega_i^T D_i D_i^T \Omega_i)^{-1} \Omega_i^T A_{1i} + \epsilon_{1i}^{-1} E_{A_{1i}}^T E_{A_{1i}} \\ &+ A_{2i}^T \Omega_i (\Gamma - \epsilon_{2i} \Omega_i^T D_i D_i^T \Omega_i)^{-1} \Omega_i^T A_{2i} + \epsilon_{2i}^{-1} E_{A_{2i}}^T E_{A_{2i}} - \pi_a^i Q_i < 0. \end{aligned} \tag{3.12}$$

Similarly, for given $\epsilon_{3i} > 0$, $\epsilon_{4i} > 0$, $Q_j^{-1} - \epsilon_{3i} D_i D_i^T > 0$, and $Q_j^{-1} - \epsilon_{4i} D_i D_i^T > 0$, we get

$$A_1(k,i)^T Q_j A_1(k,i) \leq A_{1i}^T (Q_j^{-1} - \epsilon_{3i} D_i D_i^T)^{-1} A_{1i} + \epsilon_{3i}^{-1} E_{A_{1i}}^T E_{A_{1i}}, \tag{3.13}$$
$$A_2(k,i)^T Q_j A_2(k,i) \leq A_{2i}^T (Q_j^{-1} - \epsilon_{4i} D_i D_i^T)^{-1} A_{2i} + \epsilon_{4i}^{-1} E_{A_{2i}}^T E_{A_{2i}}. \tag{3.14}$$

Thus, condition (3.7) holds if

$$\begin{aligned} &A_{1i}^T (Q_j^{-1} - \epsilon_{3i} D_i D_i^T)^{-1} A_{1i} + \epsilon_{3i}^{-1} E_{A_{1i}}^T E_{A_{1i}} \\ &+ A_{2i}^T (Q_j^{-1} - \epsilon_{4i} D_i D_i^T)^{-1} A_{2i} + \epsilon_{4i}^{-1} E_{A_{2i}}^T E_{A_{2i}} - Q_i < 0, \forall j \in \mathcal{M}_{na}^{(i)}. \end{aligned} \tag{3.15}$$

Now, applying the Schur complement, inequalities (3.12) and (3.15) are equivalent to (3.1) and (3.2), respectively. This completes the proof. □

Next, we will derive conditions by which system (2.4) with partially unknown transition probabilities (2.3) is robustly stochastically stabilizable with state feedback (2.8).

Theorem 3.2. *System (2.4) with deficient TPs (2.3) is robustly stochastically stabilizable if there exist matrices $X_i \in \mathbb{S}_n^+$, $Y_i, i \in \mathcal{M}$ such that*

$$\begin{bmatrix} -\pi_a^i X_i & \tilde{A}_{1i}\Omega_i & \tilde{A}_{2i}\Omega_i & \tilde{E}_{1i} & \tilde{E}_{2i} \\ * & \epsilon_{1i}\Omega_i^T D_i D_i^T - \tilde{X} & 0 & 0 & 0 \\ * & * & \epsilon_{2i}\Omega_i^T D_i D_i^T - \tilde{X} & 0 & 0 \\ * & * & * & -\epsilon_{1i}\mathbb{I} & 0 \\ * & * & * & * & -\epsilon_{2i}\mathbb{I} \end{bmatrix} < 0, \tag{3.16}$$

and

$$\begin{bmatrix} -X_i & \tilde{A}_{1i} & \tilde{A}_{2i} & \tilde{E}_{1i} & \tilde{E}_{2i} \\ * & \epsilon_{3i} D_i D_i^T - X_j & 0 & 0 & 0 \\ * & * & \epsilon_{4i} D_i D_i^T - X_j & 0 & 0 \\ * & * & * & -\epsilon_{3i}\mathbb{I} & 0 \\ * & * & * & * & -\epsilon_{4i}\mathbb{I} \end{bmatrix} < 0, \forall j \in \mathcal{M}_{na}^{(i)}, \tag{3.17}$$

where $\tilde{X} = diag(X_{\mu_1^i}, X_{\mu_2^i}, \dots, X_{\mu_l^i})$, $\tilde{A}_{1i} = X_i A_{1i}^T + Y_i^T B_{1i}^T$, $\tilde{A}_{2i} = X_i A_{2i}^T + Y_i^T B_{2i}^T$, $\tilde{E}_{1i} = X_i E_{A_{1i}}^T + Y_i^T E_{B_{1i}}^T$, $\tilde{E}_{2i} = X_i E_{A_{2i}}^T + Y_i^T E_{B_{2i}}^T$. The controller gains K_i, $i \in \mathcal{M}$, are given by $K_i = Y_i X_i^{-1}$.

Proof. From Definition 2.2, it is only necessary to show that the closed-loop system (2.10) is stochastically stable for all admissible uncertainties.

By Theorem 3.1, system (2.10) is stochastically stable if there exist matrices $Q_i \in \mathbb{S}_n^+, i \in \mathcal{M}$ such that

$$\begin{bmatrix} -\pi_a^i Q_i & A_{1ci}^T\Omega_i & A_{2ci}^T\Omega_i & E_{1ci}^T & E_{2ci}^T \\ * & \epsilon_{1i}\Omega_i^T D_i D_i^T - \Gamma & 0 & 0 & 0 \\ * & * & \epsilon_{2i}\Omega_i^T D_i D_i^T - \Gamma & 0 & 0 \\ * & * & * & -\epsilon_{1i}\mathbb{I} & 0 \\ * & * & * & * & -\epsilon_{2i}\mathbb{I} \end{bmatrix} < 0, \tag{3.18}$$

where $\Gamma = diag(Q_{\mu_1^i}^{-1}, Q_{\mu_2^i}^{-1}, \dots, Q_{\mu_l^i}^{-1})$, and

$$\begin{bmatrix} -Q_i & A_{1ci}^T & A_{2ci}^T & E_{1ci}^T & E_{2ci}^T \\ * & \epsilon_{3i} D_i D_i^T - Q_j^{-1} & 0 & 0 & 0 \\ * & * & \epsilon_{4i} D_i D_i^T - Q_j^{-1} & 0 & 0 \\ * & * & * & -\epsilon_{3i}\mathbb{I} & 0 \\ * & * & * & * & -\epsilon_{4i}\mathbb{I} \end{bmatrix} < 0, \text{ for all } j \in \mathcal{M}_{na}^{(i)}, \tag{3.19}$$

where $A_{1ci} = A_{1i} + B_{1i}K_i$, $A_{2ci} = A_{2i} + B_{2i}K_i$, $E_{1ci} = E_{A_{1i}} + E_{B_{1i}}K_i$, and $E_{2ci} = E_{A_{2i}} + E_{B_{2i}}K_i$.

Let $X_i = Q_i^{-1}$, $Y_i = K_i X_i$. By pre-and post-multiplying equations (3.18) and (3.19) with $diag(X_i, \mathbb{I}, \mathbb{I}, \mathbb{I}, \mathbb{I})$ we then obtain (3.16) and (3.17), respectively. This completes the proof. □

Remark 3.1. *When the transition probabilities of the jumping process are completely known, the derived conditions in Theorem 3.2 are reduced to those of Theorem 1 in [23]. Thus, the result of Theorem 3.2 in this paper can be regarded as an extension of the result of [23].*

4. Conclusion

This paper has dealt with the problem of robust stabilization of uncertain discrete-time stochastic bilinear systems with Markovian switching. Based on the analysis of the transition probability matrix, sufficient conditions have been established in terms of tractable linear matrix inequalities to design mode-dependent state feedback controllers which guarantee the robust stability of the system.

Acknowledgment. This research is funded by Vietnam Ministry of Education and Training under grant number B.2022-SP2-02.

REFERENCES

[1] L. Shaikhet, 2013. *Lyapunov Functionals and Stability of Stochastic Functional Differential Equations*. Springer, Switzerland.

[2] E.K. Boukas, 2006. *Stochastic Switching Systems, Analysis and Design*. Birkhäuser, Boston.

[3] V. Dragan, T. Morozan, A. Stoica, 2006. *Mathematical Methods in Robust Control of Linear Stochastic Systems*. Springer, New York.

[4] C.S. Kubrusly, 1986. On discrete stochastic bilinear systems stability. *J. Math. Anal. Appl.*, 113, pp. 36-58.

[5] C. S. Kubrusly, 1981. Identification of discrete-time stochastic bilinear systems. *Int. J. Control,* 33, 2, pp. 291-309.

[6] C.S. Kubrusly and O.L.V. Costa, 1985. Mean square stability conditions for discrete-stochastic bilinear systems. *IEEE Trans. Autom. Control,* 30, pp. 1082-1087.

[7] H. Gao, J. Lam, Z. Wang, 2007. Discrete bilinear stochastic systems with time-varying delay: Stability analysis and control synthesis. *Chaos, Solitons and Fractals,* 34, pp. 394-404.

[8] S. Xu, J. Lam, T. Chen, 2004. Robust H_∞ control for uncertain discrete stochastic time-delay systems. *Syst. Control Lett.*, 51, pp. 203-215.

[9] S. Xu, J. Lam, H. Gao, and Y. Zou, 2005. Robust H_∞ filtering for uncertain discrete stochastic systems with time delay. *Circuits Syst. Signal Process,* 24 (6), pp. 753-770.

[10] X. Mao, C. Yuan, 2006. *Stochastic Differential Equations with Markovian Switching*. Imperial College Press.

[11] E.K. Boukas, Z.K. Liu, 2002. *Deterministic and Stochastic Time Delay Systems*. Birkhäuser, Boston.
[12] O. L. V. Costa, M. D. Fragoso, R. P. Marques, 2005. *Discrete-time Markov Jump Linear Systems*. Springer, London.
[13] R. Elliott, F. Dufour, P. Malcom, 2005. State and mode estimation for discrete-time jump Markov systems. *SIAM J. Control Optim.*, 44 (3), pp. 1081-1104.
[14] C.E. De Souza, 2006. Robust stability and stabilization of uncertain discrete-time Markovian jump linear systems. *IEEE Trans. Autom. Control.*, 51 (5), pp. 836-841.
[14] J.C. Geromel, A.P. Gonçalves, A.R. Fioravanti, 2009. Dynamic output feedback control of discrete-time Markov jump linear systems through linear matrix inequalities. *SIAM J. Control Optim.*, 48 (2), pp. 573-593.
[16] L. Zhang, E. K. Boukas, 2009. Stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities. *Automatica*, 45, pp. 463-468.
[17] L. Zhang, E.K. Boukas, J. Lam, 2008. Analysis and synthesis of Markov jump linear systems with time-varying delay and partially known transition probabilities. *IEEE Trans. Autom. Control*, 53 (10), pp. 2458-2464.
[18] L.V. Hien, N.T. Dzung, H.B. Minh, 2016. A novel approach to state bounding for discrete-time Markovian jump systems with interval time-varying delay. *IMA Math. Control Info.*, 33 (2), pp. 293-307.
[19] N.T. Dzung and Tran Thi Thu, 2020. State-feedback control of discrete-time stochastic linear systems with markovian switching. *HNUE Journal of Science: Natural Science*, 65, (6), pp. 13-22.
[20] Y. Ding, H. Zhu, S. Zhong, Y. Zhang, Y. Zeng, 2013. H_∞ filtering for stochastic systems with Markovian switching and partly unknown transition probabilities. *Circuits Syst. Signal Process*, 32, pp. 559-583.
[21] V. Dragan, 2014. Robust stabilisation of discrete-time time-varying linear systems with Markovian switching and nonlinear parametric uncertainties. *Int. J. of Sys. Sci.*, 45 (7), pp. 1508-1517.
[22] S. Sathananthana, M. Knap, L. H. Keel, 2013. Optimal guaranteed cost control of stochastic discrete-time systems with states and input dependent noise under Markovian switching. *Stochastic Analysis and Applications*, 31, pp. 876-893.
[23] S. Xu, T. Chen, 2005. Robust H_∞ control for uncertain discrete-time stochastic bilinear systems with Markovian switching. *Int. J. Robust Nonlinear Control*, 15, pp. 201-217.
[24] S. Sathananthan , C. Beane, G.S. Ladde, L.H. Keel, 2010. Stabilization of stochastic systems under Markovian switching. *Nonlinear Analysis: Hybrid Systems*, 4, pp. 804-817.
[25] Y.S. Wang, L. Xie, C.E. De Souza, 1992. Robust control of a class uncertain systems.*Syst. Control Lett.*, 19, pp. 139-149.