A STUDY ON A TWO-DIMENSIONAL SHALLOW FLOW MODEL FOR OMPLEX TOPOGRAPHY

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Abstract: *In this paper, Finite Volume Method (FVM) is used to solve two Dimensional Shallow Water Equations (2D SWE) on structured mesh. The HLLC scheme approximate Riemann solver is invoked to compute the fluxes of mass and momentum; MUSCL procedure is employed to obtain second order accuracy. Besides, semi implicit method is utilized to solve the friction source term. The effectiveness and robustness of the above schemes is verified by three tests: hydrostatic test, a well-known Thacker's test and run up water wave on the domain with conical island. A case study of Lang Ha reservoir- Vinh Phuc is implemented to simulate breach hydrographs of dam collapsed scenarios.*
Keywords: Finite Volume Method, HLLC scheme, MUSCL procedure; Complex topography.

1. INTRODUCTION

Finite Volume Method (FVM) is a numerical scheme to compute a discrete solution of a conservation law of Partial Differential Equations (PDE) in the form of integral equations. Nowadays, it is considered the most applied numerical strategy to solve 2D SWE.

Using FVM, most complicated shallow water flow phenomena, such as: transcritical and supercritical flows; discontinuous type flow or moving wet/dry front can be simulated. However, the challenges still exist to achieve second order accuracy to preserve the C-property and obtain physical solution over complex topography. In this paper, FVM are used to solve 2D SWE on structured mesh; HLLC approximate Riemann solver is invoked to evaluate inter-cell fluxes and MUSCL procedure is employed to obtain high resolution.

The numerical scheme is usually utilized to handle the main challenges involved in the simulation of complex flows over natural topographies, such as: capturing discontinuities in the flow field without spurious oscillations, satisfying C-property to ensure the preservation of static condition and robust tracking of wet/dry fronts. The above scheme has been validated by a hydrostatic test and a well-known Thacker's tests with parabolic bed topography. Besides, a difficult test of solitary wave on domain with conical island is also reproduced. The scheme demonstrated to behave satisfactorily with respect to their effectiveness and robustness, showing good application prospects. Thus, it is applied for a real case study of Lang Ha reservoir (Tam Dao – Vinh Phuc) to obtain the breach hydrographs of dam break scenarios.

2. NUMERICAL MODEL

2.1. Governing mathematical scheme.

The conservation form of 2D SWE can be written as (Cung et al, 1980):

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}(\mathbf{U})}{\partial x}+\frac{\partial \mathbf{G}(\mathbf{U})}{\partial y}=\mathbf{S}(\mathbf{U}) \qquad (1)$$

Where: $\mathbf{U}=\begin{bmatrix} h \\ hu \\ hv \end{bmatrix}; \mathbf{F}(\mathbf{U})=\begin{bmatrix} hu \\ hu^2+0.5gh^2 \\ huv \end{bmatrix}; \mathbf{G}(\mathbf{U})=\begin{bmatrix} hv \\ huv \\ hv^2+0.5gh^2 \end{bmatrix}; \mathbf{S}(\mathbf{U})=\begin{bmatrix} 0 \\ gh(S_{0x}-S_{fx}) \\ gh(S_{0y}-S_{fy}) \end{bmatrix}$

$$S_{0x}=-\frac{\partial z_b}{\partial x}; S_{0y}=-\frac{\partial z_b}{\partial y}; S_{fx}=\frac{n^2u\sqrt{u^2+v^2}}{h^{4/3}}; S_{fy}=\frac{n^2v\sqrt{u^2+v^2}}{h^{4/3}}$$

U is the vector of conserved variables; **F** and **G** are flux vectors and **S** is source term

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accounting for bed slope term and friction term. x, y are orthogonal space coordinates on a horizontal plane and t is the time; h and z_b are water depth and bottom elevation; u, v are velocity components along x- and y- directions; S_{0x}, S_{0y}, S_{fx}, S_{fy} are bed slopes and friction slopes along the same directions; n is Manning roughness coefficient; g is gravity acceleration.

Based on Godunov type scheme, the flow variables are updated to a new time step by using the following equation:

$$\mathbf{U}_{i,j}^{n+1} = \mathbf{U}_{i,j}^{n} - \frac{\Delta t}{\Delta x}\left[\mathbf{F}_{i+1/2,j} - \mathbf{F}_{i-1/2,j}\right] - \frac{\Delta t}{\Delta y}\left[\mathbf{G}_{i,j+1/2} - \mathbf{G}_{i,j-1/2}\right] + \Delta t\mathbf{S}_{i,j} \quad (2)$$

where superscripts denote time levels; subscripts i and j are space indices along x- and y- directions; Δt, Δx, Δy are time step and space sizes of the computational cell.

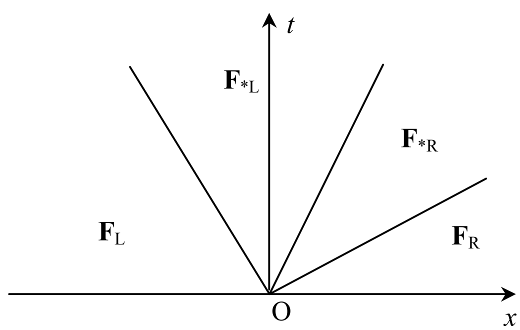


Figure 1. HLLC solution of local Riemann problem (in x direction).

Interface fluxes $F_{i\pm1/2,j}$ and $G_{i,j\pm1/2}$ are approximated by HLLC scheme (Fig. 1). For example:

$$\mathbf{F}_{i+1/2} = \begin{cases} \mathbf{F}_L & \text{if } s_1 \ge 0, \\ \mathbf{F}_{*L} & \text{if } s_1 < 0 \le s_2, \\ \mathbf{F}_{*R} & \text{if } s_2 < 0 \le s_3, \\ \mathbf{F}_R & \text{if } s_3 \le 0, \end{cases} \quad (3)$$

where, $\mathbf{U}_L$ and $\mathbf{U}_R$ are the left and the right states of Riemann problem, respectively; $\mathbf{F}_L = \mathbf{F}(\mathbf{U}_L)$ and $\mathbf{F}_R = \mathbf{F}(\mathbf{U}_R)$; s_1, s_2 and s_3 are estimates of the speeds of the left, contact and right waves, respectively. The middle region fluxes $\mathbf{F}_{*L}$ and $\mathbf{F}_{*R}$ are the numerical fluxes in the left and the right sides of the middle region of the Riemann solution which is divided by a contact wave. These values can be taken the same as (Le, 2014).

In order to achieve second order accuracy in time and space, the MUSCL-Hancock procedure with three steps: reconstruction data; evolution of extrapolated values and corrector step is employed according to (Aureli et al, 2008). In this paper, slop limiter Minmod is technique to gain physical results and satisfy the Total Variation Diminishing (TVD) property.

2.2. Stability condition, friction term and wet/dry treatment.

The stability condition for the numerical scheme described in section 2.1 is governed by the Courant–Fredrichs–Lewy (CFL) criterion, controlling the time step Δt at each time level. For Cartesian grids, CFL stability condition is given by Eq. 4:

$$\Delta t = Cr\left[\max\left(\frac{|\tilde{u}| + \sqrt{g\tilde{h}}}{\Delta x} + \frac{|\tilde{v}| + \sqrt{g\tilde{h}}}{\Delta y}\right)\right]^{-1} \quad (4)$$

where Cr is the Courant number specified in the range $0<Cr \le 1.0$.

To avoid unphysical flow inversion, friction term is discretized in a semi-implicit manner with the parameter θ is set equal to 0.5:

$$\begin{aligned} S_{fx}^* &= (1-\theta)(ghS_{fx})^n + \theta(ghS_{fx})^{n+1} \\ S_{fy}^* &= (1-\theta)(ghS_{fy})^n + \theta(ghS_{fy})^{n+1} \end{aligned} \quad (5)$$

Preservation of C-property for the cells with wet-dry interfaces can be guaranteed by the local modification technique introduced by (Liang, 2010).

The selected numerical model is written by Fortran90 and validated it with several test cases (Le, 2014). Three tests with irregular topography are chosen in the following part in order to show the robustness and effectiveness of the proposed model.

3. VALIDATION

3.1. Preservation of still water surface.

C-property is considered as one of the most important requirements that the numerical scheme should be satisfied. (Hou et al, 2013) produced a test of still water over two islands one of which is fully submerged while the other is only partly wetted. The whole domain is 1.0m×1.0m square, with the bed elevation given by the Eq. 6, (see Fig. 2a). The initial water level is 0.152m; initial velocities in *x* and *y* directions are set to zero. The computational area is uniformly divided into 100×100 cells.

$$z_b(x,y)=\begin{cases}\max\left\{0;\ 0.25-5\left[(x-0.7)^2+(y-0.5)^2\right]\right\} \text{ if } x\geq 0.45\text{m};\\ \max\left\{0;\ 0.1-10\left[(x-0.3)^2+(y-0.5)^2\right]\right\} \text{ if } x<0.45\text{m}\end{cases} \quad (6)$$

Fig. 2a shows 3D view of numerical solution of water surface at rest at t = 100*s* while Fig. 2b shows the computed water surface and unit discharge profile at centre line of domain. Water elevation remains constant meanwhile the unit discharge is equal zero, thus, the present scheme can satisfy exactly C-property. Besides, the numerical scheme achieves the correct balance between the flux term and source term also in presence of wet/dry fronts because no oscillation occurs on the interface between free surface and partly submerged island.

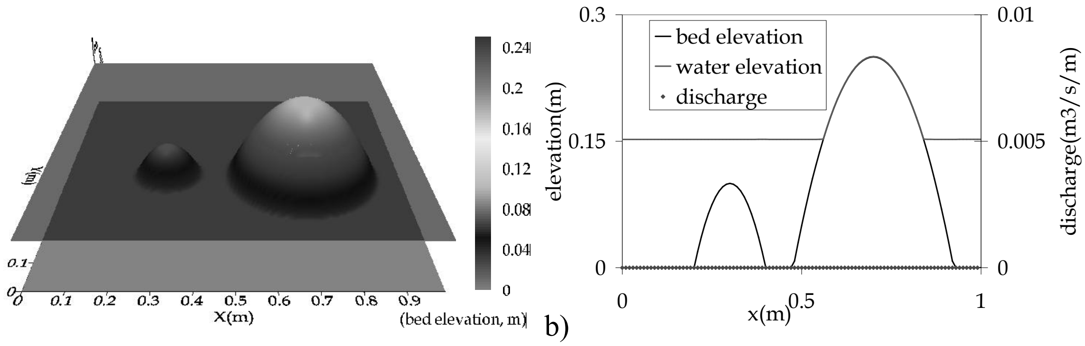


Figure 2. a) Bed topography; b) Still water over two bumps.

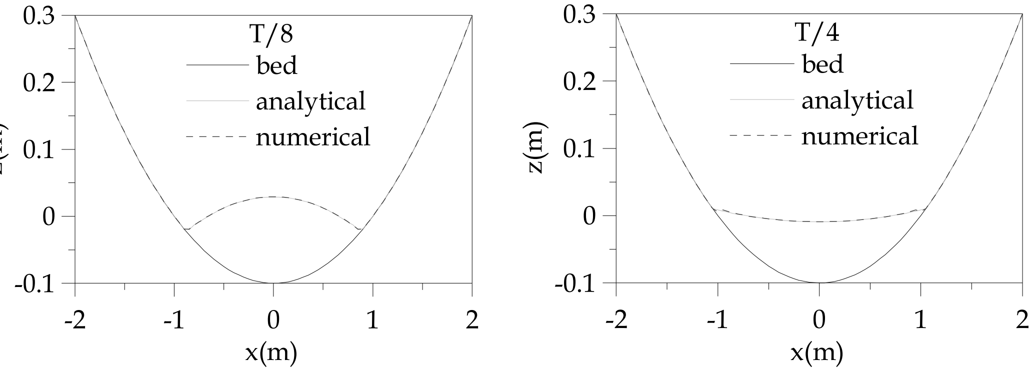
3.2. Thacker's long wave resonance in parabolic basin.

(Thacker, 1981) provided an analytical solution corresponding to free surface oscillation in a parabolic basin that can be used to verify the behavior of 2D numerical models in dealing with wetting and drying fronts. For this case, the bed topography is defined as:

$$z_b(x,y)=h_0\left(\frac{x^2+y^2}{a^2}-1\right) \quad (7)$$

The initial water elevation is specified by Eq. 8:

$$\eta(x,y)=\max\left[z_b(x,y);\ h_o\left(\frac{\sqrt{1-A^2}}{1-A}-\frac{x^2+y^2}{a^2}\left(\frac{1-A^2}{(1-A)^2}-1\right)-1\right)\right] \quad (8)$$



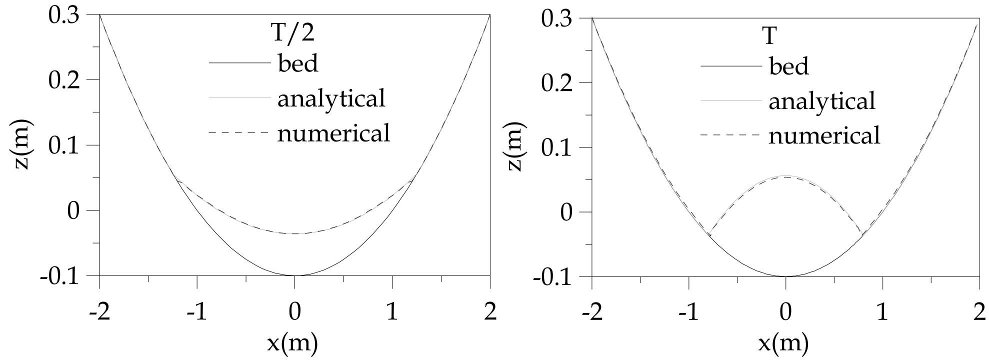


Figure 3. Water surface elevation at different times

The analytical solution of water elevation is:

$$\eta(x,y,t) = \max\left[z_b(x,y);\ h_0\left(\frac{\sqrt{1-A^2}}{1-A\cos(\omega c)^2} - \frac{x^2+y^2}{a^2}\left(\frac{1-A^2}{(1-A\cos(\omega c))^2} - 1 \right) - 1 \right) \right] \quad (9)$$

where $T=2\pi/\omega$; $\omega = \sqrt{8gh_0}/a$ and $A = \left(a^4 - r_0^4\right)/\left(a^4 + r_0^4\right)$. The parameters used for this numerical test are a=1.0, r_0=0.8m and h_0=1.0m.

On the computational domain [-2m,2m] × [-2m, 2m], where h_0 is the depth of the water at the centre and a is a radial distance from the centre point to the zero elevation of the shoreline. The numerical solutions of water elevation at different times simulating by the proposed scheme are displayed in Fig. 3.

As can be seen from Fig. 3, the computed water surface elevation profile are in excellent agreement with analytical solution at t = T/8, T/4, T/2. But a small displacement can be seen between them at t = T. This is possibly due to numerical damping in the scheme and the step–like representation of the bed geometry (Song et al, 2011).

3.3. Run-up of a solitary wave on a conical island.

This complicated test is set up to study the run-up of waves on a conical island in the US Army Engineer Waterways Experiment Station. As the moving wet-dry fronts on irregular bed are produced in this experiment, when the wave runs up on the island, it can be applied to test model’s capability to cope with wetting and drying over complex bed.

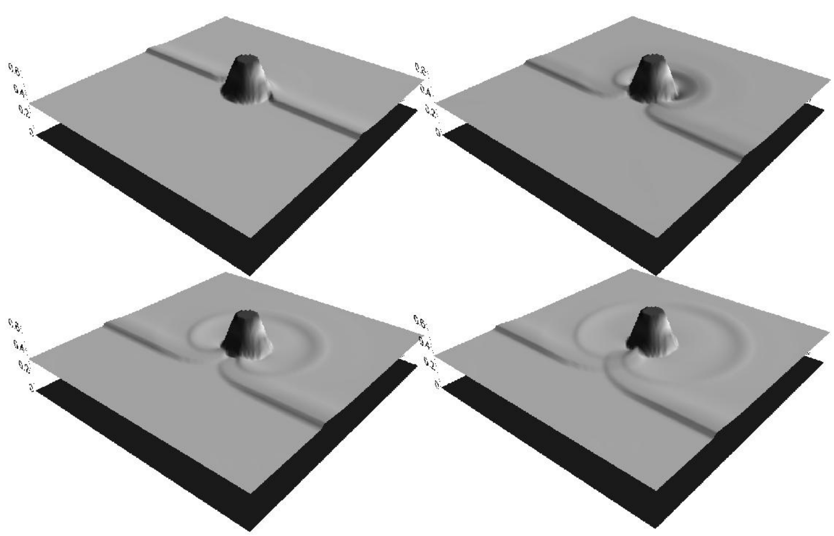


Figure 4. Water wave run up at t=9s, 10s, 11s, 13s.

A 25.92m× 27.6m computational domain is chosen. A conical island is located at the center O(0,0) of domain, which its elevation is following by Eq. 10:

$$z_b(x,y) = \max\left[0;\ 0.9 - 0.25.\sqrt{x^2 + y^2}\right] \text{if } z_b(x,y) > 0.625\text{m};\ z_b(x,y) = 0.625\text{m} \quad (10)$$

Still water with a depth of 0.32m is initiated in the domain. The left boundary is set to be the wave inflow boundary, where water level η and velocity components u and v are imposed:

$$\eta(t) = H.\text{sech}^2\left[\sqrt{\frac{3H}{4D^3}}C(t-T)\right] + D; \qquad u(t) = \frac{C.(\eta - D)}{\eta}; \qquad v(t) = 0 \qquad (11)$$

Where H is the amplitude of the incident wave, H=0.064m and D is the still water depth, D=0.32m; T denotes the time when the crests reach domain, T=2.45s and C is the wave celerity, with $C = \sqrt{g(D+H)}$.

The simulated evolution of the solitary wave around island is sketched in Fig. 4. In addition, the computed and measured water levels at four gauges are plotted in Fig. 5. Obviously, the presented model gives a good prediction the wave heights and arrival time at most gauges. Besides, comparison with other numerical results in different works, such as (Hou et al, 2013), the close agreement can be observed.

Thus, with three above tests, the robustness and effectiveness of the numerical model is demonstrated when it overcomes the challenges, such as: ensuring C-property, tracking wet/dry front in complex topography conditions.

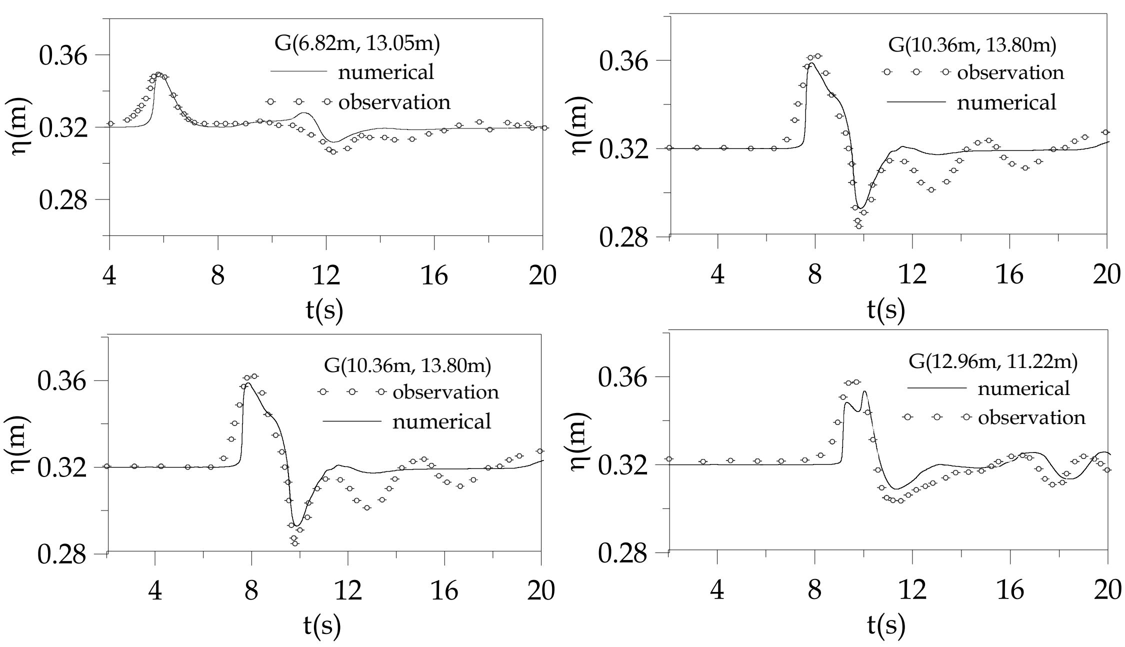


Figure 5. Time histories surface displacements at different gauges.

4. APPLICATION

Lang Ha reservoir constructed since 1986 is located in Tam Đao -Vinh Phuc province (Fig. 6a). Its responsibility is to irrigate 400ha of agricultural area (Công ty xây dựng và chuyển giao công nghệ Thủy lợi, 2006). Recently, the main earth dam of this reservoir is degraded seriously, so simulating dam break flood propagation to build early warning scenarios for reservoir's downstream when dam collapses should be considered.

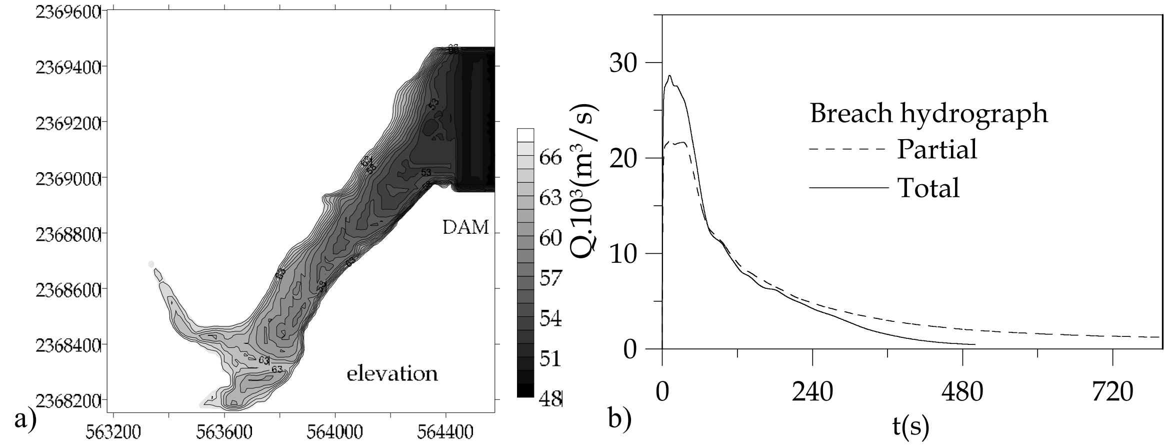


Figure 6. a) Contour map of Lang Ha reservoir; b) Breach hydrographs of total and partial dam break scenarios.

In this work, the breach hydrographs of two scenarios: total and partial dam break are simulated. The ratio between partial dam collapsed area with total one is 0.82. The most dangerous hypotheses are set up: extreme water elevation +66.3m in upstream; water depth at downstream is set to equal zero and the main earth dam instantaneously collapsed. The Manning coefficient is taken to 0.03.

According to (Fig. 6b), in case of total dam break scenario, the peak of discharge is 27546 m^3/s and emptying time is around 480 seconds. Meanwhile, these values are 21632 m^3/s and around 720s, respectively in solution of partial dam break scenario.

These hydrographs can be used as input data for hydraulic simulation of flood propagation in downstream if reservoir downstream's geometry is available.

5. CONCLUSIONS

In this paper, FVM is used to solve 2D SWE on structured mesh. HLLC approximate Riemann solver is applied to solve flux terms. Second order accuracy is obtained by MUSCL procedure. By three tests with uneven bed presented in this paper, the scheme demonstrated to behave satisfactorily with respect to their effectiveness and robustness in simulating complex flows on irregular topographies. The numerical difficulties such as: C property, wet/dry front. However, a small shortcoming is still exists in this scheme when numerical damping occurs on Thacker'test.

A real case study of Lang Ha reservoir, Vinh Phuc is utilized to apply the presented model. The breach hydrographs of two scenarios are calculated. In further study, if geometry of its downstream is available, the flooding map will be generated.

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Tóm tắt:

NGHIÊN CỨU MỘT MÔ HÌNH NƯỚC NÔNG HAI CHIỀU CHO DẠNG ĐỊA HÌNH PHỨC TẠP

Trong bài báo này, phương pháp thể tích hữu hạn được sử dụng để giải hệ phương trình nước nông hai chiều trên hệ lưới có cấu trúc. Phương pháp HLLC được dùng để tính xấp xỉ các giá trị của thông lượng trong hệ phương trình nước nông. Qui trình MUSCL được dùng để có được kết quả chính xác bậc hai. Ngoài ra, phương pháp tính bán ẩn được dùng để giải số hạng ma sát trong hệ phương trình này. Bài báo dùng 3 ví dụ để kiểm tra tính đúng đắn và hiệu quả của mô hình lựa chọn: thủy tĩnh, Thacker và sóng lan truyền trên miền tính toán có trụ hình nón cụt. Phần cuối, áp dụng mô hình tính đường quá trình lưu lượng cho 2 kịch bản vỡ đập của hồ chứa Làng Hà – Vĩnh Phúc.

Từ khóa: Thể tích hữu hạn, phương pháp HLLC, qui trình MUSCL, địa hình phức tạp.

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