1D MODELLING OF INFRAGRAVITY WAVE PROPAGATION ON FRINGING REEF USING SWASH

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Abstract: *This paper presents wave propagation process on an idealized fringing reef profile using numerical model, in which infra-gravity wave motion is of major concern. The results show that SWASH is capable to accurately predict infra-gravity wave initiated at the reef face and reef crest for the case of high reflection coefficient at back reef with the model skill in range between 0.94 and 0.98. Over 24 scenarios, SWASH tends to slightly underestimate the spectra wave height in compared to measured one. Moreover, for low relative reef flat submergence, H_{m0}^{i}/d, SWASH does not account correctly the breaking position which affects the wave energy dissipation.*

Keywords: Fringing reefs, infra-gravity waves, wave breaking, wave set-up, SWASH.

1. INTRODUCTION

A typical fringing reef is characterized by an abruptly steep face slope and a shallow reef flat connecting to the shoreline. Sea swell wave energy mostly dissipates, only infra-gravity wave energy dominates in the mid and back reef. Infra-gravity waves (IG) generated on the reef flat by the breaking-point mechanism (Longuet-Higgins and Stewart 1962) with period ranging from 20s up to 250s. They are responsible for several hydrodynamic effects such as infra-gravity wave resonance, entrapped energy on the flat, swash waves which make infra-gravity waves an important aspect for coastal engineering.

There is a need to have an accurate prediction with proper description of the generation and nearshore transformation of IG waves on the fringing reef. Several types of numerical models have been applied in studying IG waves such as "surf beat model" (Roelvink et al. 2009, Dongeren et al. 2012), Boussinesq model (Madsen et al. 1991, Nwogu 2010, Yao et al. 2012, Lin and Liu 1998). However, it is computationally more expensive and time consuming to apply for complex bathymetry. Alternatively, a non-hydrostatic model using shallow water equation is rational solution in terms of time and expenses.

SWASH (Simulating WAves till Shores) is a non-hydrostatic wave-flow model takes its starting point as Navier Stoke equation to calculate the surface elevation and currents. To simplify the problem, the free surface is described by a single value function that allows non-hydrostatic models to efficiently compute free surface flow. Zijlema performed model validation based on experimental data from Demirbilek et al. (2007) with a fringing reef of 6m length (corresponding 120m in prototype) and mild back reef slope (1/12). Rinjdorp (2012) compared the model results in case of bi-chromatic wave propagate on mild and steep beach slope. In recent study, model validation was carried out for a fringing reef of 10m width and fore reef slope 1/20 and no back reef slope with model scale 1:40 (Pham Lan Anh et al. 2020). In fact, the reef dimensions (fore reef slope, reef flat length, beach slope) have a great impact on the wave transformation across reef flat. The back reef slope steepness can range from minimum value of 1/12 up to maximum of vertical (Buckley 2018). Most of recent studies focus on the variation of flat length or fore reef slope,

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howbeit, neglecting the variation of back reef slope, otherwise leaving it in a mild steepness. Hence, this paper aims to demonstrate the robustness of SWASH in simulating wave propagation on an idealized fringing reef flat (10m in model scale) in case of the steeply inclined back reef slope (1/5). Therefore, SWASH capabilities in simulating IG waves are investigated by comparing model predictions with experimental data of report of Pham Lan Anh (2020) for infra-gravity wave characteristics on fringing reef.

This paper is structured as followed: The first part focuses on SWASH basic equation and its sensitive parameters. It is followed by the model setup description in SWASH based on the laboratory setup in a wave flume (Pham Lan Anh 2020) to research on infra-gravity wave characteristics. The third part displays the results and discussion on the wave spectrum, sea-swell waves and infra-gravity waves.

2. FUNDAMENTAL EQUATIONS

Swash derived from incompressible Navier Stokes equations that describe conservation of mass and momentum. In this study unidirectional waves are considered in 2 dimensional plain. The free surface z=ζ(x,t) and the bottom z=–d(x), t is the time, x and z are the Cartesian coordinates with z defines downward and z=0 located in the still water level.

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{p_h+p_{nh}}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}+\frac{\partial \tau_{xx}}{\partial x} \qquad (2)$$

$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial w^2}{\partial z}=-\frac{1}{\rho}\frac{p_{nh}}{\partial z}+\frac{\partial \tau_{zz}}{\partial z}+\frac{\partial \tau_{zx}}{\partial x} \qquad (3)$$

Where $u(x,z,t)$ and $w(x,z,t)$ are the horizontal and vertical velocities respectively; $p_h(x,z,t)$ and $P_{nh}(x,z,t)$ are the hydrostatic and non-hydrostatic pressure respectively; $\tau_{xx}, \tau_{xz}, \tau_{zx}, \tau_{zz}$ are the turbulent stresses and ρ is the density of water. Considering a unit water column, an equation of surface water elevation is derived by balancing the mass conservation equation:

$$\frac{\partial \xi}{\partial x}+\frac{\partial}{\partial x}\int_{-d}^{\xi} u\,dz=0 \qquad (4)$$

The friction term is added via the bottom shear stress following the quadratic friction law:

$$\tau_b=c_f\frac{|U|U}{h} \qquad (5)$$

c_f is the friction coefficient, U the depth average flow velocity, $h=d+\zeta$ is the total water depth including wave set-up.

To capture wave breaking a condition of the wave front is applied to initiate the breaking process $m_t\zeta > \alpha\sqrt{gh}$ in which, α represents the maximum surface steepness and determines the onset of breaking process. To display persistence of wave breaking the condition $m_t\zeta > \beta\sqrt{gh}$ is applied with β<α and β is the threshold to stop breaking.

3. MODEL SETUP

Laboratory experiments was carried out in Holland flume in Thuyloi university which is 45m long 1.2m high and 1.0 m wide. The geometry of an idealized fringing reef is shown in figure 1 characterizing a 1:40 scale model. Six waves gauges were installed to measure water surface elevation at the shoreline, along the reef flat and in deep water. A paddle wave maker is located 22m from the location of model structure creating irregular wave field based on JONSWAP spectrum with peak enhancement factor γ = 1.25. Tested wave heights (0.06m – 0.15m) and periods (1.0s-1.6s) is found most suitable to Viet Nam storm wave condition in East Sea. For further experiment details, it can be made a reference to Pham Lan Anh (2020).

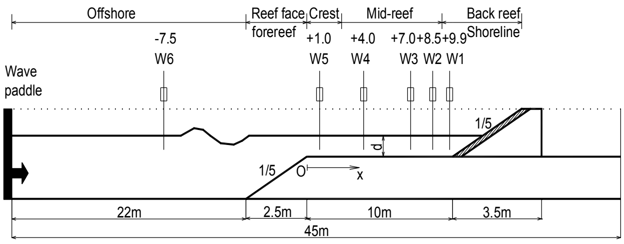

Figure 1. Experiment setup for an idealized fringing reef

The input wave conditions for simulating wave transformation in SWASH are listed below.

Table 1. Experiment scenarios

Flat depth d (m) Model	Prototype	Back reef steepness	Model H_{m0}^{i} (m)	T_p (s)	Prototype H_{m0}^{i} (m)	T_p (s)	Wave steepness
		1/5	0.06	1.35	1.2	8.54	0.01
0.05	2.0		0.07	1.50	2.8	9.50	0.02
0.1	4.0		0.09	1.40	3.6	8.85	0.03
0.15	6.0		0.09	1.60	3.6	10.12	0.02
0.2	8.0		0.12	1.60	4.8	10.12	0.03
			0.15	1.80	6.0	11.38	0.03

H_{m0}^{i} incoming spectrum wave height; T_p peak period of incoming waves

Numerical simulation was performed with a computational domain ranging within 21m, which is narrower than the experimental domain. The purpose of domain restriction is to keep the hydraulic boundary as close to the reef face as possible. To ensure spatial resolution, a grid size of 1cm was chosen which was meant that, on average, there were from 285 grid cells to 506 grid cells per wavelength. On the west boundary, time series of wave gauge 6 in deep water was applied. This satisfied the condition that area of interest should be kept at least two wave length from the boundaries. On the east boundary, wave naturally reflected at a hard beach slope of 1/5 without a sponge layer. Two vertical layers were chosen which is the best choice in increasing the frequency dispersion and an initial time step is 1ms. The friction coefficient is 0.01 for Manning option. This value is not known beforehand and must be obtained after calibration. It was a similar implementation for the threshold of wave starting breaking α=0.6 and the threshold of wave stopping breaking β=0.3. Reflection from offshore boundary was minimized by a weakly reflected boundary.

4. RESULTS AND DISCUSSION

There are 24 tests chosen from the whole data experiment in the report (Pham Lan Anh 2020) which represent the variation in water depth, in coming wave height and period. Figure 2 shows the transformation water surface water elevation from deep water WG6 to reef crest WG5, mid-reef WG2, WG3, WG4 and at the shoreline WG1.

It can be seen that within 200 sec capturing most of wave gauges show a common tendency of fluctuation between SWASH calculation and measurement. Near the reef face and move toward mid-reef (WG5, WG4) there is several difference at peak due to phase lag or magnitude difference, however, this is linearly minimized once waves moves toward shoreline (WG3, WG2). In WG6, wave shape stays in symmetry form which is typical for deep water waves, whereas from WG5 to WG1 waves transform in asymmetric shape with high positive peaks but low and flat negative peaks.

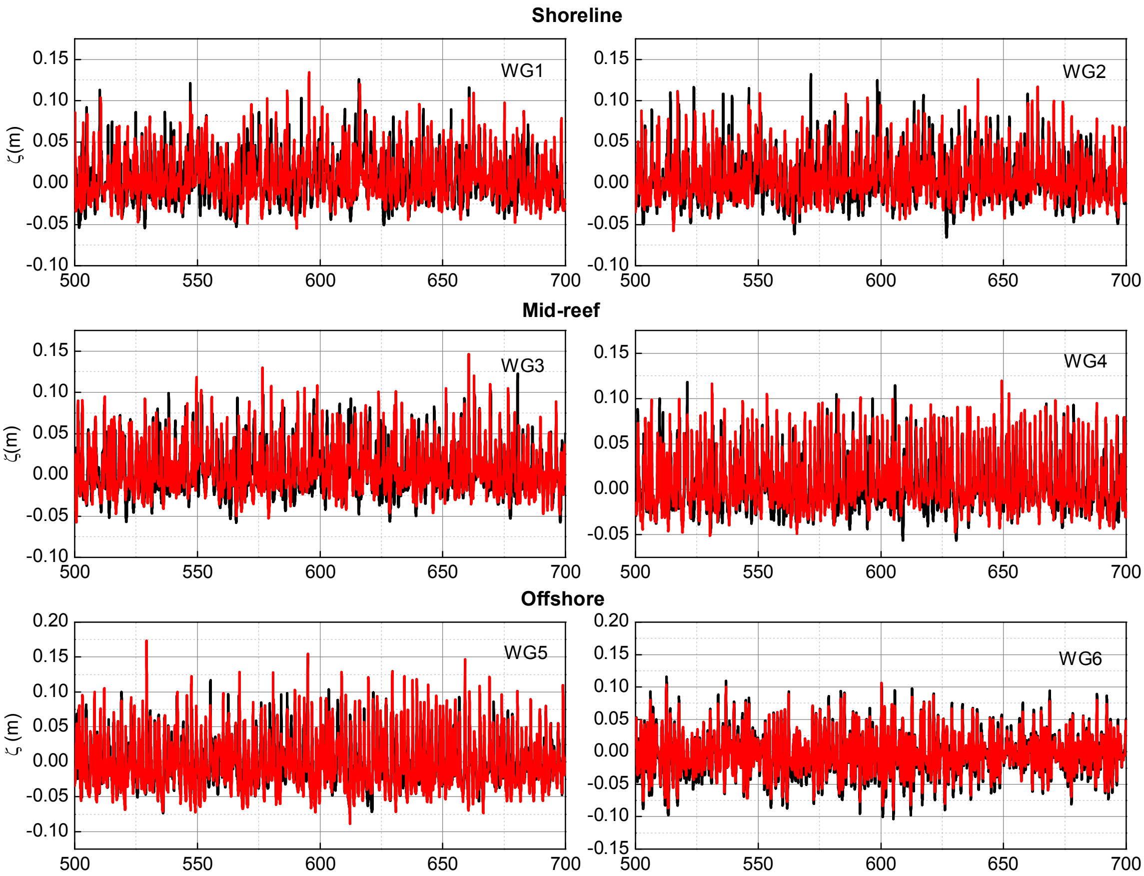

Figure 2. Measured and SWASH calculated water elevation under d=0.2m, H_{m0}^{i}=15cm, T_p=1.8s; in black represent SWASH calculated, in red represent measured.

Figure 3 displays the wave spectra evolution from reef face (WG5) to the shore line (WG1) within low frequency band ($f_{IG} \leq 1/2\, f_p$, f_{IG} is the IG frequency, f_p is the peak frequency of incident waves). The spectra evolution, in general, agree well with the measured data, except for the very low IG frequency. The computed energy (black) underestimates at high harmonics and overestimates at lower harmonics which seems compensate each other and still lead to equally correlative in H_{m0IG} (figure 6a, b, c). These could be corrected by refining the grid size down to 0.5cm to improve the spatial resolution in the model.

The spectra wave height in SWASH and in measurement is estimated as follow:

$$H_{m0} = 4.\sqrt{\int S(f)df} \quad (5)$$

Moreover, wave height variation across the reef flat indicates that the model reproduces properly the wave dissipation process via breaking at the reef crest and reef face. Waves rapidly attenuate at the reef crest which is around half of the incident wave height. The measurement and predicted model agree very well to this point (figure 4), except D70H07T150 (d=0.2m, H_{m0}^{i}=7cm, T_p=1.5s) and D70H09T140 (d=0.2m, H_{m0}^{i}=9cm, T_p=1.4s). The general tendency of wave height distribution for these two scenarios is almost reverse to the decreasing tend of wave height. This seems matching well with set-up distribution across the reef flat in D70H07T150 and D70H09T140 (figure 5). Explanation is presented based on table 2 and figure 6d below.

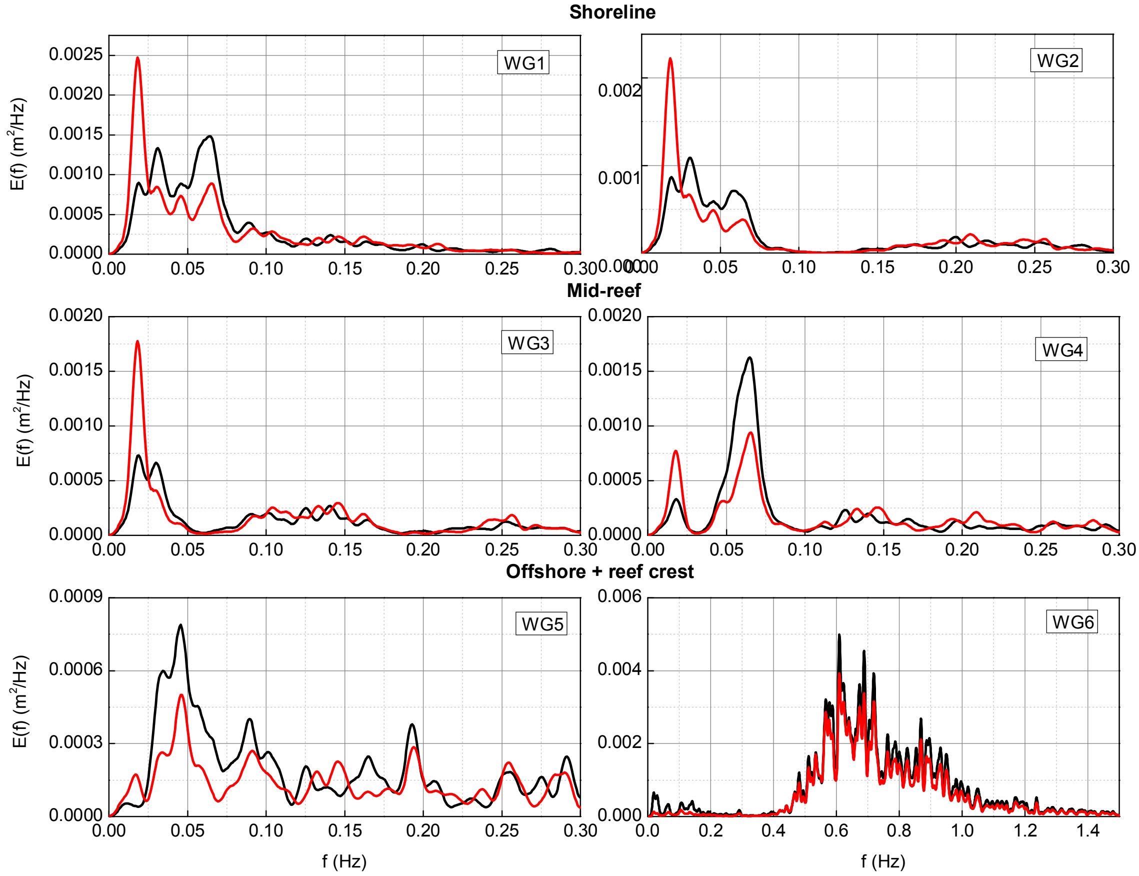

Figure 3. Comparison of measured and calculated wave spectra at the IG wave frequency band under d=0.05m, $H_{m0}{}^{i}$=9cm, Tp=1.6s; in black represent SWASH, in red represent measured.

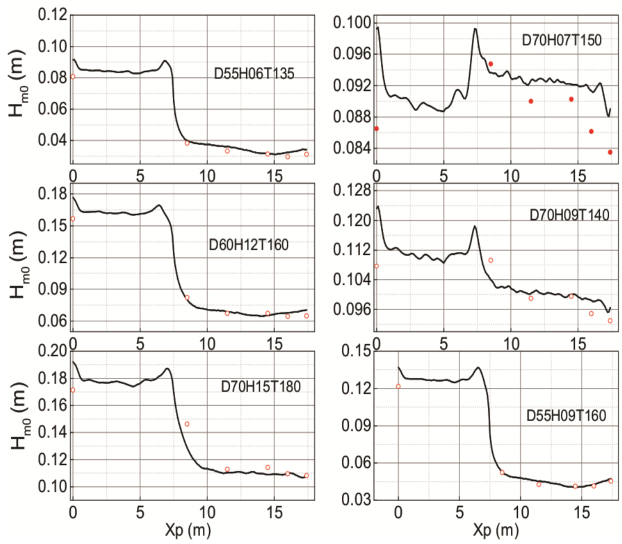

Figure 4. Wave height (H_{m0}) distribution in deep water and along the reef flat, black and red represent SWASH and measured, respectively

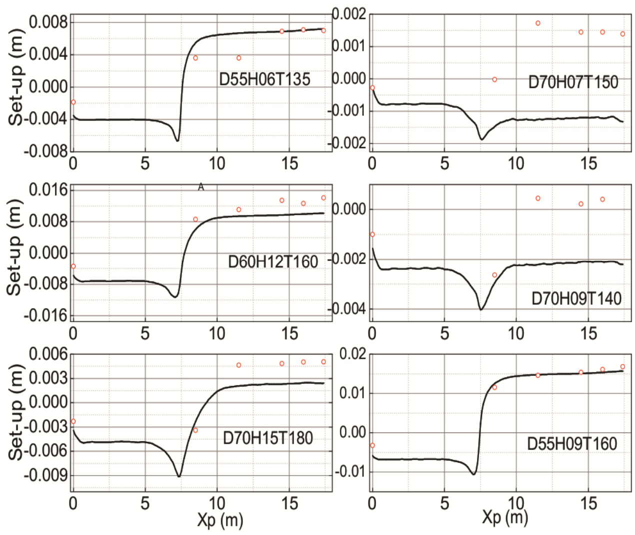

Figure 5. Set-up along the reef flat, black and red represent SWASH and measured, respectively

Table 2. Comparison of wave breaking position between experiment observation and SWASH calculation with 10 consecutive waves

Scenarios		Breaking position		Type of breaking	
		SWASH	Lab observation	SWASH	Lab observation
D55H06T135	d=0.05m; H_{m0}^{i}=6cm; T_p=1.35s	Reef crest	Reef crest	-	Spilling
D55H09T160	d=0.05m; H_{m0}^{i}=9cm; T_p=1.60s	Reef face	Reef face	-	Plunging
D60H12T160	d=0.1m; H_{m0}^{i}=12cm; T_p=1.6s	Reef face	Reef face & reef crest	-	Spilling & plunging
D70H07T150	d=0.2m; H_{m0}^{i}=7cm; T_p=1.5s	No breaking	Reef crest	-	Spilling
D70H09T140	d=0.2m; H_{m0}^{i}=9cm; T_p=1.4s	No breaking	Reef crest	-	Spilling
D70H15T180	d=0.2m; H_{m0}^{i}=15cm; T_p=1.8s	Reef face	Reef face	-	Plunging

In order to explain further the above difference, ten consecutive incoming waves to the reef were considered after 500 sec elapsed to investigate the position of wave breaking. It can be seen that SWASH assesses incorrectly the breaking position of scenarios with relative small spectra waves. In this case relative spectra wave height is made dimensionless to reef flat water depth, H_{m0}^{i}/d. The ratio H_{m0}^{i}/d reflects the shallowness of the water depth or the reef flat submergence which affects the depth-induced wave breaking. Based on table 2, figure 6d, if $H_{m0}^{i}/d \leq 0.45$ SWASH does not account for breaking occurrence on the reef crest, while in fact it happens in laboratory observation (gray shading). In cases $H_{m0}^{i}/d > 0.45$ the breaking position is determined correctly in comparison with laboratory observation. Hence, although the dispersive relation has been improved by applying the vertical layers, SWASH still has short-coming in energy dissipation calculation in different wave breaking conditions (figure 6d).

Infra-gravity spectra wave height H_{m0IG} and short spectra wave height H_{m0SS} (Yao 2019) generated by varying breaking point mechanism on the flat are estimated as

$$H_{m0SS} = 4\sqrt{\int_{f_c}^{\infty} S(f)\,df},\ H_{m0IG} - 4\sqrt{\int_{0}^{f_c} S(f)\,df} \quad (6)$$

S(f) variance wave energy density spectrum. Wave spectra is divided into short wave-high frequency ($f > f_c$) and IG wave - low frequency ($f \leq f_c$) relied on the demarcating frequency f_c, where $f_c = 0.5 f_p$ and f_p is the peak frequency of the incident wave.

In figure 6 a, b, c the results show good agreement between measurement and prediction. There is minimal scatter in relation between measured and calculated IG wave height (figure 6c). These high harmonics frequencies have underestimates in calculated energy spectra, hence leading to some skew points at the shoreline and mid-reef.

Finally, the error estimator for SWASH and measurements has been calculated to confirm such good agreement. The model skill was determined by bias and scatter index (SI) (table 3)

$$Bias = \frac{1}{N}\sum_{i}^{N}\left(X_{calculated}^{i} - X_{measured}^{i}\right) \quad (7)$$

$$SI = 1 - \frac{\sum \left| X_{calculated} - X_{measured} \right|^2}{\sum \left(\left| X_{calculated} - \bar{X}_{measured} \right| + \left| X_{measured} - \bar{X}_{measured} \right| \right)^2} \quad (8)$$

$X_{calculated}$ and $X_{measured}$ are the corresponding values for SWASH and measured parameters; N is the total number of data set considered. In table, there are two quantities that are brought to compare, namely the spectra wave height H_{m0}, the sea-swell (short) wave height H_{m0SS} and low frequency wave height H_{m0IG}.

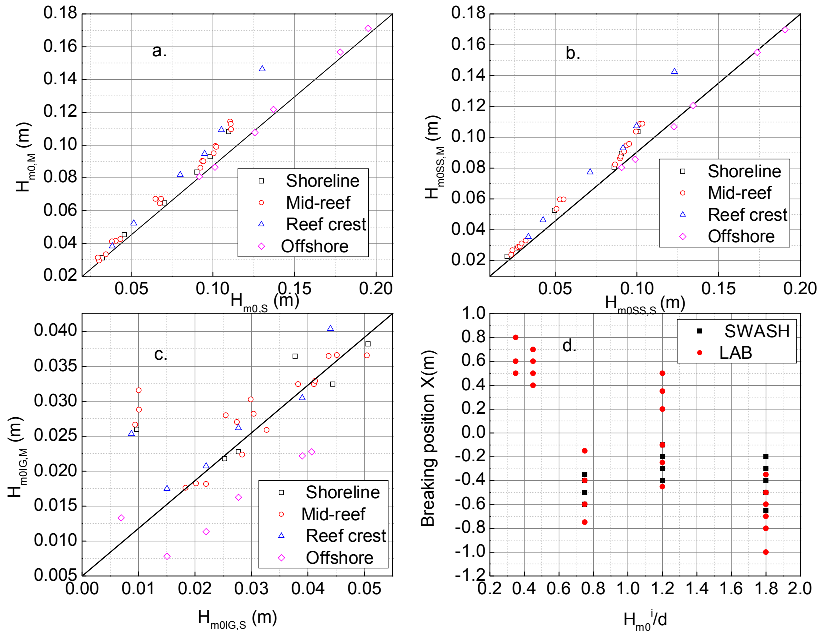

Figure 6. Calculated spectra wave height (subscript S) versus measured spectra wave height (subscript M) (a). the total spectra wave height (a) short wave height (c) IG wave height (d) wave breaking position versus flat submergence H_{m0}^i/d.

Table 3. Error estimators for calculated and measured wave height and wave set-up

Gauge	Bias H_{m0} (m)	SI H_{m0}	Bias $H_{m0SS\ (m)}$	SI H_{m0SS}	Bias $H_{m0IG\ (m)}$	SI H_{m0IG}	Bias set-up (m)	SI set-up
WG1	0.00363	0.94	-0.0004	0.95	0.003	0.931	-0.0014	0.96
WG2	0.0029	0.96	-0.0004	0.96	0.0021	0.94	-0.00003	0.96
WG3	-0.0007	0.96	-0.0026	0.96	-0.0016	0.94	-0.001	0.96
WG4	0.0016	0.96	-0.0017	0.96	0.002	0.94	-0.002	0.97
WG5	-0.0036	0.98	-0.0067	0.96	0.0014	0.95	0.0018	0.98
WG6	0.0018	0.98	0.0016	0.96	0.0019	0.95	-0.002	0.98

Overall the comparison between SWASH runs and measurement shows a good agreement at all locations under 24 scenarios. The maximum average difference is approximately 4mm at WG1 and WG5 which is totally small compared to typical wave height ranging from 6cm to 15cm. For the scatter index, the lowest value happens at WG1 under low frequency wave height (93.1%) which is highly acceptable.

5. CONCLUSION

SWASH are, in general, capable to reproduce sufficiently and correctly the infra-gravity wave transformation across the fringing reef in case of steeply inclined back reef slope. For complicated breaking process, SWASH can robustly capture the vertical steepness in decaying wave height distribution over reef flat happening at the reef face. Moreover, results also show that the model slightly underestimates the calculated wave height. The difference ranges between 2mm to 4mm in compared to typical wave height ranging from 6cm to 15cm and the skill model ranges between 0.94 and 0.98 which is highly reliable. However, for low H_{m0}^i/d (≤0.45), SWASH does not account correctly the breaking position which affects the wave energy dissipation. Therefore, it is highly recommended to simulate the model with relatively high H_{m0}^i/d (>0.45) to gain a proper result. It is worth to say that 0.45 is a relative value inferred from 24 runs of this modelling. Further study should be taken to determine correctly this transitional value which reflects the relative submergence.

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Tóm tắt:

MÔ PHỎNG QUÁ TRÌNH TRUYỀN SÓNG NGOẠI TRỌNG LỰC TRÊN THỀM ĐẢO NỔI BẰNG MÔ HÌNH SWASH 1D

Bài báo trình bày mô phỏng quá trình biến đổi sóng, đặc biệt là sóng ngoại trọng lực trên thềm đảo nổi xa bờ bằng mô hình SWASH 1D. Kết quả cho thấy SWASH mô phỏng tương đối chính xác sự hình thành và biến đổi sóng ngoại trọng lực với độ chính xác mô hình SI từ 0.94 tới 0.98. SWASH có xu hướng cho kết quả chiều cao sóng nhỏ hơn thí nghiệm một chút. Với những giá trị độ nông tương đối trên thềm thấp (H_{m0}^{i}/d nhỏ), SWASH chưa tính đến chính xác vị trí sóng vỡ, vì vậy ảnh hưởng tới tiêu hao năng sóng và độ lớn nước dâng, chiều cao sóng ở trường hợp này.

Từ khóa: Đảo nổi, sóng ngoại trọng lực, sóng vỡ, nước dâng, SWASH.

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