DETERMINED ABSORPTION COEFFICIENT OF ^{85}Rb ATOM IN THE Y-CONFIGURATION

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Abstract: *In this work, we establish a system of equations of density and derive analytical expression for the absorption coefficient of ^{85}Rb atomin the Y -configuration for a weak probe laser beam induced by two strong coupling laser beams. Our results show possible ways to control absorption coefficient by frequency detuning probe laser and intensity of the coupling laser.*

Keywords: *Electromagnetically induced transparency, absorption coefficient.*

1. Introduction

The manipulation of subluminal and superluminal light propagation in optical medium has attracted many attentions due to its potential applications during the last decades, such as controllable optical delay lines, optical switching [2], telecommunication, interferometry, optical data storage and optical memories quantum information processing, and so on [6]. The most important key to manipulate subluminal and superluminal light propagations lies in its ability to control the absorption and dispersion properties of a medium by a laser field.

As we know that coherent interaction between atom and light field can lead to interesting quantum interference effects such as electromagnetically induced transparency (EIT) [1]. The EIT is a quantum interference effect between the probability amplitudes that leads to a reduction of resonant absorption for a weak probe light field propagating through a medium induced by a strong coupling light field [5]. Basic configurations of the EIT effect are three-level atomic systems including the Λ-Ladder and V-type configurations. In each configuration, the EIT efficiency is different, in which the Λ-type configuration is the best, whereas the V-type configuration is the worst [4], [7], therefore, the manipulation of light in each configuration are also different. This suggests that we choose to use the analytical model to determine the absorption coefficient for the Y configuration of the ^{85}Rb atomic system.

2. The density matrix equation for ^{85}Rb atomic system configure Y

We first consider a Y-configuration of ^{85}Rb atom as shown in Fig. 1. State $|1\rangle$ is the ground states of the level $5S_{1/2}$ (F=3). The $|2\rangle$, $|3\rangle$ and $|4\rangle$ states are excited states of the levels $5P_{3/2}$ (F'=3), $5D_{5/2}$ (F"=4) and $5D_{5/2}$ (F"=3) [7].

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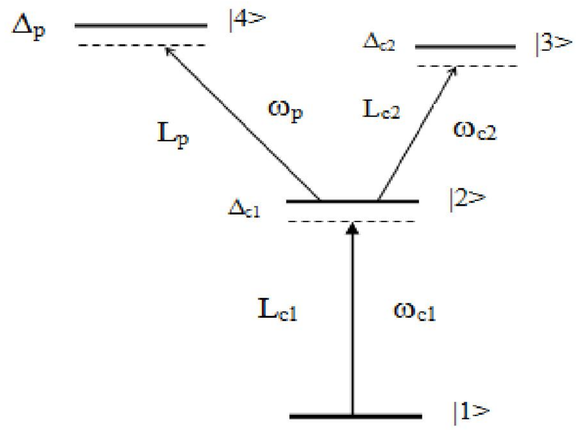

Figure 1. *Four-level excitation of the Y- configuration.*

Put this Y-configuration into three laser beams atomic frequency and intensity appropriate: a week probe laser L_p has intensity E_p with frequency ω_p applies the transition $\left|2\right\rangle \ \forall \ \left|4\right\rangle$ and the Rabi frequencies of the probe $\Omega_p = \dfrac{\mu_{42}E_p}{\hbar}$; Two strong coupling laser L_{c1} and L_{c2} couple the transition $\left|1\right\rangle \ \forall \left|2\right\rangle$ and $\left|2\right\rangle \forall \left|3\right\rangle$ the Rabi frequencies of the two coupling fields $\Omega_{c1} = \dfrac{\mu_{21}E_{c1}}{\hbar}$ and $\Omega_{c2} = \dfrac{\mu_{32}E_{c2}}{\hbar}$, where μ_{ij} is the electric dipole matrix element $\left|i\right\rangle \leftrightarrow \left|j\right\rangle$. The evolution of the system, which is represented via the density operator ρ is determined by the following Liouville equation [2]:

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}\left[H,\rho\right] + \Lambda\rho, \tag{1}$$

where, H represents the total Hamiltonian and $\Lambda\rho$ represents the decay part. Hamilton of the systerm can be written by matrix form:

$$H = \begin{pmatrix} \hbar\omega_1 & \frac{\hbar\Omega_{c1}}{2}e^{i\omega_{c1}t} & 0 & 0 \\ \frac{\hbar\Omega_{c1}}{2}e^{-i\omega_{c1}t} & \hbar\omega_2 & \frac{\hbar\Omega_{c2}}{2}e^{i\omega_{c2}t} & \frac{\hbar\Omega_p}{2}e^{i\omega_p t} \\ 0 & \frac{\hbar\Omega_{c2}}{2}e^{-i\omega_{c2}t} & \hbar\omega_3 & 0 \\ 0 & \frac{\hbar\Omega_p}{2}e^{-i\omega_p t} & 0 & \hbar\omega_4 \end{pmatrix} \tag{2}$$

We consider the slow variation and put: $\rho_{43} = \tilde{\rho}_{43}e^{-i(\omega_p - \omega_{c2})t}$, $\rho_{42} = \tilde{\rho}_{42}e^{-i\omega_p t}$, $\rho_{41} = \tilde{\rho}_{41}e^{-i\left(\omega_p + \omega_{c1}\right)t}$, $\rho_{32} = \tilde{\rho}_{32}e^{-i\omega_{c2}t}$, $\rho_{31} = \tilde{\rho}_{31}e^{-i\left(\omega_{c1} + \omega_{c2}\right)t}$, $\rho_{21} = \tilde{\rho}_{21}e^{-i\omega_{c1}t}$. In the framework of the semiclassical theory, the density matrix equations can be written as:

$$\dot{\rho}_{44} = \frac{i\Omega_p}{2}\left(\tilde{\rho}_{42} - \tilde{\rho}_{24}\right) - \Gamma_{43}\rho_{44} \tag{3.1}$$

$$\dot{\tilde{\rho}}_{41} = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{42} - \frac{i\Omega_p}{2}\rho_{21} + [i(\Delta_{c1} + \Delta_p) - \gamma_{41}]\tilde{\rho}_{41} \tag{3.2}$$

$$\dot{\tilde{\rho}}_{42} = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{41} + \frac{i\Omega_{c2}}{2}\tilde{\rho}_{43} + \frac{i\Omega_p}{2}(\rho_{44} - \rho_{22}) + (i\Delta_p - \gamma_{42})\tilde{\rho}_{42} \tag{3.3}$$

$$\dot{\tilde{\rho}}_{43} = \frac{i\Omega_{c2}}{2}\tilde{\rho}_{42} - \frac{i\Omega_p}{2}\tilde{\rho}_{23} + [i(\Delta_p - \Delta_{c2}) - \gamma_{43}]\tilde{\rho}_{43} \tag{3.4}$$

$$\dot{\rho}_{33} = \frac{i\Omega_{c2}}{2}\left(\tilde{\rho}_{32} - \tilde{\rho}_{23}\right) + \Gamma_{43}\rho_{44} - \Gamma_{32}\rho_{33} \tag{3.5}$$

$$\dot{\tilde{\rho}}_{31} = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{32} - \frac{i\Omega_{c2}}{2}\tilde{\rho}_{21} + [i(\Delta_p + \Delta_{c1}) - \gamma_{31}]\tilde{\rho}_{31} \tag{3.6}$$

$$\dot{\tilde{\rho}}_{32} = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{31} + \frac{i\Omega_{c2}}{2}\left(\rho_{33} - \rho_{22}\right) + \frac{i\Omega_p}{2}\tilde{\rho}_{34} + (i\Delta_{c2} - \gamma_{32})\tilde{\rho}_{32} \tag{3.7}$$

$$\dot{\tilde{\rho}}_{34} = \frac{i\Omega_p}{2}\tilde{\rho}_{32} - \frac{i\Omega_{c2}}{2}\tilde{\rho}_{24} + [-i(\Delta_p - \Delta_{c2}) - \gamma_{43}]\tilde{\rho}_{34} \tag{3.8}$$

$$\dot{\rho}_{22} = \frac{i\Omega_{c1}}{2}\left(\tilde{\rho}_{21} - \tilde{\rho}_{12}\right) + \frac{i\Omega_{c2}}{2}\left(\tilde{\rho}_{23} - \tilde{\rho}_{32}\right) + \frac{i\Omega_p}{2}\left(\tilde{\rho}_{24} - \tilde{\rho}_{42}\right) + \Gamma_{32}\rho_{33} - \Gamma_{21}\rho_{22} \tag{3.9}$$

$$\dot{\tilde{\rho}}_{21} = \frac{i\Omega_{c1}}{2}\left(\rho_{22} - \rho_{11}\right) - \frac{i\Omega_{c2}}{2}\tilde{\rho}_{31} - \frac{i\Omega_p}{2}\tilde{\rho}_{41} + (i\Delta_{c1} - \gamma_{21})\tilde{\rho}_{21} \tag{3.10}$$

$$\dot{\tilde{\rho}}_{23} = \frac{i\Omega_{c2}}{2}\left(\rho_{22} - \rho_{33}\right) - \frac{i\Omega_{c1}}{2}\tilde{\rho}_{13} - \frac{i\Omega_p}{2}\tilde{\rho}_{43} + (-i\Delta_{c2} - \gamma_{32})\tilde{\rho}_{23} \tag{3.11}$$

$$\dot{\tilde{\rho}}_{24} = \frac{i\Omega_p}{2}(\rho_{22} - \rho_{44}) - \frac{i\Omega_{c1}}{2}\tilde{\rho}_{14} - \frac{i\Omega_{c2}}{2}\tilde{\rho}_{34} + (-i\Delta_p - \gamma_{42})\tilde{\rho}_{24} \tag{3.12}$$

$$\dot{\rho}_{11} = \frac{i\Omega_{c1}}{2}\left(\tilde{\rho}_{12} - \tilde{\rho}_{21}\right) + \Gamma_{21}\rho_{22} \tag{3.13}$$

$$\dot{\tilde{\rho}}_{12} = \frac{i\Omega_{c1}}{2}\left(\rho_{11} - \rho_{22}\right) + \frac{i\Omega_{c2}}{2}\tilde{\rho}_{13} + \frac{i\Omega_p}{2}\tilde{\rho}_{14} + (i\Delta_{c1} - \gamma_{21})\tilde{\rho}_{12} \tag{3.14}$$

$$\dot{\tilde{\rho}}_{13} = \frac{i\Omega_{c2}}{2}\tilde{\rho}_{12} - \frac{i\Omega_{c1}}{2}\tilde{\rho}_{23} + [-i(\Delta_{c1} + \Delta_{c2}) - \gamma_{31}]\tilde{\rho}_{13} \tag{3.15}$$

$$\dot{\tilde{\rho}}_{14} = \frac{i\Omega_p}{2}\tilde{\rho}_{12} - \frac{i\Omega_{c1}}{2}\tilde{\rho}_{24} + [-i(\Delta_p + \Delta_{c1}) - \gamma_{41}]\tilde{\rho}_{14} \tag{3.16}$$

(where, the frequency detuning of the probe and L_{c1}, L_{c2} coupling lasers from the relevant atomic transitions are respectively determined by $\Delta_p = \omega_p - \omega_{42}$, $\Delta_{c1} = \omega_{c1} - \omega_{21}$. In addition, suppose the initial atomic system is at a level $|2\rangle$ therefore: $\rho_{11} \approx \rho_{33} \approx \rho_{44} \approx 0, \rho_{22} = 1$.

Now, we analytically solve the density matrix equations under the steady-state condition by setting the time derivatives to zero:

$$\frac{d\rho}{dt} = 0\,, \tag{4}$$

Therefore the equations (3.2), (3.3) and (3.4), we have:

$$0 = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{42} - \frac{i\Omega_p}{2}\rho_{21} + [i(\Delta_{c1} + \Delta_p) - \gamma_{41}]\tilde{\rho}_{41} \tag{5.1}$$

$$0 = \frac{i\Omega_{c1}}{2}\tilde{\rho}_{41} + \frac{i\Omega_{c2}}{2}\tilde{\rho}_{43} + \frac{i\Omega_p}{2}(\rho_{44} - \rho_{22}) + (i\Delta_p - \gamma_{42})\tilde{\rho}_{42} \tag{5.2}$$

$$0 = \frac{i\Omega_{c2}}{2}\tilde{\rho}_{42} - \frac{i\Omega_p}{2}\tilde{\rho}_{23} + [i(\Delta_p - \Delta_{c2}) - \gamma_{43}]\tilde{\rho}_{43} \tag{5.3}$$

Because of $\Omega_p << \Omega_{c1}$ and Ω_{c2} so that we ignore the term $\frac{i\Omega_p}{2}\tilde{\rho}_{21}$ and $\frac{i\Omega_p}{2}\tilde{\rho}_{23}$ in the equations (4) and (5). Slove the equations (4) – (5), we have:

$$\tilde{\rho}_{42} = \frac{-i\Omega_p/2}{\gamma_{42} - i\Delta_p + \dfrac{\Omega_{c1}^2/4}{\gamma_{41} - i(\Delta_p + \Delta_{c1})} + \dfrac{\Omega_{c2}^2/4}{\gamma_{43} - i(\Delta_p - \Delta_{c2})}}\,, \tag{6}$$

3. Absorption coefficient of the atomic medium

We start from the susceptibility of atomic medium for the probe light that is determined by the following relation:

$$\chi = -2\frac{Nd_{21}}{\varepsilon_0 E_p}\rho_{21} = \chi' + i\chi''\,, \tag{7}$$

The absorption coefficient α of the atomic medium for the probe beam is determined through the imaginary part of the linear susceptibility (7):

$$\alpha = \frac{\chi''\omega_p}{c} = -\frac{\omega_p}{c}\frac{2N\mu_{42}^2}{\Omega_p\hbar\varepsilon_0}\mathrm{Im}(\tilde{\rho}_{42}) \tag{8}$$

We considere the case of ^{85}Rb atomic: γ_{42} = 3MHz, γ_{41} = 0.3MHz and γ_{43} = 0.03MHz, the atomic density $N = 10^{11}/\mathrm{cm}^3$. The electric dipole matrix element is $d_{42} = 2.54.10^{-29}$ Cm, dielectric coefficient $\varepsilon_0 = 8.85.10^{-12}$ F/m, $\hbar = 1{,}05.10^{-34}$ J.s, and frequency of probe beam $\omega_p = 3.84.10^{14}$ Hz. Fixed frequency Rabi of coupling laser beam L_{c1} in value Ω_{c1} = 16MHz (correspond to the value that when there is no laser L_{c2} then the transparency of the probe beam near 100%) and the frequency coincides with the frequency of the transition $|1\rangle \leftrightarrow |2\rangle$, it means $\Delta_{c1} = 0$. Consider the case of the frequency deviation of the coupling laser beam L_{c2} is Δ_{c2} = 10MHz . We plot a three-dimensional graph of the absorption coefficient α at the intensity of the coupling laser beam L_{c2} (Rabi frequency Ω_{c2}) and and the frequency deviation of the probe laser beam L_p, the result is shown in Fig 2.

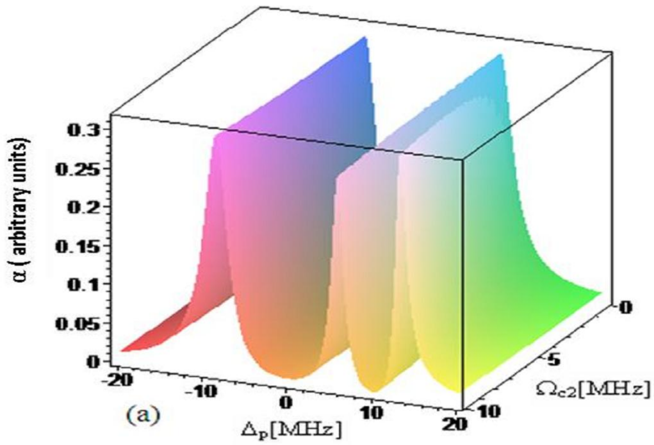

***Figure 2**. Three-dimensional graph of the absorption coefficient α according to Δp and Ωc2 with Δc1 = 0 MHz*

As shown in Fig 2, we see that when there is no coupling laser beam, it makes L_{c2} (Ω_{c2} = 0) then our model is only a three-step configuration [5], [6], we have only one transparent window at the resonant frequency of the probe laser beam. When presenting in the coupling laser beam L_{c2} (with the frequency deviation chosen is Δ_{c2} = 10MHz) and gradually increasing Rabi frequency Ω_{c2}, we see a window appear more during time the absorber envelopes at frequency deviation of probe beam Δ_p = 10MHz (satisfy the condition of two-photon resonance with the laser beam L_p and L_{c2} is $\Delta_p - \Delta_{c2} = 0$), and the depth and width of this transparent window also increases with the increase of Ω_{c2}.

To be more specific, we plot a two-dimensional graph of Figure 3 with some specific values of Rabi frequency Ω_{c2}.

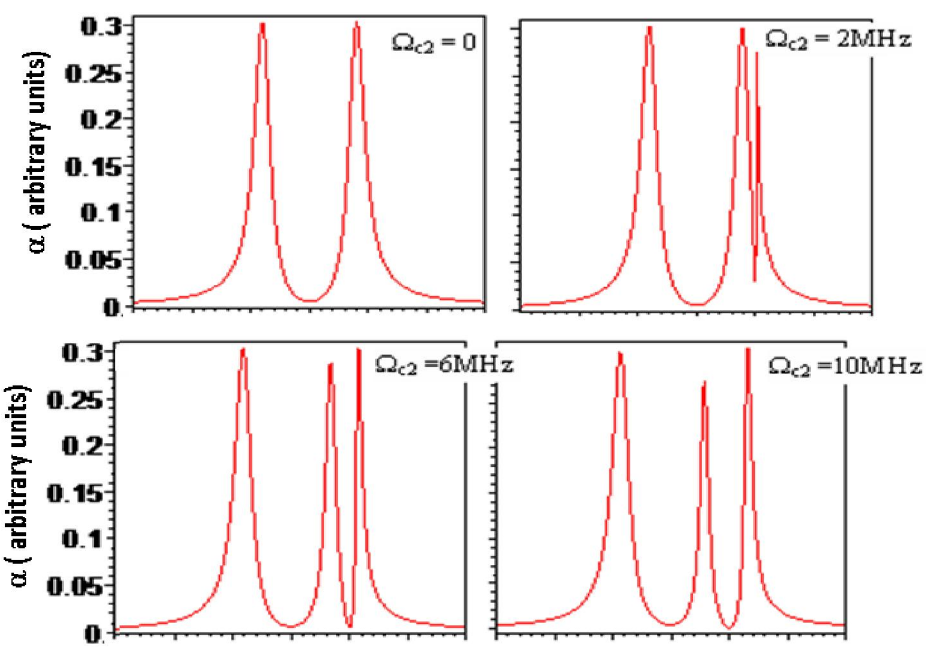

***Figure 3.** Two-dimensional graph* of *the absorption coefficient α according to* Ω_{c2} with Ω_{c1} *= 16MHz ,* Δ_{c1} *= 0 and* Δ_{c2} *= 10MHz.*

4. Conclusion

In the framework of the semi-classical theory, we have cited the density matrix equation for the ^{85}Rb atomic system in the Y-configuration under the simultaneous effects of two laser probe and coupling beams. Using approximate rotational waves and approximate electric dipoles, we have found solutions in the form of analytic for the absorption coefficient of atoms when the probe beam has a small intensity compared to the coupling beams. Drawing the absorption coefficient expression will facilitate future research applications. Consequently, we investigated the absorption of the probe beam according to the intensity of the coupling beam Ω_{c1}, Ω_{c2} and the deviation of the probe beam Δ_p. The results show that a Y-configuration appears two transparent windows for the probe laser beam. The depth and width or position of these windows can be altered by changing the intensity or frequency deviations of the coupling laser fields.

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