A VECTOR MODEL FOR GRAVITATION FIELD

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ABSTRACT: *In this paper, we introduce a vector model to describe gravitational field. The model is based on the assumption that the gravitational mass is Lorentz invariant . We introduce a non- relativistic equation system to describe the gravitational field, then we generalize it to obtain a relativistic equation system .*

1. INTRODUCTION

It is known that free fall acceleration (gravitation acceleration) is the same for all bodies at the same observation point. This is the Galileo's law of free fall and its generalization is the proportion between inertial mass m_i and gravitational mass m_g of a body.

We also know that inertial mass m_i is dependen=-t on each inertial frame of reference in which it is measured. Is gravitational mass dependent on the frame of reference? The weak equivalent principle identifies m_g with m_i so m_g is also dependent on each inertial frame of reference.

In fact, all experiments confirming proportion between inertial and gravitational mass were realized only for macro objects at rest and micro particles which slowly moved. There are not any experiments which confirm that gravitational mass is dependent on velocity.

In this model we recognize that gravitational mass is Lorentz invariant and also has two signs as electrical charge. A question is arisen that whether inertial mass of a particle with negative gravitational mass is positive or negative .

The weak equivalent principle affirms that it is negative. But if there were some form of matter with a negative inertial mass, how would it behave? The first insight is that it would move in a direction opposite to that in which it is pushed. So, if we have a body with the negative inertial mass, and we push it (i.e. apply a force) to the right, it will move (i.e. accelerate) to the left. The harder we try to push it to the right, the more fervently it would move to the left. This is derived from Newton's Law, $F = m_i a$, which clearly shows that if a positive force, F, is applied to a negative inertial mass,- m_i, the acceleration,a, must be negative (i.e. opposite to the direction of force) for the equation to hold.

But, how would a negative inertial mass behave in the Earth's gravity?

Some may be surprised to know that a particle with a negative inertial mass , would fall down at the same rate as any particle with a positive inertial mass due to the simultaneous action of laws $F = m_i a$ and $F=-Gm_{g1}m_{g2}/r^2$.

A difficult problem arises for negative inertial mass is "paradox of negative inertial mass" in the theory of gravity as discussed by Bondi[6], Schiff[7] and Will[8]: Suppose that a body (with mass $m_{i1} < 0$) is brought close to a normal body (with mass $m_{i2} > 0$) .According to above equations, the body with the positive inertial mass (m_{i2}) would attract the body with the negative inertial mass (m_{i1})whereas the body m_{i1} would repel the body m_{i2}. The pair (a "gravitational dipole")would accelerate itself off, without any outside help and use of propulsion! The conservation law of momentum and that of energy would all be violated.

To get rid of this "negative inertial- mass paradox", a "positive- energy theorem" was posed in the middle of 1960s saying that the total asymptotically determined mass of any isolated body in general relativity (GR) must be nonnegative. This theorem had been proved since 1979 in a variety of ways and is in total conformity with Einstein's equation $E=m_ic^2$, because the observed inertial mass and energy are always positive definite.

This model does not originate from the equivalence principle, therefore it is not constrained by the fact that a particle with negative gravitational mass must has negative inertial mass. However, if negative inertial mass exists really (the possibility of existence of negative inertial mass also comes from the mass- energy equivalence $E = m_ic^2$ when we know that gravitational energy is negative[10]), " negative inertial mass paradox " can also avoid by considering a basic symmetry between m_i and $-m_i$ as is suggested by Guang-Jiong Ni[9]: he suggested a generalization of Newton's gravitation law into the following form :

$$F = \pm G m_{g1} m_{g2} / r^2$$

Where the minus sign holds for m_{g1} and m_{g2} with the same sign whereas -the plus sign holds for m_{g1} and m_{g2} with different sign.

With this generalization, we regard the difference between positive and negative inertial masses merely relative. We merely perform a symmetry transformation $m_i \to -m_i$ to show the equal existence of the positive gravitational mass versus negative gravitational mass, but eventually there is no negative inertial mass at all.

On the other hand, if a particle with negative gravitational mass has positive inertial mass , it will "fall up" in the earth 's gravitational field. Why do we find no any particle with negative gravitational mass in nature? There are some directions which show that antimatter has negative gravitational mass.

There is no direct experimental evidence about the nature of the gravitational interaction between matter and antimatter , although it is commonly agreed that antimatter has the same gravitational properties as ordinary matter. The various theoretical difficulties that led to the early rejection of the idea of antigravity have been critically reviewed by Nieto et al[11].

The idea of universe with matter and antimatter domains was studied by Brown and Stecker[12].These authors suggested that grand unified field theories with spontaneous symmetry breaking the early big bang could lead more naturally to a baryon- symmetric cosmology with a domain structure than to a baryon-asymmetric cosmology. Alfven also studied a similar cosmological model[13].

Guang-Jiong Ni[9] also proposed a cosmological model with matter and antimatter clusters kept apart by their mutual repulsive gravitational interaction for explanation of the recently observed expansion of the universe.

A universe with matter and antimatter clusters implies the possibility of large scale annihilation resulting in gamma ray bursts. The precise location of the sources of Gamma Ray Bursts (GRB) and rough estimations of total emitted energy in 1998 has showed that the energy liberated in one recent event was of the order of the rest mass of two stars with the size of the sun[14,15]. Events with a duration of 30 ms to 1.6 hours are observed daily , and appear to occur at cosmologically large distances ($> 10^9$ light years). There are few plausible processes capable of liberating such an energy density. Besides matter – antimatter annihilation, collisions between high density bodies such as neutron stars seem also good candidates to explain GRBs. It has been argued that the weakness of the 0.5 MeV line due to electron-positron annihilation in gamma – ray spectra rules out large scale annihilation. But this line should only be expected for the annihilation of particles at rest. Perhaps it can be argued that few particles would be at rest when a star and an anti-star collide. The annihilation of relativistic particles yields continuous gamma ray spectra.

One of the main problems of modern cosmology is that the existence of virtual particle-antiparticle pairs in vacuum leads to an extremely large cosmological constant which does not fit observations. The hypothesis of antigravity might solve this problem as the gravitational field of virtual particles would be compensated by that of virtual antiparticles. The cosmological constant would be zero and the accelerated expansion of the universe would be explained by the mutual repulsion between matter and antimatter. Ripalda [16], Chardin [17] have also noted that repulsive gravity would lead to a cosmological constant of the correct order.

On the other hand, CP violation in neutral Kaon decay and similar experiments have brought up the idea that perhaps time reversal is not one of the fundamental symmetries of nature. Chardin [18,19] has proposed that CP violation in neutral Kaon decay might be explained by the hypothesis of a repulsive effect of the earth's gravitational field on antiparticles.

Gr∅n [20] shows that Poincare stresses are explained as due to vacuum polarization in connection with a recently presented class of electromagnetic mass models in general relativity. The gravitational blue-shift of light, noted in an earlier solution of the Einstein-Maxwell equations, is explained as due to repulsive gravitation produced by the negative gravitational mass of the polarized vacuum. It is pointed out that the electron model of Lopez, which includes spin, and which is a source of the Kerr- Newmann field, gives rise to repulsive gravitation.

Tsvi Piran[21] shows that small negative fluctuations – small dimples in the primordial density field-that act as if they have an effective negative gravitational mass , play a dominant role in shaping our Universe. These initially tiny perturbations repel matter surrounding them, expand and grow to become voids in the galaxy distribution. These voids – regions with a diameter of 40 h^{-1} Mpc which are almost devoid of galaxies - are the largest objects in the Universe.

According to Feymann, time flows backward for antiparticles (a consequence of T-symmetry) there the antigravity absence can mean (like in the case of weak interaction) the violation of T- invariance (the " time arrow" existence).

This complete paper consists of the following parts:

1/. A vector model for gravitational field.

2/. An approach to the equivalence principle and nature of inertial forces.

3/. An approach to the classical tests of GRT in a vector model of gravitational field.

2. QUANTITIES CHARACTERIZE FOR GRAVITATIONAL FIELD

2.1 The gravitational field strength

From expression of Newton's law of universal gravitation seen as static gravitational force between gravitational charges m_{g1} and m_{g2}

$$\vec{F} = -G_0 \frac{m_g^1 m_g^2}{r^3}\vec{r}$$

We define gravitational field strength denoted by $\vec{E}_g$ is generated by m_{g1} at a point M with radius vector $\vec{r}$ as follows:

$$\vec{E}_g(\vec{r}) \equiv \frac{\vec{F}}{m_g^2} = -\frac{m_g^1}{4\pi\varepsilon_g^0}\vec{r}$$

Here we define: $G = \dfrac{1}{4\pi\varepsilon_g^0}$

We also define vector of gravitational induction as follows: $\vec{D}_g \equiv \varepsilon_g^0 \vec{E}_g$

2.2 Divergence of gravitational field strength

We have divergence of $\vec{D}_g$:

$$div\ \vec{D}_g = -\rho_g$$

Here ρ_g is density of gravitational charge

2.3 Density of gravitational current – gravitational current strength

Consider a charge m_g which moves with velocity v , we introduce following definitions :

- Density of gravitational current: $\vec{J}_g \equiv n_0 m_g \vec{v}$

Here n_0 is density of particles carrying gravitational charge m_g.

- Gravitational current strength: $I_g \equiv \int_s \vec{J}_g\, d\,\vec{s}$

Here S is a surface element that the current passes.

If gravitational charge is conserved as normal charges , we also have continuous equation :

$$div\ \vec{J}_g + \frac{\partial \rho_g}{\partial t} = 0$$

2.4 Magneto-gravitational field:

Due to gravitational charges are Lorentz invariant, when a gravitational charge moves, it also generates a field $\vec{B}_g$ similar to case that an electric charge moves which generates field $\vec{B}$. This is a consequence the special theory of relativity .

We call the field by term "magneto-gravitational field"

We also suppose that $\vec{B}_g$ has no sources ,i.e.:$\nabla \vec{B}_g = 0$

We also define vector $\vec{H}_g$ as follows : $\vec{H}_g \equiv \frac{1}{\mu_g}\vec{B}_g$

3.A SYSTEM OF AXIOMS

We recognize following axioms:

- Gravitational charge is Lorentz invariant.Gravitational charge is conserved i.e. we recognize continuous equation: $div\ \vec{J}_g + \frac{\partial \rho_g}{\partial t} = 0$
- Magneto-gravitational field $\vec{B}_g$ exists and $\vec{B}_g$ satisfies:$\nabla \vec{B}_g = 0$
- Wave front of gravitational field propagates with velocity of light c .

4. LAGRANGIAN AND NON- RELATIVISTIC EQUATIONS OF AVITATIONAL FIELD

In order to introduce Lagrangian , we shall use the generalized convolution product that L.N. TAO [5]has used for other fields .

4.1 Fundamentals of convolution product:

Generalized convolution product of two scalar– vector and vector – vector functions is defined as follows:

$$[g * \vec{A}](\vec{x}, t) \equiv \int_0^t g(\vec{x}, t-\tau)\vec{A}(\vec{x}, \tau)\, d\tau$$

$$[\vec{A}(\vec{x},t)\overset{*}{.}\vec{B}(\vec{x},t)] \equiv \int_0^t \vec{A}(\vec{x},t-\tau)\vec{B}(\vec{x},\tau)d\tau$$

We see easily that the generalized convolution product also possesses- properties as: commutative , associative and distributive property .

4.2 Lagrangian and non – relativistic equations of gravitational field

We recognize the following action :

$$I(G)=\int_V \begin{array}{l}[-1*(\nabla*\vec{H}_g)*\vec{E}_g+1*\vec{J}_g*\vec{E}_g-\vec{D}_g*\vec{E}_g-\vec{B}_g*\overrightarrow{H}_g+\vec{D}_g^0*\vec{E}_g \\ +\vec{B}_g^0*\overrightarrow{H}_g-\dfrac{1}{26_g}\vec{J}_g*\vec{J}_g++\dfrac{1}{2\varepsilon_g}\vec{D}_g*\vec{D}_g+\dfrac{1}{2\mu_g}\vec{B}_g*\vec{B}_g](\vec{x},t)dV\end{array}$$

Here σ_g is a constant, V is a space region in which we investigate gravitational field. From δI(G) =0 , we obtain following system of non – relativistic equations to describe gravitational field after some simple calculations :

$$\nabla\times\vec{E}_g=\frac{\partial\vec{B}_g}{\partial t} \qquad (1)$$

$$\nabla\times\vec{H}_g=\vec{J}_g-\partial\vec{D}_g/\partial t \qquad (2)$$

$$\nabla\vec{D}_g=-\rho_g \qquad (3)$$

$$\nabla\vec{B}_g=0 \qquad (4)$$

$$\vec{J}_g=6_g\vec{E}_g \qquad (5)$$

5. RELATIVISTIC EQUATIONS OF GRAVITATIONAL FIELD

In order to write the field equations in 4- dimensional form ,we introduce gravitational potentials as follows :

Vector potential : $\vec{B}_g=curl\vec{A}_g$ (6)

Scalar potential : $\vec{E}_g=-grad\varphi_g+\dfrac{\partial\vec{A}_g}{\partial t}$ (7)

In 4 - dimensional manifold of CARTERSIAL coordinates:

$$x^0=ct;x^1=x;x^2=y;x^3=z$$

We introduce:

4-dimensional potential vector :

$$A_g^k=(A_g^0,\vec{A}_g)\text{ with }A_g^0=-\frac{\varphi_g}{c};A_g^1=A_{gx};A_g^2=A_{gy};A_g^3=A_{gz}$$

4 - dimensional current density vector :

$$J_g^k\equiv(J_g^0,\vec{J}_g)\text{ with }J_g^0=\rho_g c;J_g^1=\rho_g v_x;J_g^2=\rho_g v_y;J_g^3=\rho_g v_z$$

The non- relativistic equations (1),(2),(3),(4),(5) can be generalized to relativistic form as follows :

The first group: $$\nabla \vec{D}_g = -\rho_g \qquad \nabla \times \vec{H}_g = \rho_g \vec{v} - \frac{\partial \vec{D}_g}{\partial t}$$ becomes $\partial_i D_g^{ik} = J_g^k$

with:
$$D_g^{ik} = \begin{pmatrix} 0 & cD_{gx} & cD_{gy} & cD_{gz} \\ -cD_{gx} & 0 & -H_{gz} & H_{gy} \\ -cD_{gy} & H_{gz} & 0 & -H_{gx} \\ -cD_{gz} & -H_{gy} & H_{gx} & 0 \end{pmatrix}$$

The second group: $$\nabla \vec{B}_g = 0 \qquad \nabla \times \vec{E}_g = \frac{\partial \vec{B}_g}{\partial t}$$ becomes $\partial_k E_{gmn} + \partial_m E_{gnk} + \partial_n E_{gkm} = 0$

With :
$$E_{gik} = \begin{pmatrix} 0 & -\frac{E_{gx}}{c} & -\frac{E_{gy}}{c} & -\frac{E_{gz}}{c} \\ \frac{E_{gx}}{c} & 0 & -B_{gz} & B_{gy} \\ \frac{E_{gy}}{c} & B_{gz} & 0 & -B_{gx} \\ \frac{E_{gz}}{c} & -B_{gy} & B_{gx} & 0 \end{pmatrix}$$

The third group: $$\vec{D}_g = \varepsilon_g \vec{E}_g \qquad \vec{H}_g = \frac{1}{\mu_g} \vec{B}_g$$ becomes $D_g^{ik} = \varepsilon^{im} \varepsilon^{kn} E_{gmn}$

with : $$\varepsilon^{ik} = \frac{1}{\sqrt{\mu_g}} diag(1,-1,-1,-1)$$

6. CONCLUSION

Thus, with assumption that the gravitational mass is Lorentz invariant, we introduced a vector model to describe gravitational field. We obtained a non-relativistic equations system and then generalized it to obtain a relativistic equation system.

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TÓM TẮT: *Trong bài báo này, chúng tôi đưa vào một mô hình véctơ để diễn tả trường hấp dẫn. Mô hình này được dựa trên giả thuyết rằng khối lượng hấp dẫn là bất biến Lorentz. Chúng tôi đưa vào một hệ phương trình phi tương đối tính để mô tả trường hấp dẫn, kế đến tương đối tính hóa nó để thu được một hệ phương trình tương đối tính*

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