# AN APPROACH TO THE EQUIVALENCE PRINCIPLE AND THE NATURE OF INERTIAL FORCES

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*(Manuscript Received on December 19th, 2005, Manuscript Revised March 2 th, 2006)*

***ABSTRACT**: In this paper we prove that inertial forces which exist in non inertial systems of reference are just gravitational forces. Thus the Equivalence Principle is a consequence of this model.*

## 1.INTRODUCTION.

It is known that inertial forces only exist in non inertial systems of reference. They were discovered from very long time ago but their nature was unclear. Main viewpoints of inertial forces are as follows [1,2,3,4,5]:

Newton's viewpoint: we see clearly Newton's viewpoint of inertial forces by means of his discussion of the rotation of water basin. Newton believed that inertial forces only existed in systems which were accelerated with respect to his absolute space.

Mach's viewpoint: Mach opposed Newton's viewpoint of the absolute space and believed that inertial forces only existed when systems were accelerated with respect to all matters of universe. He thought that inertial forces were just gravitational forces caused by all distance matter but did not point out by which way they acted on.

Einstein's viewpoint: Einstein recognized that inertial force field was equivalent with gravitational force field by the Equivalence Principle.

In this model we shall point out that inertial forces are just gravitational forces.

## 2. QUASI-EQUIPOTENTIAL SPACE.

We consider a space region in which only consists of gravitational charges with the same signs, say , positive sign. Static gravitational potential generated by all gravitational charges of this region at a point M is :

$$\varphi_g(M) = -\sum_i \frac{Gm_{gi}}{r_i}$$

Where $r_i$ is distance from $m_{gi}$ to M. Because this region's all gravitational charges have the same signs , so that $\varphi_g(M) \neq 0$.

If this region 's gravitational charges are distributed homogeneous and isotropic , we can believe that $\varphi_g(M)$= constant in this region. Our observed universe can be considered as a quasi-equipotential space with the background gravitational potential $\varphi_{g0}(M)$= constant, due to recent observations point out that it is flat[6,7,8,9].

## 3. AN APPROACH TO NATURE OF INERTIAL FORCES.

The background gravitational potential of our observed universe is $\varphi_{g0}$= constant.

An observer A is fixed with respect to our universe ,the gravitational potentials in his system are :

$$A \begin{cases} \varphi_g = \varphi_{go} \\ \vec{A}_g = 0 \end{cases}$$

An different observer B stands in a system which moves with velocity V with respect to A on X- direction ( V is measured by A ). Gravitational potentials in system of B, following the Lorentz 's transformation , are :

$$B\begin{cases}\varphi'_g = \dfrac{\varphi_g}{\sqrt{1-\beta^2}} \\ A'_{gx} = \dfrac{v}{c^2}\dfrac{\varphi_{g0}}{\sqrt{1-\beta^2}} \\ A'_{gy} = \mathrm{A}'_{gz} = 0\end{cases}$$

from above formulas, we have :

$$\begin{cases}\vec{E}'_g = -grad'\varphi'_g + \dfrac{\partial \vec{A}'_g}{\partial t'} \\ \vec{B}'_g = curl'\vec{A}_g'\end{cases}$$

If B is fixed or moves uniformly in a straight line with respect to A , we have :

$$\begin{cases}\vec{E}'_g = -grad'\varphi_g' + \dfrac{\partial \vec{A}_g'}{\partial t'} = 0 \\ \vec{B}'_g = curl'\vec{A}_g' = 0\end{cases}$$

because V does not depend on t .

If B is accelerated with respect to A ,i.e. : $v = \dfrac{at}{\sqrt{1+\dfrac{a^2t^2}{c^2}}}, v_o = 0$ we have :

$$\boldsymbol{E'_{gx} = -\frac{\partial \varphi'_g}{\partial x'} + \frac{\partial A'_{gx}}{\partial t'},\ E'_{gy} = E'_{gz} = 0}$$

$$\boldsymbol{with : \frac{\partial}{\partial x'} = \frac{\partial x}{\partial x'}.\frac{\partial}{\partial x} + \frac{\partial t}{\partial x'}.\frac{\partial}{\partial t}; \frac{\partial}{\partial t'} = \frac{\partial x}{\partial t'}.\frac{\partial}{\partial x} + \frac{\partial t}{\partial t'}.\frac{\partial}{\partial t}}$$

and $$x = \frac{x'+vt}{\sqrt{1-\dfrac{v^2}{c^2}}}; t = \frac{t' + \dfrac{v}{c^2}x}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Put $E'_{gx} = -A + B$ (1)

Where $A \equiv -\dfrac{\partial \varphi'_g}{\partial x'} = (\dfrac{\partial x}{\partial x'}\dfrac{\partial \varphi'_g}{\partial x} + \dfrac{\partial t}{\partial x'}\dfrac{\partial \varphi'_g}{\partial t}) = C + D$ (2)

With $C \equiv \frac{\partial x}{\partial x'}\frac{\partial \varphi'_g}{\partial x} = (1-\frac{v^2}{c^2})^{-1/2}.0 \ = 0$ due to $\varphi'_{gx}$ is independent with respect to x.

And $D = \frac{\partial t}{\partial x'}\frac{\partial \varphi'_g}{\partial t} = \frac{v}{c^2}(1-\frac{v^2}{c^2})^{-1/2}\frac{\partial \varphi'_g}{\partial v}\frac{\partial v}{\partial t}$ (3)

We have $\frac{\partial \varphi'_g}{\partial v} = \frac{\partial}{\partial v}(1-\frac{v^2}{c^2})^{-1/2}.\varphi_{g0} = -\frac{1}{2}(1-\frac{v^2}{c^2})^{-3/2}.(-2\frac{v}{c^2}).\varphi_{g0}$

$$= \varphi_{g0}\frac{v}{c^2}(1-\frac{v^2}{c^2})^{-3/2} \quad (4)$$

And $\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}[at.(1+a^2t^2/c^2)^{-1/2}] = a.(1+a^2t^2/c^2)^{-1/2} + (-\frac{1}{2}).(1+a^2t^2/c^2)^{-3/2}.\frac{2a^2t}{c^2}at$

$$= a.(1+a^2t^2/c^2)^{-3/2}[(1+a^2t^2/c^2)^{-3/2}[1+a^2t^2/c^2 - a^2t^2/c^2]$$

$$= a.(1+\frac{a^2t^2}{c^2})^{-3/2} \quad (5)$$

Thus A becomes :

$$A = D = \frac{v}{c^2}(1-\frac{v^2}{c^2})^{-1/2}.\varphi_{g0}.\frac{v}{c^2}(1-\frac{v^2}{c^2})^{-3/2}.a(1+\frac{a^2t^2}{c^2})^{-3/2}$$

$$= \frac{\varphi_{g0}}{c^2}.a.(1-\frac{v^2}{c^2})^{-2}(1+\frac{a^2t^2}{c^2})^{-3/2}\frac{v^2}{c^2} \quad (6)$$

Then we account B:

$$B = \frac{\partial A'_{gx}}{\partial t'} = \frac{\partial x}{\partial t'}\frac{\partial A'_{gx}}{\partial x} + \frac{\partial t}{\partial t'}\frac{\partial A'}{\partial t} = E + F$$

Where: $E = \frac{\partial t}{\partial x'}\frac{\partial A'_{gx}}{\partial x} = v.(1-\frac{v^2}{c^2})^{-1/2}.0 = 0$ due to $A'_{gx}$ also is independent with respect to x.

And:

$$F \equiv \frac{\partial t}{\partial t'}\frac{\partial A'_{gx}}{\partial t} = (1-\frac{v^2}{c^2})^{-1/2}\frac{\partial A'_{gx}}{\partial v}\frac{\partial v}{\partial t} = a.(1-\frac{v^2}{c^2})^{-1/2}(1+\frac{a^2t^2}{c^2})^{-3/2}\frac{\partial A'_{gx}}{\partial v} \quad (7)$$

We have :

$$\frac{\partial A'_{gx}}{\partial v} = \frac{\partial}{\partial v}[\frac{v}{c^2}\varphi_{g0}(1-\frac{v^2}{c^2})^{-1/2}] = \frac{\varphi_{g0}}{c^2}[(1-\frac{v^2}{c^2})^{-1/2} + (-\frac{1}{2}).((1-\frac{v^2}{c^2})^{-3/2}.(-2\frac{v}{c^2})v]$$

$$= \frac{\varphi_{g0}}{c^2}[(1-\frac{v^2}{c^2})^{-1/2} + \frac{v^2}{c^2}(1-\frac{v^2}{c^2})^{-3/2}] = \frac{\varphi_{g0}}{c^2}(1-\frac{v^2}{c^2})^{-3/2}[1-\frac{v^2}{c^2}+\frac{v^2}{c^2}]$$

$$= \frac{\varphi_{g0}}{c^2}(1-\frac{v^2}{c^2})^{-3/2} \quad (8)$$

Thus

$$B=\frac{\partial A'_{gx}}{\partial t'}=F=a.(1+\frac{a^2t^2}{c^2})^{-3/2}(1-\frac{v^2}{c^2})^{-3/2}\frac{\varphi_{g0}}{c^2}(1-\frac{v^2}{c^2})^{-1/2} \qquad (9)$$

From (1),(6),(9), we have:

$$E'_{gx}=-A+B=-\frac{\varphi_{g0}}{c^2}.a.(1-\frac{v^2}{c^2})^{-2}(1+\frac{a^2t^2}{c^2})^{-3/2}\frac{v^2}{c^2}+$$

$$+a.(1+\frac{a^2t^2}{c^2})^{-3/2}(1-\frac{v^2}{c^2})^{-3/2}\frac{\varphi_{g0}}{c^2}(1-\frac{v^2}{c^2})^{-1/2}$$

$$=\frac{\varphi_{g0}}{c^2}.a.(1-\frac{v^2}{c^2})^{-2}(1+\frac{a^2t^2}{c^2})^{-3/2}[1-\frac{v^2}{c^2}]$$

$$=\frac{\varphi_{g0}}{c^2}.a.(1-\frac{v^2}{c^2})^{-1}(1+\frac{a^2t^2}{c^2})^{-3/2} \qquad (10)$$

From

$$v=\frac{at}{\sqrt{1+\frac{a^2t^2}{c^2}}} \quad \text{so that} \quad 1-\frac{v^2}{c^2}=(1+\frac{a^2t^2}{c^2})^{-1}$$

(10) becomes:

$$E'_{gx}=\frac{\varphi_{g0}}{c^2}.a.(1-\frac{v^2}{c^2})^{-1/2}$$

We also have:

$\vec{B}'_g=0$ due to $\vec{A}'_g$ is independent with respect to x',y',z'.

At zero order of $v^2/c^2$, we have : $E'_{gx}=(\frac{\varphi_{g0}}{c^2})a$

Gravitational force acts on a particle with gravitational charge $m_g$ in system B is:

$$F'_{gx}=m_g\,E'_{gx}=(\frac{\varphi_{g0}}{c^2})m_g a \quad .$$

This force just is inertial force in system B when we recognize that :

$$F'_{gx}=(\frac{\varphi_{g0}}{c^2})m_g a=-m_i a$$

Or

$$\frac{m_i}{m_g}=-\frac{\varphi_{g0}}{c^2}$$

From experiments , we have : $\frac{m_i}{m_g}\cong 1$ so $\frac{\varphi_{g0}}{c^2}\cong -1$.

Thus gravitational field exists in non inertial systems of reference is :

$$E'_{gx} \cong -\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.a$$

Where a is acceleration of system.

## 4.DISCUSIONS

From above results we find that Newton, Mach and Einstein's viewpoints of inertial forces are satisfied in this model.

Firstly, in this model inertial forces ( uniform gravitational forces) exist only in systems which are accelerated with respect to "the background gravitational potential font" of universe $\varphi_{g0}$. Whether this font is just "the absolute space of Newton"! However the background font is not actually absolute but vanishes if all matters do not exist.

Secondly, in this model inertial forces are just uniform gravitational forces which are caused by all matter of universe by means of the background font $\varphi_{g0.}$ This satisfies Mach 's viewpoint and it also points out way that all matters of universe cause inertial forces.

Finally, in this model inertial forces field are just uniform gravitational field as Einstein's the principle of equivalence. However in this model inertial forces also vanish at infinite when the background font vanishes. This uniform gravitational field is not equivalent to central gravitational field by tidal forces so that the principle of equivalence holds only for small regions of space. We also find that when the system of frame moves with velocity approaching c, inertial forces will be infinite.

***Acknowledgements:*** *We extend our thanks to the professors in laboratory of theoretical physics of University of Natural Sciences, VNU-HCM, especially to the professor NGUYEN NGOC GIAO for helpful remarks .*

## MỘT TIẾP CẬN ĐẾN NGUYÊN LÝ TƯƠNG ĐƯƠNG VÀ BẢN CHẤT CỦA CÁC LỰC QUÁN TÍNH

**Võ Văn Ớn**

Khoa Vật Lý-Đại Học Khoa Học Tự Nhiên – ĐHQG-HCM

***TÓM TẮT :*** *Trong bài báo này chúng tôi chứng minh rằng các lực quán tính tồn tại trong các hệ qui chiếu phi quán tính chính là các lực hấp dẫn.Như vậy nguyên lý tương đương là một hệ quả của mô hình này.*

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