Inverse dynamic analyzing of flexible link manipulators with translational and rotational joints

Bien Xuan Duong, My Anh Chu, and Khoi Bui Phan

***Abstract*— Inverse dynamic problem analyzing of flexible link robot with translational and rotational joints is presented in this work. The new model is developed from single flexible link manipulator with only rotational joint. The dynamic equations are built by using finite element method and Lagrange approach. The approximate force of translational joint and torque of rotational joint are found based on rigid model. The simulation results show the values of driving forces at joints of flexible robot with desire path and errors of joint variables between flexible and rigid models. Elastic displacements of end-effector are shown, respectively. There are remaining issues which need be studied further in future work because the error joints variables in algorithm to solve inverse dynamic problem of flexible with translational joint has not been mentioned yet.**

***Index Terms*—Inverse Dynamic , flexible link manipulator, translational joint, elastic displacements.**

1 INTRODUCTION

Dynamic analysis of mechanisms, especially robots, is very important. The dynamic equations of motion represent the behavior of system, so accurate modeling and equations are essential to successfully design of the control system. The analysis of robots considering the elastic characteristics of its members has been considerable attention in recent years. Flexibility in robots can affect position accuracy. Inverse dynamic of flexible robots is very essential for selecting the actuator and designing the proper control strategy. Most of the investigations on the dynamic modeling of robot manipulators with elastic arms have been confined to manipulators with only revolute joints.

In the literature, most of the investigations on the inverse dynamics of the flexible robot manipulator copies with manipulators constructed with only rotational joints [1-3]. Kwon and Book [1] present a single link robot which is described and modeled by using assumed modes method (AMM). Inverse equation is derived in a state space form from direct dynamic equations and using definitions concepts which are causal system, anti-causal system and Non-causal system. Based on these concepts, the time-domain inverse dynamic method was interpreted in the frequency-domain in detail by using the two sided Laplace transform in the frequency-domain and the convolution integral. This method is limited to linear system. Stable inversion method is studied for the same robot configuration but the nonlinear effect is taken into account [2]. An inversion-based approach to exact nonlinear output tracking control is presented. Non-causal inversion is incorporated into tracking regulators and is a powerful tool for control. Eliodoro and Miguel [3] propose a new method based on the finite difference approach to discretize the time variable for solving the inverse dynamics of the robot. This method is a non-recursive and non-iterative approach carried out in the time domain in contrast with methods previously proposed. By using either the finite element method (FEM) or AMM, some other authors consider the dynamic modeling and analysis of the flexible robots with translational joint [4-8]. Pan et al [4] presented a model R-P with FEM approach. The

Manuscript Received on July 13th, 2016. Manuscript Revised December 06th, 2016.

Bien Xuan Duong, My Anh Chu are with Military Technical Academy. Email: xuanbien82@yahoo.com.

Khoi Bui Phan is with Ha Noi University of Science and Technology, Ha Noi.

result is differential algebraic equations which are solved by using Newmark method. Al-Bedoor and Khulief [5] presented a general dynamic model for R-P robot based on FEM and Lagrange approach. They defined a concept which is translational element. The stiffness of translational element is changed. The prismatic joint variable is distance from origin coordinate system to translational element. The number of element is small because it is hard challenge to build and solve differential equations. Khadem [6] studied a three-dimensional flexible n-degree of freedom manipulator having both revolute and prismatic joint. A novel approach is presented using the perturbation method. The dynamic equations are derived using the Jourdain's principle and the Gibbs-Appell notation. Korayem [7] also presented a systematic algorithm capable of deriving equations of motion of N-flexible link manipulators with revolute-prismatic joints by using recursive Gibbs-Appell formulation and AMM. However, the inverse dynamics modeling and analysis of the generalized flexible robot constructed with translational joint has not been much mentioned yet.

The objective of the described work in what follows was to present surveying inverse dynamics problem of flexible link robot with translational and rotational joints. The Lagrange approach in conjunction with the finite element method is employed in deriving the equations of motion. Inverse dynamics problem of model with flexible link can be approximately solved based on model with rigid links. The forward kinematic, inverse kinematic and inverse dynamics of rigid model are used to find joints values from desire path and driving force and torque which are inputs data for flexible model problems. The force and torque of joints can be found in such a way that the end point of link 2 can track the desire path even though link 2 is deformed.

2 DYNAMIC MODELING

2.1 Dynamic model

In this work, we concern the dynamic model of two link flexible robot which motions on horizontal plane with translational joint for first rigid link and rotational joint for second flexible link to formulate the inverse dynamics problem. It is shown as Fig 1.

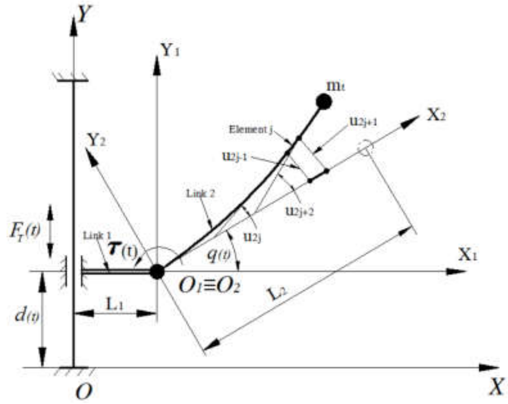

Figure 1. Flexible links robot with translational and rotational joints

The coordinate system XOY is the fixed frame. Coordinate system $X_1O_1Y_1$ is attached to end point of link 1. Coordinate system $X_2O_2Y_2$ is attached to first point of link 2. The translational joint variable $d(t)$ is driven by $F_T(t)$ force. The rotational joint variable $q(t)$ is driven by $\tau(t)$ torque. Both joints are assumed rigid. Link 1 with length L_1 is assumed rigid and link 2 with length L_2 is assumed flexibility. Link 2 is divided n elements. The elements are assumed interconnected at certain points, known as nodes. Each element has two nodes. Each node of element j has 2 elastic displacement variables which are the flexural (u_{2j-1}, u_{2j+1}) and the slope displacements (u_{2j}, u_{2j+2}). Symbol m_t is the mass of payload on the end-effector point. The coordinate $\mathbf{r}_{01}$ of end point of link 1 on XOY is computed as

$$\mathbf{r}_{01} = \begin{bmatrix} L_1 & d(t) \end{bmatrix}^T \tag{1}$$

The coordinate $\mathbf{r}_{2j}$ of element j on $X_2O_2Y_2$ can be given as

$$\mathbf{r}_{2j} = \begin{bmatrix} (j-1)l_e + x_j & w_j(x_j, t) \end{bmatrix}^T ; (x_j = 0..l_e) \tag{2}$$

Where, length of each element is $l_e = L_2/n$ and $w_j(x_j, t)$ is the total elastic displacement of element j which is defined by [10]

$$w_j(x_j, t) = \mathbf{N}_j(x_j)\mathbf{Q}_j(t) \tag{3}$$

The vector of shape function $\mathbf{N}_j(x_j)$ is defined as

$$\mathbf{N}_j\left(x_j\right)=\left[\boldsymbol{f}_1\left(x_j\right)\quad \boldsymbol{f}_2\left(x_j\right)\quad \boldsymbol{f}_3\left(x_j\right)\quad \boldsymbol{f}_4\left(x_j\right)\right] \quad (4)$$

Mode shape function $\boldsymbol{f}_i\left(x_j\right);(i=1...4)$ can be presented in [10]. The elastic displacement $\mathbf{Q}_j\left(t\right)$ of element j is given as

$$\mathbf{Q}_j\left(t\right)=\left[u_{2j-1}\quad u_{2j}\quad u_{2j+1}\quad u_{2j+2}\right]^T \quad (5)$$

The coordinate $\mathbf{r}_{21j}$ of element j on $X_1O_1Y_1$ can be written as

$$\mathbf{r}_{21j}=\mathbf{T}_2^1.\mathbf{r}_{2j} \quad (6)$$

Where, $\mathbf{T}_2^1=\begin{bmatrix}\cos q\left(t\right) & -\sin q\left(t\right)\\ \sin q\left(t\right) & \cos q\left(t\right)\end{bmatrix}$ is the transformation matrix from $X_2O_2Y_2$ to $X_1O_1Y_1$. The coordinate $\mathbf{r}_{02j}$ of element j on XOY can be computed as

$$\mathbf{r}_{02j}=\mathbf{r}_1+\mathbf{r}_{21j} \quad (7)$$

The elastic displacement $\mathbf{Q}_n\left(t\right)$ of element n is given as

$$\mathbf{Q}_n\left(t\right)=\left[u_{2n-1}\quad u_{2n}\quad u_{2n+1}\quad u_{2n+2}\right]^T \quad (8)$$

The coordinate $\mathbf{r}_{0E}$ of end point of flexible link 2 on XOY can be computed as

$$\mathbf{r}_{0E}=\begin{bmatrix}L_1+L_2\cos q\left(t\right)-u_{2n+1}\sin q\left(t\right)\\ d\left(t\right)+L_2\sin q\left(t\right)+u_{2n+1}\cos q\left(t\right)\end{bmatrix} \quad (9)$$

If assumed that robot with all of links are rigid, the coordinate $\mathbf{r}_{0E\_rigid}$ on XOY can be written as

$$\mathbf{r}_{0E\_rigid}=\begin{bmatrix}L_1+L_2\cos q\left(t\right)\\ d\left(t\right)+L_2\sin q\left(t\right)\end{bmatrix} \quad (10)$$

The kinematic energy of link 1 can be computed as

$$T_1=\frac{1}{2}m_1.\dot{\mathbf{r}}_{01}^2 \quad (11)$$

Where m_1 is the mass of link 1. The kinetic energy of element j is determined as

$$T_{2j}=\frac{1}{2}\int_0^{l_e}m_2\left[\frac{\partial\mathbf{r}_{02j}}{\partial t}\right]^2dx_j=\frac{1}{2}\dot{\mathbf{Q}}_{jg}^T\left(t\right)\mathbf{M}_j\dot{\mathbf{Q}}_{jg}\left(t\right) \quad (12)$$

Where m_2 is mass per meter of link 2. The generalized elastic displacement $\mathbf{Q}_{jg}\left(t\right)$ of element j is given as

$$\mathbf{Q}_{jg}\left(t\right)=\left[d\left(t\right)\quad q\left(t\right)\quad u_{2j-1}\quad u_{2j}\quad u_{2j+1}\quad u_{2j+2}\right]^T \quad (13)$$

Each element of inertial mass matrix $\boldsymbol{M}_j$ can be computed as

$$\mathbf{M}_j\left(s,e\right)=\int_0^{l_e}m_2\left[\frac{\partial\mathbf{r}_{02j}}{\partial Q_{js}}\right]^T\left[\frac{\partial\mathbf{r}_{02j}}{\partial Q_{je}}\right]dx_j;s,e=1,2,..,6 \quad (14)$$

Where Q_{js} and Q_{je} are the s^{th},e^{th} element of Q_{jg} vector. It can be shown that $\mathbf{M}_j$ is of the form

$$\mathbf{M}_j=\begin{bmatrix}m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16}\\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26}\\ m_{31} & m_{32} & & & & \\ m_{41} & m_{42} & & \mathbf{M}_{j\_base} & & \\ m_{51} & m_{52} & & & & \\ m_{61} & m_{62} & & & & \end{bmatrix} \quad (15)$$

Where,

$$\mathbf{M}_{j\_base}=\begin{bmatrix}\frac{13}{35}m_2l_e & \frac{11}{210}m_2l_e^2 & \frac{9}{70}m_2l_e & -\frac{13}{420}m_2l_e^2\\ \frac{11}{210}m_2l_e^2 & \frac{1}{105}m_2l_e^3 & \frac{13}{420}m_2l_e^2 & -\frac{1}{140}m_2l_e^3\\ \frac{9}{70}m_2l_e & \frac{13}{420}m_2l_e^2 & \frac{13}{35}m_2l_e & -\frac{11}{210}m_2l_e^2\\ -\frac{13}{420}m_2l_e^2 & -\frac{1}{140}m_2l_e^3 & -\frac{11}{210}m_2l_e^2 & \frac{1}{105}m_2l_e^3\end{bmatrix} \quad (16)$$

And,

$$m_{11}=m_2.l_e;m_{12}=-\frac{1}{12}m_2.l_e\begin{bmatrix}(6u_{2j-1}+6u_{2j+1}+l_eu_{2j}\\ -l_eu_{2j+2})\sin q+6l_e(1-2j)\cos q\end{bmatrix};$$

$$m_{13}=m_{15}=\frac{1}{2}m_2l_e\cos q;m_{14}=m_{16}=\frac{1}{2}m_2l_e^2\cos q;m_{21}=m_{12};$$

$$m_{23}=\frac{1}{20}m_2l_e^2(10j-7);m_{24}=\frac{1}{60}m_2l_e^3(5j-3);m_{25}=\frac{1}{20}m_2l_e^2(10j-3);$$

$$m_{22}=\frac{1}{210}m_2l_e\begin{pmatrix}210l_e^2j(j-1)+l_e^2(2u_{2j}^2-3u_{2j}u_{2j+2}+2u_{2j+2}^2)\\ +22l_e(u_{2j-1}u_{2j}-u_{2j+1}u_{2j+2})+13l_e(u_{2j}u_{2j+1}-u_{2j-1}u_{2j+2})\\ +78(u_{2j-1}^2+u_{2j+1}^2)+70l_e^2+54u_{2j-1}u_{2j+1}\end{pmatrix};$$

$$m_{26}=-\frac{1}{60}m_2l_e^3(5j-2);m_{31}=m_{13};m_{32}=m_{23};m_{41}=m_{14};m_{42}=m_{24};$$

$$m_{51}=m_{15};m_{52}=m_{25};m_{61}=m_{16};m_{62}=m_{26}$$

The total elastic kinetic energy of link 2 is yielded as

$$T_{dh}=\sum_{j=1}^{n}T_{2j}=\frac{1}{2}\dot{\mathbf{Q}}^T\left(t\right)\mathbf{M}_{dh}\dot{\mathbf{Q}}\left(t\right) \quad (17)$$

The inertial mass matrix $\mathbf{M}_{dh}$ is constituted from matrices of elements follow FEM theory, respectively. Vector $\mathbf{Q}\left(t\right)$ represents the generalized coordinate of system and is given as

$$\mathbf{Q}\left(t\right)=\left[d\left(t\right)\quad q\left(t\right)\quad u_1\quad .\quad .\quad u_{2n+1}\quad u_{2n+2}\right]^T \quad (18)$$

The kinetic energy of payload is given as

$$T_P = \frac{1}{2} m_t . \dot{\mathbf{r}}_{0E}^2 \quad (19)$$

The kinetic energy of system is determined as

$$T = T_1 + T_{dh} + T_P = \frac{1}{2} \dot{\mathbf{Q}}^T(t) \mathbf{M} \dot{\mathbf{Q}}(t) \quad (20)$$

Matrix $\mathbf{M}$ is mass matrix of system. The gravity effects can be ignored as the robot movement is confined to the horizontal plane. Defining E and I are Young's modulus and inertial moment of link 2, the elastic potential energy of element j is shown as P_j with the stiffness matrix $\mathbf{K}_j$ and presented as [10]

$$P_j = \frac{1}{2} \int_0^{l_e} EI \left[\frac{\partial^2 w_j(x_j, t)}{\partial x_j^2} \right]^2 dx_j = \frac{1}{2} \mathbf{Q}_j^T(t) \mathbf{K}_j \mathbf{Q}_j(t) \quad (21)$$

Where,

$$\mathbf{K}_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12EI}{l_e^3} & \frac{6EI}{l_e^2} & -\frac{12EI}{l_e^3} & \frac{6EI}{l_e^2} \\ 0 & 0 & \frac{6EI}{l_e^2} & \frac{4EI}{l_e} & -\frac{6EI}{l_e^2} & \frac{2EI}{l_e} \\ 0 & 0 & -\frac{12EI}{l_e^3} & -\frac{6EI}{l_e^2} & \frac{12EI}{l_e^3} & -\frac{6EI}{l_e^2} \\ 0 & 0 & \frac{6EI}{l_e^2} & \frac{2EI}{l_e} & -\frac{6EI}{l_e^2} & \frac{4EI}{l_e} \end{bmatrix} \quad (22)$$

The total elastic potential energy of system is yielded as

$$P = \sum_{j=1}^{n} P_j = \frac{1}{2} \mathbf{Q}^T(t) \mathbf{K} \mathbf{Q}(t) \quad (23)$$

The stiffness matrix $\mathbf{K}$ is constituted from matrices of elements follow FEM theory similar $\mathbf{M}$ matrix, respectively.

2.2 Dynamic equations of motion

Fundamentally, the method relies on the Lagrange equations with Lagrange function $L = T - P$ are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{Q}}(t)} \right) - \frac{\partial L}{\partial \mathbf{Q}(t)} = \mathbf{F}(t) \quad (24)$$

Vector $\mathbf{F}(t)$ is the external generalized forces acting on specific generalized coordinate $\mathbf{Q}(t)$ and is determined as

$$\mathbf{F}(t) = \begin{bmatrix} F_T(t) & \tau(t) & 0 & . & . & 0 & 0 \end{bmatrix}^T \quad (25)$$

Size of matrices $\mathbf{M}, \mathbf{K}$ is $(2n+4) \times (2n+4)$ and size of $\mathbf{F}(t)$ and $\mathbf{Q}(t)$ is $(2n+4) \times 1$. The rotational joint of link 2 is constrained so that the elastic displacements of first node of element 1 on link 2 can be zero. Thus variables u_1, u_2 are zero. By enforcing these boundary conditions and FEM theory, the generalized coordinate $\mathbf{Q}(t)$ becomes

$$\mathbf{Q}(t) = \begin{bmatrix} d(t) & q(t) & u_3 & . & . & u_{2n+1} & u_{2n+2} \end{bmatrix}^T \quad (26)$$

So now, size of matrices $\mathbf{M}, \mathbf{K}$ is $(2n+2) \times (2n+2)$ and size of $\mathbf{F}(t)$ and $\mathbf{Q}(t)$ is $(2n+2) \times 1$. When kinetic and potential energy are known, it is possible to express Lagrange equations as shown

$$\mathbf{M}(\mathbf{Q})\ddot{\mathbf{Q}} + \mathbf{C}(\mathbf{Q}, \dot{\mathbf{Q}})\dot{\mathbf{Q}} + \mathbf{D}\dot{\mathbf{Q}} + \mathbf{K}\mathbf{Q} = \mathbf{F}(t) \quad (27)$$

Where structural damping $\mathbf{D}$ and coriolis force $\mathbf{C}$ matrices are calculated as

$$\mathbf{C}(\mathbf{Q}, \dot{\mathbf{Q}})\dot{\mathbf{Q}} = \dot{\mathbf{M}}(\mathbf{Q})\dot{\mathbf{Q}} - \frac{1}{2}\left(\frac{\partial}{\partial \mathbf{Q}} (\dot{\mathbf{Q}}^T \mathbf{M}(\mathbf{Q}) \dot{\mathbf{Q}}) \right) \quad (28)$$

$$\mathbf{D} = \alpha \mathbf{M} + \beta \mathbf{K} \quad (29)$$

Where α and β are the damping ratios of the system which are determined by experience.

3 INVERSE DYNAMIC ANALYZING

Solving inverse dynamics problem can be computed a feed-forward control to follow a trajectory more accurately. Inverse dynamics of flexible robot is the process of determining load profiles to produce given displacement profiles as function of time. Forward dynamics of flexible robot is process of finding displacements given the loads. This is much simpler than inverse dynamics process because elastic displacements do not to know before if there are not external forces which effect on system. Unlike the rigid link, the inverse dynamics of flexible robot is more complex because of links deformations. We need to determine the force and torque of actuators in such a way that the end point of link 2 can still track the desire path even though link 2 is deformed. Inverse dynamics problem of model with flexible link can be approximately solved based on model with rigid links. Steps to solve are shown as Fig 2. The detail of blocks in Fig 2 is presented in Fig 3, Fig 4, Fig 5 and Fig 6.

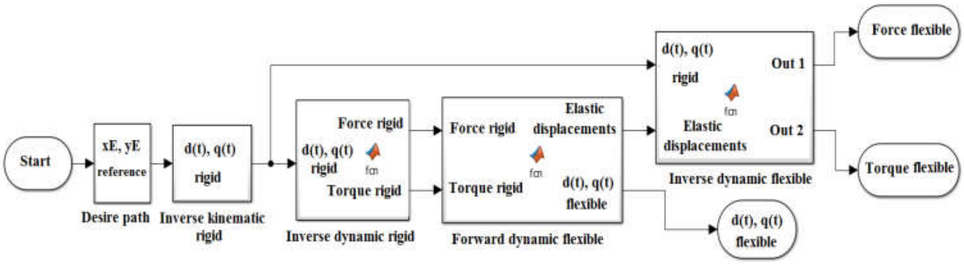

Figure 2. General diagram of inverse dynamic flexible robot algorithm

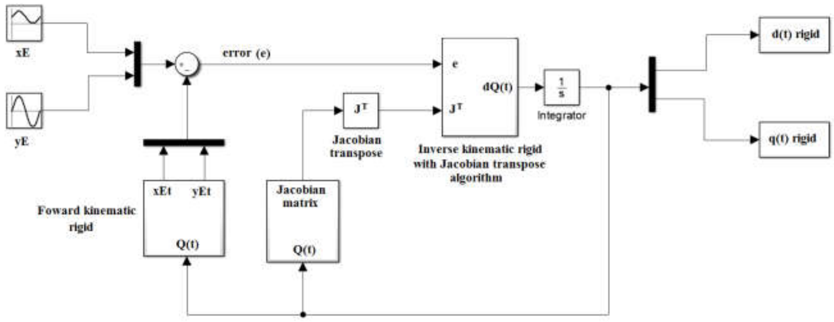

Figure 3. Diagram of inverse kinematic rigid model block

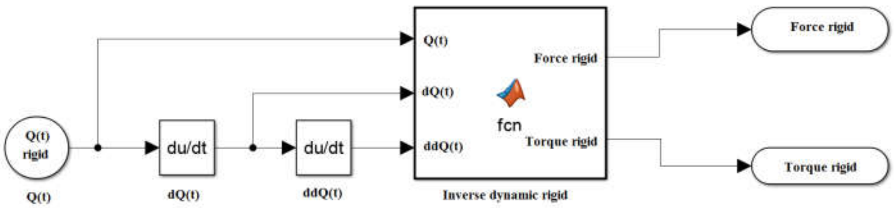

Fig. 4.Diagram of inverse dynamic rigid model block

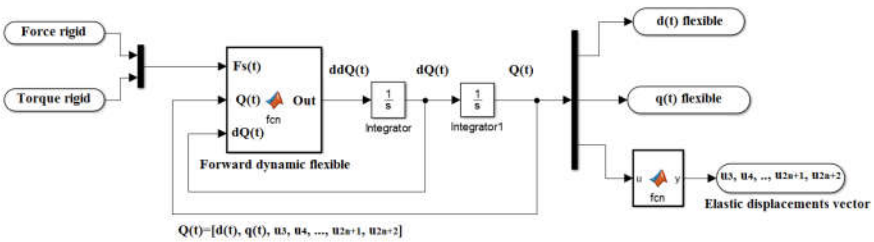

Figure 5. Diagram of forward dynamic flexible robot block

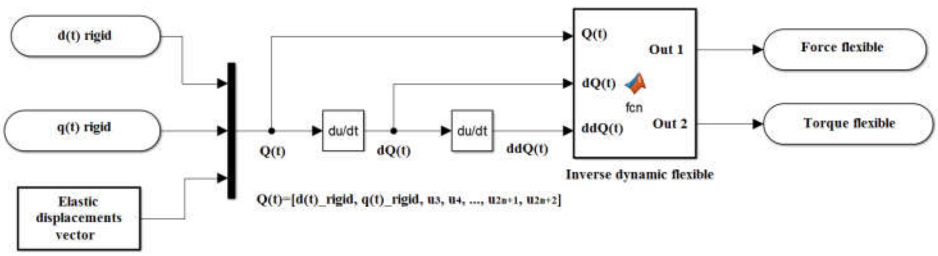

Figure 6. Diagram of inverse dynamic flexible robot block

Firstly, assuming that two link is rigid. The translational and rotational joints of rigid model are computed from desire path by solving inverse kinematic rigid problem [9] which is shown in Fig. 3. Then driving force and torque at joints of rigid model are computed by solving inverse dynamic rigid [9] (Fig. 4). Results are input data for forward dynamic flexible model follow equation (27) and are shown in Fig. 5. Finally, the approximates force and torque of joints are found by solving inverse dynamic flexible problem with inputs data which are joints values of rigid model and elastic displacements. It is presented by block in Fig. 6.

4 NUMERICAL SIMULATIONS

Simulation specifications of flexible model are given by Table 1.

TABLE 1
PARAMETERS OF DYNAMIC MODEL

Property	Symbol	Value
Length of link 1 (m)	L_1	0.05
Mass of link 1 and base (kg)	m_1	1.4
Parameters of link 2		
Length of link (m)	L_2	0.3
Width (m)	b	0.02
Thickness (m)	h	0.001
Number of element	n	5
Cross section area (m^2)	A=b.h	2.10^{-5}
Mass density (kg/m^3)	ρ	7850
Mass per meter (kg/m)	m=ρ.A	0.157
Young's modulus (N/m^2)	E	2.10^{10}
Inertial moment of cross section (m^4)	I=b.h^3/12	1.67×10^{-12}
Damping ratios	α, β	0.005;0.007
Mass of payload (g)	m_t	10
Desire path on workspace in OX axis (m)	xE	0.25-0.1sin(t-π/2)
Desire path on workspace in OX axis (y)	yE	0.1sin(t)
Time simulation (s)	T	2

Simulation results for inverse dynamic of flexible robot with translational and rotational joints are shown from Fig 7 to Fig 16. It is noteworthy to mention that we need to find the initial values of joints variable at t=0 when inverse kinematic of rigid model is solved.

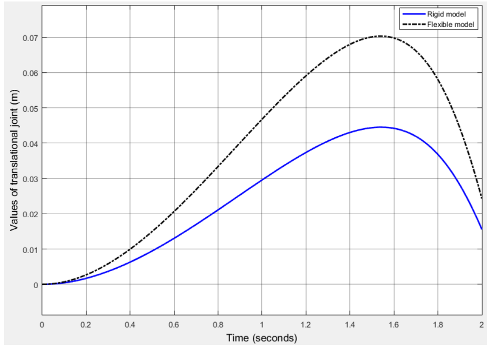

Figure 7. Translational joint values of rigid and flexible model

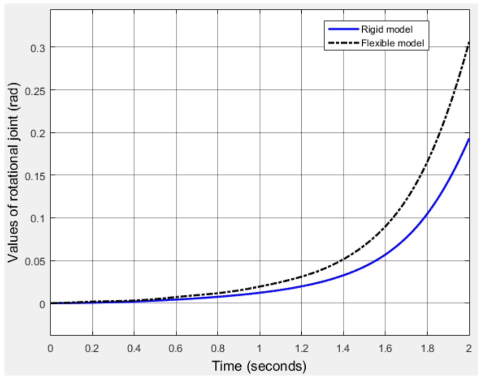

Figure 8. Rotational joint values of rigid and flexible model

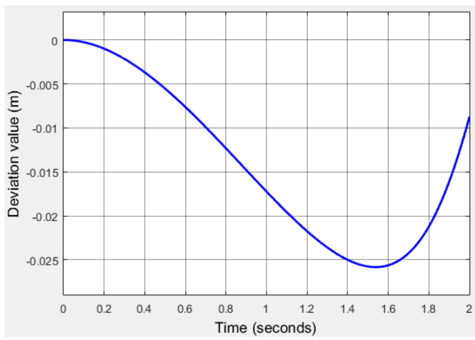

Figure 9. Deviation of translational joint variables between rigid and flexible model

Fig. 7 and fig. 8 show the values of joint variables between rigid and flexible model. Translational and rotational joints values are small because of short time simulation. Fig. 9 and fig. 10 describe deviation of these values. Maximum deviation value of translational joint is 25 mm and rotational joint variable is 0.17 rad. These deviations appear from effect of elastic displacements and error of numerical method which is used to solving problems.

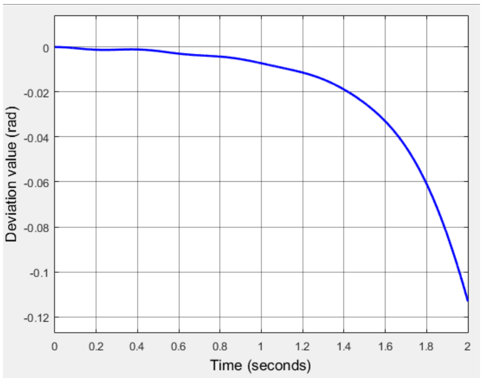

Figure 10. Deviation of rotational joint variables between rigid and flexible model

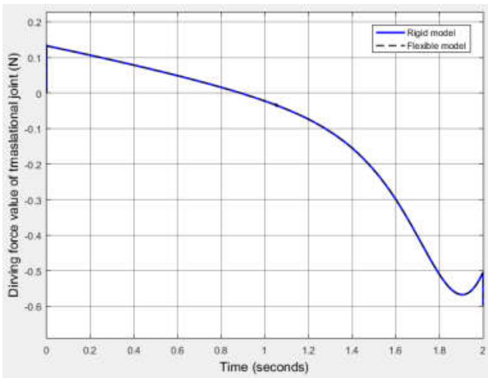

Figure 11. Driving force values of rigid and flexible model

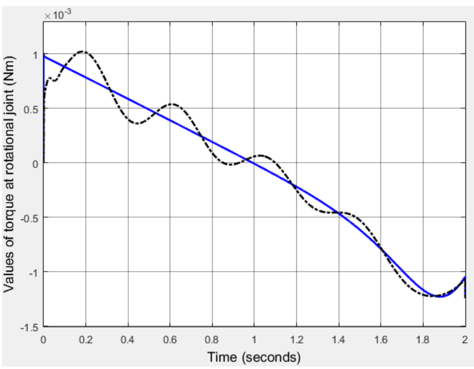

Figure 12. Driving torque values of rigid and flexible model

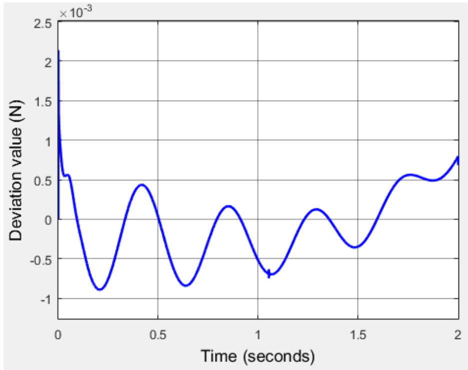

Figure 13. Deviation of driving force between rigid and flexible model

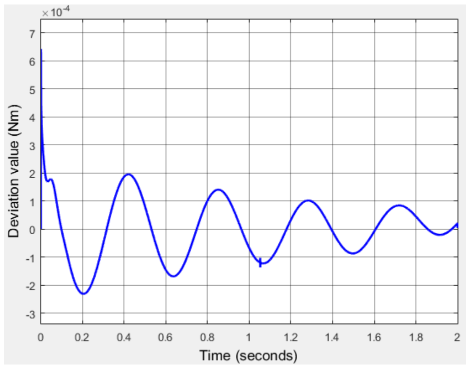

Figure 14. Deviation of driving force between rigid and flexible model

Fig. 10 to fig 14 present values of driving forces at joints and these deviations between rigid and flexible model. The values of driving force at translational joint are not too difference because first link of both models is assumed rigid. Maximum force is 0.6 N. Driving torque values at rotational joint are more difference because of effect of elastic displacements of flexible link.

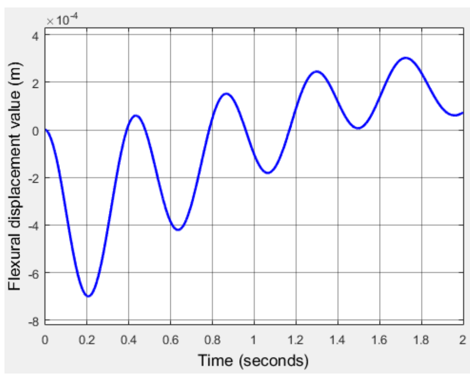

Figure 15. Flexural displacement value at end-effector point in flexible model

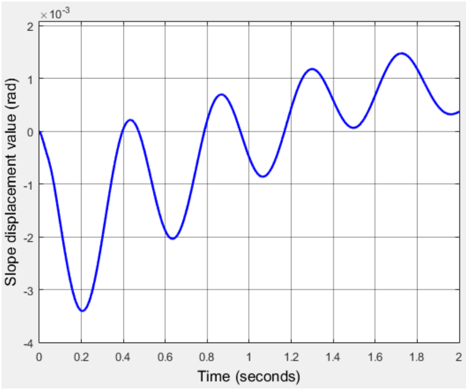

Figure 16. Slope displacement value at end-effector point in flexible model

Fig. 15 shows flexural displacement value at end-effect point. Maximum value is 0.7 mm. Fig. 16 shown slope displacement at end-effect point. Maximum value is 0.035 rad. Both values are small because of short time simulation and small values of joint variables.

In general, simulation results show that elastic displacements of flexible link effect on dynamic behaviors of system. Different between rigid model and flexible model are clearly visible.

5 CONCLUSION

Nonlinear dynamic modeling and equations of motion of flexible manipulators with translational and rotational joints are built by using finite element method and Lagrange approach. Model is developed based on single link manipulator with only rotational joint. Inverse dynamic problem of flexible link manipulator is surveyed with an algorithm which is based on rigid model. Approximate driving force and torque at joints of flexible link manipulator are found with desire path. Derivation values of these also are shown. Elastic displacements at end-effector point are presented. However, there are remaining issues which need be studied further in future work because the error joints variables in algorithm to solve inverse dynamic problem of flexible with translational joint has not been mentioned yet.

REFERENCES

[1] D. S. Kwon and W. J. Book, "A time-domain inverse dynamic tracking control of a single link flexible manipulator", *Journal of Dynamic Systems, Measurement and Control*, vol. 116, pp. 193–200, 1994.

[2] S. Devasia, D. Chen and B. Paden, "Non-linear inversion-based output tracking", *IEEE Transactions on Automatic Control,* vol. 41, no. 7, 1996.

[3] C. Eliodoro and Miguel. A. Serna, "Inverse dynamics of flexible robots", *Mathematics and computers in simulation*, 41, pp. 485-508, 1996.

[4] B. O. Al-Bedoor and Y. A. Khulief, "General planar dynamics of a sliding flexible link", *Sound and Vibration*. **206**(5), pp. 641–661, 1997.

[5] Y. C. Pan, R. A. Scott, "Dynamic modeling and simulation of flexible robots with prismatic joints", *J. Mech. Design*, **112**, pp. 307–314, 1990.

[6] S. E. Khadem and A. A. Pirmohammadi, "Analytical development of dynamic equations of motion for a three-dimensional flexible manipulator with revolute and prismatic joints", *IEEE Trans. Syst. Man Cybern. B Cybern*, **33**(2), pp. 237–249, 2003.

[7] M. H. Korayem, A. M. Shafei and S. F. Dehkordi, "Systematic modeling of a chain of N-flexible link manipulators connected by revolute–prismatic joints using recursive Gibbs-Appell formulation", *Archive of Applied Mechanics,* Volume 84, Issue 2, pp. 187–206, 2014.

[8] W. Chen, "Dynamic modeling of multi-link of flexible robotic manipulators", *Computers and Structures*, 79, pp. 183 -195, 2001.

[9] Nguyen Van Khang and Chu Anh My, *Fundamentals of Industrial Robot*. Education Publisher, Ha Noi, Viet Nam, 2010, pp. 82-112.

[10] S. S. Ge, T. H. Lee and G. Zhu, "A Nonlinear feedback controller for a single link flexible manipulator based on a finite element method", *Journal of robotics system*, 14(3), pp. 165-178, 1997.

[11] M. O. Tokhi and A. K. M. Azad, Flexible robot manipulators–Modeling, simulation and control. Published by Institution of Engineering and Technology, London, United Kingdom, 2008, pp. 113-117.

Phân tích động lực học rô bốt có khâu đàn hồi với các khớp tịnh tiến và khớp quay

Dương Xuân Biên, Chu Anh Mỳ, Phan Bùi Khôi

***Tóm tắt* - Bài báo này trình bày việc phân tích bài toán động lực học ngược của hệ rô bốt có khâu đàn hồi với các khớp tịnh tiến và khớp quay. Mô hình động lực học mới được phát triển từ hệ rô bốt có 1 khâu đàn hồi với chỉ một khớp quay. Hệ phương trình động lực học được xây dựng dựa trên phương pháp Phần tử hữu hạn và hệ phương trình Lagrange. Lực dẫn động cho khớp tịnh tiến và mô men dẫn động cho khớp quay được tính xấp xỉ dựa trên mô hình rô bốt với các khâu giả thiết cứng tuyệt đối. Kết quả mô phỏng việc phân tích động lực học ngược mô tả giá trị lực/mô men dẫn động giữa mô hình cứng và mô hình đàn hồi cùng với giá trị sai lệch giữa chúng. Giá trị chuyển vị đàn hồi tại điểm thao tác cuối cũng được thể hiện. Tuy nhiên, vẫn còn rất nhiều vấn đề cần nghiên cứu thêm trong tương lai bởi giá trị sai lệch của biến khớp trong thuật toán giải động lực học ngược vẫn chưa được xét đến trong bài báo này.**

***Từ khóa* - Động lực học ngược, khâu đàn hồi, khớp tịnh tiến, chuyển vị đàn hồi**