# DEVELOPMENT OF AN AUTOMATED STORAGE/RETRIEVAL SYSTEM 

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ABSTRACT: This paper shows the mathematical model of an automated storage/retrieval system (AS/RS) based on innitial condition. We iditificate oscillation modes and kinematics displacement of system on the basis model results. With the use of the present model, the automated warehouse cranes system can be design more efficiently. Also, a AS/RS model with the control system are implemented to show the effectiveness of the solution. This research is part of $R / D$ research project of HCMC Department of Science and Technology to meet the demand of the manufacturing of automated warehouse in VIKYNO corporation, in particular, and in VietNam corporations, in general.

Keywords: automated storage/retrieval system , AS/RS model

## 1. INTRODUCTION

An AS/RS is a robotic material handling system (MHS) that can pick and deliver material in a direct - access fashion. The selection of a material handling system for a given manufacturing system is often an important task of mass production in industry. One must carefully define the manufacturing environment, including nature of the product, manufacturing process, production volume, operation types, duration of work time, work station characteristics, and working conditions in the manufacturing facility.

Hence, manufacturers have to consider several specifications: high throughput capacity, high IN/OUT rate, hight reliability and better control of inventory, improved
safety condition, saving investerment costs, managing professionally and efficiently. This system has been used to supervise and control for automated delivery and picking [1], [2], [3].

In this paper, several design hypothesis is given to propose a mathematical model and emulate to iditificate oscillation modes and kinematic displacement of system based on innitial conditions of force of load. As a results, we decrease error and testing effort before manufacturing [4], [5]. No existing AS/RS met all the requirements. Instead of purchasing an existing AS/RS, we chose to design a system for our need of study period and present manufacturing in VietNam.

This works was implemented at Robotics Division, National Laboratory of Digital Control and System Engineering (DCSELAB).

## 2. MODELLING OF AS/RS

An AS/RS is a robot that composed of (1) a carriage that moves along a linear track ( x axis), (2) one/two mast placed on the carriage, (3) a table that moves up and down along the mast (y-axis) and (4) a shuttle-picking device that can extend its length in both direction is put on the table. The motion of picking/placing an object by the shuttle-picking device is performed horizontally on the z -axis.

In this paper, an $\mathrm{AS} / \mathrm{RS}$ is considered a none angular deflection construction in cross section in place where having concentrated mass [4], [5], [6]. There are several assumtions as follows:

The weight of construction post is concentrated mass in floor level (Fig. 1).

Structural deformation is not depend on bar axial force. Assume that the mass of each part in $\mathrm{AS} / \mathrm{RS}$ is given as $m_{1}, m_{2}, m_{3}$, and $m_{L}$ is lifting mass.

When operation, there are two main motions: translating in horizontal direction with load $f_{1}$; translating in vertical direction with load $f_{2}$. The innitial conditions of AS/RS are lifting mass, lifting speed, lifting height, moving speeds, inertia force, resistance force, which can be used to establish mathematical model of AS/RS and verify the system behavior.

The assumed parameters of the AS/RS are given in Table 1.


Fig. 1: Model of AS/RS

Table 1 Parameters of the AS/RS

| Parameter | Value |  |
| :--- | :--- | :--- |
| m 1 | 150 | $[\mathrm{~kg}]$ |
| m 2 | 30 | $[\mathrm{~kg}]$ |
| mL | $100-500$ | $[\mathrm{~kg}]$ |
| m 3 | 20 | $[\mathrm{~kg}]$ |
| $\xi$ | 2 | $[\%]$ |
| L | 20 | $[\mathrm{~m}]$ |
| E | 21 x 106 | $\left[\mathrm{~N} / \mathrm{cm}^{2}\right]$ |
| I | $2.8 \times 103$ | $\left[\mathrm{~cm}^{4}\right]$ |
| k 1 | 352.8 | $[\mathrm{~N} / \mathrm{cm}]$ |
| k 2 | 352.8 | $[\mathrm{~N} / \mathrm{cm}]$ |
| kc | 6594 | $[\mathrm{~N} / \mathrm{cm}]$ |
| d | 20 | $[\mathrm{~mm}]$ |

### 2.1 Mathemmatical Model

Case 1: Horizontal moving along steel rail with load $f_{1}$ [7]

It is assume that (1) Structural deformation is not depend on bar axial force; (2) The mass in each part of automated warehouse cranes is given as $m_{1}, m_{2}, m_{3}$, in there, $m_{L}$ is lifting mass; (3)When the system moves, there are two main motions: travelling along steel rail underload $f_{1}$ and lifting body vertical direction underload $\mathrm{f}_{2}$.

The following model for traveling can be obtained:

$$
\begin{gathered}
m_{13}+k_{1}\left(x_{1}-x_{2}\right)+C_{1} \& \&_{1}=f_{1} \\
m_{23}+k_{1}\left(x_{2}-x_{1}\right)+k_{2}\left(x_{2}-x_{3}\right)=0 \\
m_{3}+k_{2}\left(x_{3}-x_{2}\right)+C_{2} \&_{3}=0
\end{gathered}
$$

where $\mathrm{m}_{13}=\mathrm{m}_{1}+\mathrm{m}_{2}+\mathrm{m}_{\mathrm{L}}+\mathrm{m}_{3}$

$$
k_{1}=\frac{6 E I}{x_{4}^{3}}, k_{2}=\frac{6 E I}{\left(L-x_{4}\right)^{3}}
$$

$$
\text { with } \mathrm{x}_{4}=\frac{L}{2} \text {, then } \mathrm{k}_{1}=\mathrm{k}_{2}
$$

where E: elastic coefficient of material

I: second moment of area
$\mathrm{k}_{1}, \quad \mathrm{k}_{2} \quad: \quad$ stiffness proportionality

## Case 2: Vertical moving with $\operatorname{load} f_{2}$ [8]

$m_{2 L}+k_{c} x_{4}+C_{3}{ }^{\&}{ }_{4}=f_{2} \quad$ where $\mathrm{m}_{2 \mathrm{~L}}=\mathrm{m}_{2}+$ $\mathrm{m}_{\mathrm{L}}$
$\mathrm{k}_{\mathrm{c}} \quad:$ stiffness of cable ${ }_{k_{C}}=\frac{A \mathrm{E}}{l}=\frac{\pi d^{2} E}{4\left(L-x_{4}\right)}$
D : diameter of cable

### 2.2. Solution of motion equation

a. Travelling along steel rail underload $f_{1}$

If resistance force is skipped, the motion equation can be written as:
$\left[\begin{array}{ccc}\mathrm{m}_{13} & 0 & 0 \\ 0 & \mathrm{~m}_{23} & 0 \\ 0 & 0 & \mathrm{~m}_{3}\end{array}\right]\left[\begin{array}{c} \\ 0\end{array}\right]+\left[\begin{array}{ccc}\mathrm{k}_{1} & -\mathrm{k}_{1} & 0 \\ -\mathrm{k}_{1} & \mathrm{k}_{1}+\mathrm{k}_{2} & -\mathrm{k}_{2} \\ 0 & -\mathrm{k}_{2} & \mathrm{k}_{2}\end{array}\right]\left[\begin{array}{l}\mathrm{x}_{1} \\ \mathrm{x}_{2} \\ \mathrm{x}_{3}\end{array}\right]=\left[\begin{array}{l}\mathrm{f}_{1} \\ 0 \\ 0\end{array}\right]$
or in the matrix form $k x=F$
$\omega_{1}=0(\mathrm{rad} / \mathrm{s})$,
Solution of Eq. (1) can be solved by superposition method [9] as the followings:

Eigen problem: $k \phi=M \phi \omega^{2} \Rightarrow\left(k-M \omega^{2}\right) \phi=0$
that satisfy $\operatorname{det}\left(k-M \omega^{2}\right)=0$
where $\phi \quad: \mathrm{n}$ level vector
$\omega \quad$ : vibration frequency $(\mathrm{rad} / \mathrm{s})$.
$\operatorname{det}\left[\begin{array}{ccc}\mathrm{k}_{1}-\mathrm{m}_{13} \omega^{2} & -k_{1} & 0 \\ -k_{1} & \mathrm{k}_{1}+\mathrm{k}_{2}-\mathrm{m}_{23} \omega^{2} & -\mathrm{k}_{2} \\ 0 & -k_{2} & \mathrm{k}_{2}-m_{3} \omega^{2}\end{array}\right]=0$
At the position $x_{4}=\frac{L}{2}$
Substituting constant values in Table 1 into

$$
\omega_{2}=1.065(\mathrm{rad} / \mathrm{s}), \omega_{3}=4.254(\mathrm{rad} / \mathrm{s}) .
$$

The solutions $\phi_{i}$ from the equation $\left(k-M \omega_{i}^{2}\right) \phi_{i}=0$ are as follows:
$\omega_{1}^{2}=0(\mathrm{rad} / \mathrm{s}) \quad: \phi_{1}$ values is any
$\omega_{2}^{2}=1.135(\mathrm{rad} / \mathrm{s}) \quad:$
$\phi_{2}=\left\{\begin{array}{lll}1 & -1.252 & -1.338\end{array}\right\}^{T}$
$\omega_{3}^{2}=18.095(\mathrm{rad} / \mathrm{s}) \quad:$
$\phi_{3}=\left\{\begin{array}{lll}1 & -34.9 & 153.073\end{array}\right\}^{T}$
These $\phi_{i}$ need to be satisfied $\phi_{i}{ }^{T} k \phi_{i}=\omega_{i}{ }^{2}$
$-\phi_{2}{ }^{T} k \phi_{2}=\omega_{2}{ }^{2}$

Eq. (3), we have
$a\left\{\begin{array}{lll}\phi_{21} & \phi_{22} & \phi_{23}\end{array}\right\}\left[\begin{array}{ccc}\mathrm{k}_{1} & -\mathrm{k}_{1} & 0 \\ -\mathrm{k}_{1} & \mathrm{k}_{1}+\mathrm{k}_{2} & -\mathrm{k}_{2} \\ 0 & -\mathrm{k}_{2} & \mathrm{k}_{2}\end{array}\right] a\left\{\begin{array}{l}\phi_{21} \\ \phi_{22} \\ \phi_{23}\end{array}\right\}=\omega_{2}{ }^{2} \Rightarrow a= \pm 0.025$
$-\phi_{3}{ }^{T} k \phi_{3}=\omega_{3}{ }^{2}$
${ }^{b}\left\{\begin{array}{lll}\phi_{31} & \phi_{32} & \phi_{33}\end{array}\right\}\left[\begin{array}{ccc}\mathrm{k}_{1} & -\mathrm{k}_{1} & 0 \\ -\mathrm{k}_{1} & \mathrm{k}_{1}+\mathrm{k}_{2} & -\mathrm{k}_{2} \\ 0 & -\mathrm{k}_{2} & \mathrm{k}_{2}\end{array}\right] b\left\{\begin{array}{l}\phi_{31} \\ \phi_{33} \\ \phi_{33}\end{array}\right\}=\omega_{3}{ }^{2}$
$\Rightarrow b= \pm 1.183 \times 10^{-3}$
If a and b are positive, $\phi_{i}$ values is as follows:
$\phi_{2}=\left\{\begin{array}{lll}0.025 & -0.031 & -0.033\end{array}\right\}^{T}, \phi_{3}=\left\{\begin{array}{lll}0.001 & -0.041 & 0.181\end{array}\right\}^{T}$

It can be seen that the condition $\phi^{T} M \phi=I$ is satisfied.

If resistance force is skipped, the motion equation will be written as the followings:

$$
(t)+\Omega^{2} x(t)=\phi^{T} f(t)
$$

and n individual equation can be written:

$$
\omega_{i}^{2}(t)+\omega_{i}^{2} x_{i}(t)=R_{i}(\tau)
$$

$$
\mathrm{x}_{i}(t)=\frac{1}{\omega_{i}} \int_{0}^{t} R_{i}(\tau) \sin \omega_{i}(t-\tau) d \tau+\alpha_{i} \sin \omega_{i} t+\beta_{i} \cos \omega_{i} t
$$

$$
\alpha_{l}(t)=\frac{R_{i}(\tau)}{\omega_{i}} \sin \omega_{i} t-\alpha_{i} \omega_{i} \cos \omega_{i} t-\beta_{i} \omega_{i} \sin \omega_{i} t
$$

$\alpha_{\mathrm{i}}$ and $\beta_{\mathrm{i}}$ can be specified from initial conditions
$\left.x_{i}\right|_{t=0}=\phi_{i}^{T} M^{\circ} u$
抟 $\left.\right|_{t=0}=\phi_{i}^{T} M^{\circ}$ 纫
Geometric inversion can be defined by principle of superposition:
$u(t)=[\phi][x(t)]=\left[\phi_{1}\right]\left[x_{1}(t)\right]+\left[\phi_{2}\right]\left[x_{2}(t)\right]+\ldots+\left[\phi_{n}\right]\left[x_{n}(t)\right]$
Displacement of point is defined by principle of superposition [9]

$$
\mathrm{u}_{i}(\mathrm{t})=\sum_{\mathrm{i}=1}^{\mathrm{n}} \phi_{\mathrm{i}} \mathrm{x}_{i}(\mathrm{t})
$$

If resistance forces are considered
Using integral Duhamel to find motion equation [9]:

$$
\begin{aligned}
& \mathrm{x}_{\mathrm{i}}(\mathrm{t})=\frac{1}{\bar{\omega}_{\mathrm{i}}} \int_{0}^{\mathrm{t}} \mathrm{R}_{\mathrm{i}}(\tau) \mathrm{e}^{-\xi_{\mathrm{i}} \omega_{\mathrm{i}}(\mathrm{t}-\tau)} \sin \bar{\omega}_{\mathrm{i}}(\mathrm{t}-\tau) \mathrm{d} \tau+ \\
& \\
& \quad \mathrm{e}^{-\xi_{i} \omega_{\mathrm{i}} \mathrm{t}}\left(\alpha_{\mathrm{i}} \sin \bar{\omega}_{\mathrm{i}} \mathrm{t}+\beta_{\mathrm{i}} \cos \bar{\omega}_{\mathrm{i}} \mathrm{t}\right)
\end{aligned}
$$

$$
\begin{aligned}
& R_{i}(\tau)=\phi_{i}^{T} f(t) \\
& R_{2}(\tau)=0.025 f_{1}(t) \\
& R_{3}(\tau)=0.001 f_{1}(t)
\end{aligned}
$$

Using integral Duhamel to find motion equation [9]
where : $\bar{\omega}_{i}=\omega_{i} \sqrt{1-\xi_{i}^{2}}$

$$
\xi_{i}: \text { damping ratio }
$$

$\&(t)=\frac{R_{i}(\tau) e^{-\xi^{-} \omega_{1} t}}{\bar{\omega}_{1}^{2}+\xi_{i}^{2} \omega_{i}^{2}}\left(\frac{\xi_{i}^{2} \omega_{1}^{2}}{\bar{\omega}_{1}}+\bar{\omega}_{i}\right) \sin \bar{\omega}_{i} t-$

$$
\mathrm{e}^{\varepsilon_{i j} \omega_{1} \mathrm{t}}\left(\left(\xi_{\mathrm{i}} \omega_{i} \alpha_{i}+\beta_{\mathrm{i}} \bar{\omega}_{\mathrm{i}}\right) \sin \bar{\omega}_{\mathrm{i}} \mathrm{t}\left(\xi_{\mathrm{i}} \omega_{i} \beta_{\mathrm{i}}-\alpha_{i} \bar{\omega}_{\mathrm{i}}\right) \cos \bar{\omega}_{\mathrm{i}} \mathrm{t}\right)
$$

We find $\alpha_{\mathrm{i}}$ and $\beta_{\mathrm{i}}$ value based on initial condition

Displacement of point is defined by principle of superposition (Eq. (8))

Influential dynamic load act (\&) warehouse cranes in some cases

The acting force is a constant and system has influential resistance force

It is assumed that $\mathrm{f}_{1}=W_{t}=423.6(\mathrm{~N})$
From Eq. (4) and Eq. (5), we have
$R_{2}(\tau)=0.025 f_{1}=0.025 * 423.6=10.59(\mathrm{~N})$
$R_{3}(\tau)=0.001 f_{1}=0.001 * 423.6=0.42(\mathrm{~N}(9)$
From Eq. (10)
$\bar{\omega}_{2}=\omega_{2} \sqrt{1-\xi_{2}^{2}}=1.065 \sqrt{1-0.02^{2}}=1.06(\mathrm{rad} / \mathrm{s})$
$\bar{\omega}_{3}=\omega_{3} \sqrt{1-\xi_{3}^{2}}=4.254 \sqrt{1-0.02^{2}}=4.25(\mathrm{rad} / \mathrm{s})$
Substituting the values: $R_{2}(\tau), \bar{\omega}_{2}, \xi_{2}$ into Eq. (9), and from initial condition:
$\left.x_{2}\right|_{t=0}=\phi_{2}^{T} M^{\circ}\left\{\begin{array}{l}0 \\ 0 \\ 0\end{array}\right\}=0$
$\left.\mathcal{k g}_{2}\right|_{t=0}=\phi_{2}^{T} M^{\circ}\left\{\begin{array}{l}0 \\ 0 \\ 0\end{array}\right\}=0$

Substituting $R_{3}(\tau), \bar{\omega}_{3}, \xi_{3}$ into Eq. (9) and from Eq. (9) and Eq. (11), we have $\alpha_{3}=0$,

$$
\beta_{3}=0:
$$

$$
\begin{aligned}
& \mathrm{x}_{3}(\mathrm{t})=0.023\left(1-\mathrm{e}^{-0.085 \mathrm{t}}(\cos 4.25 \mathrm{t}+0.02 \sin 4.25 \mathrm{t})\right) \\
& \quad \text { with } \\
& \left\{\begin{array}{lll}
\omega_{1} & \omega_{2} & \omega_{3}
\end{array}\right\}^{T}=\left\{\begin{array}{lll}
0 & 1.06 & 4.25
\end{array}\right\}^{T}(\mathrm{rad} / \mathrm{s})
\end{aligned}
$$

As the results, the motion equation can be derived as:

From Eq. (6) and Eq. (7) with $\alpha_{2}=0$,
$\beta_{2}=0:$

$$
\mathrm{x}_{2}(\mathrm{t})=9.42\left(1-\mathrm{e}^{-0.02 \mathrm{t}}(\cos 1.06 \mathrm{t}+0.02 \sin 1.06 \mathrm{t})\right)
$$

with $\omega_{3}=4.25$ :

$$
\left\{\begin{array}{l}
x_{1}(t)  \tag{12}\\
x_{2}(t) \\
x_{3}(t)
\end{array}\right\}=\left\{\begin{array}{l}
0 \\
9.42\left(1-e^{-0.02 t}(\cos 1.06 t+0.02 \sin 1.06 t)\right) \\
0.023\left(1-e^{-0.085 t}(\cos 4.25 t+0.02 \sin 4.25 t)\right)
\end{array}\right\}
$$

With the force of load is periodic, resistance force of the system is assumed to be $f_{1}=A \cos \omega t$
From Eq. (4) and (5), we have
$R_{2}(\tau)=0.025 f_{1}=0.025 A \cos \omega t$
$R_{3}(\tau)=0.001 f_{1}=0.001 \mathrm{~A} \cos \omega t$
Solution $\mathrm{x}_{2}(\mathrm{t})$
Substituting $R_{2}(\tau), \bar{\omega}_{2}, \xi_{2}$ into Eq. (9) and initial condition into Eq. (9) and Eq. (11), we have $\mathrm{x}_{2}(t)=22.24 \times 10^{-3} A \cos \omega t\left(1-e^{-0.02 t}(\cos 1.06 t+0.02 \sin 1.06 t)\right)$

Solution $\mathrm{x}_{3}(\mathrm{t})$
Substituting $R_{3}(\tau), \bar{\omega}_{3}, \xi_{3}$ into Eq. (9) and initial condition into Eq. (9) and Eq. (11), we have $\mathrm{x}_{3}(\mathrm{t})=5.534 \times 10^{-5} \mathrm{~A} \cos \omega \mathrm{t}\left(1-\mathrm{e}^{-0.085 \mathrm{t}}(\cos 4.25 \mathrm{t}-0.02 \sin 4.25 \mathrm{t})\right)$
Trang 30

The motion equation under periodic load can be derived as folows:

$$
\left\{\begin{array}{l}
x_{1}(\mathrm{t})  \tag{13}\\
\mathrm{x}_{2}(\mathrm{t}) \\
\mathrm{x}_{3}(\mathrm{t})
\end{array}\right\}=\left\{\begin{array}{l}
0 \\
22.24 \times 10^{-3} \mathrm{~A} \cos \omega \mathrm{t}^{*} \\
\left(1-\mathrm{e}^{-0.02 \mathrm{t}}(\cos 1.06 \mathrm{t}+0.02 \sin 1.06 \mathrm{t})\right) \\
5.534 \times 10^{-5} \mathrm{Acos} \omega \mathrm{t}^{*} \\
\left(1-\mathrm{e}^{-0.085 \mathrm{t}}(\cos 4.25 \mathrm{t}-0.02 \sin 4.25 \mathrm{t})\right)
\end{array}\right\}
$$

It is assumed that $f_{1}=423.6 \cos 40 t$
Equation (13) becomes

$$
\left\{\begin{array}{l}
x_{1}(\mathrm{t})  \tag{14}\\
\mathrm{x}_{2}(\mathrm{t}) \\
\mathrm{x}_{3}(\mathrm{t})
\end{array}\right\}=\left\{\begin{array}{l}
0 \\
9.42 \cos 40 \mathrm{t}\left(1-\mathrm{e}^{-0.02 \mathrm{t}}(\cos 1.06 \mathrm{t}+0.02 \sin 1.06 \mathrm{t})\right) \\
0.023 \cos 40 \mathrm{t}\left(1-\mathrm{e}^{-0.085 t}(\cos 4.25 \mathrm{t}-0.02 \sin 4.25 \mathrm{t})\right)
\end{array}\right\}
$$

b. Lifting carrier in vertical direction under load $f_{2}$

The model of liffting carrier can be written as $m_{2 L}+k_{c} x_{4}+C_{3} \varepsilon_{4}=f_{2}$
and $\alpha_{4}, \beta_{4}$ are defined from initial condition.
when $\mathrm{t}=0: \mathrm{x}_{4}=10(\mathrm{~m}), \quad x_{4}=1(\mathrm{~m} / \mathrm{s})$.
as $m_{2 L}+k_{c} x_{4}+C_{3}{ }^{2}{ }_{4}=f_{2} \quad$ (15) Substituting the values into Eq. (16), Eq. (17),

Skipping resistance force and $f_{2}=0$, we have $\mathrm{x}_{4}(t)=\alpha_{4} \sin \omega_{4} t+\beta_{4} \cos \omega_{4} t$
we have $\beta_{4}=10, \alpha_{4}=0.083$
Oscillation system is of a harmonic motion
$\&_{4}(t)=\alpha_{4} \omega_{4} \cos \omega_{4} t-\beta_{4} \omega_{4} \sin \omega_{4} t \quad(17)$
as
$\mathrm{X}_{4}(t)=0.083 \sin 11.989 t+10 \cos 11.989 t$
where $\omega_{4}=\frac{k_{c}}{m_{2 L}}=\frac{6594}{550}=11.989(\mathrm{rad} / \mathrm{s})$
If resistance forces are considered, using integral Duhamel to find the motion equation
$\mathrm{x}_{\mathrm{i}}(\mathrm{t})=\frac{1}{\bar{\omega}_{\mathrm{i}}} \int_{0}^{\mathrm{t}} \mathrm{R}_{\mathrm{i}}(\tau) \mathrm{e}^{-\xi_{\mathrm{i}} \omega_{\mathrm{i}}(\mathrm{t}-\tau)} \sin \bar{\omega}_{\mathrm{i}}(\mathrm{t}-\tau) \mathrm{d} \tau+$

$$
\begin{equation*}
\mathrm{e}^{-\xi_{\mathrm{i}} \omega_{\mathrm{i}}^{\mathrm{i}}} \mathrm{t}\left(\alpha_{\mathrm{i}} \sin \bar{\omega}_{\mathrm{i}} \mathrm{t}+\beta_{\mathrm{i}} \cos \bar{\omega}_{\mathrm{i}} \mathrm{t}\right) \tag{19}
\end{equation*}
$$

where

$$
\bar{\omega}_{i}=\omega_{i} \sqrt{1-\xi_{i}^{2}}, \bar{\omega}_{i}=11.989 \sqrt{1-0.02^{2}}=11.987(\mathrm{rad} / \mathrm{s})
$$

$\alpha_{i}, \beta_{i}$ can be derived from the initial condition.

$$
\begin{array}{r}
x_{4}(t)=\frac{R_{4}(\tau)}{\bar{\omega}_{4}^{2}+\xi_{4}^{2} \omega_{4}^{2}}\left(1-e^{-\xi_{4} \omega_{4} t}\left(\cos \bar{\omega}_{4} t+\frac{\xi_{4} \omega_{4}}{\bar{\omega}_{4}} \sin \bar{\omega}_{4} t\right)+\right.  \tag{20}\\
e^{-\xi_{4} \omega_{4} t}\left(\alpha_{4} \sin \bar{\omega}_{4} t+\beta_{4} \cos \bar{\omega}_{4} t\right)
\end{array}
$$

$\&_{4}(\mathrm{t})=\frac{\mathrm{R}_{4}(\tau) \mathrm{e}^{-\xi_{4} \omega_{4} \mathrm{t}}}{\bar{\omega}_{4}^{2}+\xi_{4}^{2} \omega_{4}^{2}}\left(\frac{\xi_{4}^{2} \omega_{4}^{2}}{\bar{\omega}_{4}}+\bar{\omega}_{4}\right) \sin \bar{\omega}_{4} \mathrm{t}-$
$e^{-\xi_{4} \omega_{4} t}\left(\left(\xi_{4} \omega_{4} \alpha_{4}+\beta_{4} \bar{\omega}_{4}\right) \sin \bar{\omega}_{4} t+\left(\xi_{4} \omega_{4} \beta_{4}-\alpha_{4} \bar{\omega}_{4}\right) \cos \bar{\omega}_{4} t\right)$
when $t=0: x_{4}=10(\mathrm{~m}), x_{4}=1(\mathrm{~m} / \mathrm{s})$.
Substituting above values into Eq. (20), we have $\beta_{4}=10$.
Substituting above values into Eq. (21), we have $1=\left(\xi_{4} \omega_{4} \beta_{4}-\alpha_{4} \bar{\omega}_{4}\right)$
$\Rightarrow \alpha_{4}=\frac{\xi_{4} \omega_{4} \beta_{4}-1}{\bar{\omega}_{4}}=\frac{0.02 * 11.989-1}{11.987}=-63.4 \times 10^{-4}$
Substituting $\alpha_{4}, \beta_{4}, \omega_{4}, \bar{\omega}_{4}$ into Eq. (20), we have

$$
\begin{align*}
x_{4}(t)= & \frac{R_{4}(\tau)}{143.75}\left(1-e^{-0.24 t}(\cos 11.987 t+0.02 \sin 11.987 t)+\right.  \tag{22}\\
& e^{-0.24 t}\left(-64.3 \times 10^{-4} \sin 11.987 t+10 \cos 11.987 t\right)
\end{align*}
$$

With the force of load is a costant, the resistance force is assumed to be $\mathrm{f}_{2}=\mathrm{S}_{\max }=1736.76 \mathrm{~N}$
Substituting $\mathrm{R}_{\mathrm{i}}(\tau)=\mathrm{f}_{2}=1736.76(\mathrm{~N})$ into Eq. (22):
$\mathrm{x}_{4}(\mathrm{t})=12.08\left(1-\mathrm{e}^{-0.24 t}(\cos 11.987 \mathrm{t}+0.02 \sin 11.987 \mathrm{t})+\right.$ $\mathrm{e}^{-0.24 \mathrm{t}}\left(-64.3 \times 10^{-4} \sin 11.987 \mathrm{t}+10 \cos 11.987 \mathrm{t}\right)$

Or $\mathrm{x}_{4}(\mathrm{t})=12.08\left(1-\mathrm{e}^{-0.24 \mathrm{t}}(0.172 \cos 11.987 \mathrm{t}+0.02 \sin 11.987 \mathrm{t})\right)$
The force of load is periodic, the resistance force of the system is assumed to be $f_{2}=\mathbf{1 7 3 6 . 7 6} \cos 40 t$

Substituting $\mathrm{R}_{\mathrm{i}}(\tau)=f_{2}=\mathbf{1 7 3 6 . 7 6} \cos 40 t$ into Eq. (22)
$x_{4}(t)=\frac{\mathbf{1 7 3 6} .76 \cos 40 t}{143.75}\left(1-e^{-0.24 t}(\cos 11.987 t+0.02 \sin 11.987 t)+\right.$ $\mathrm{e}^{-0.24 t}\left(-64.3 \times 10^{-4} \sin 11.987 \mathrm{t}+10 \cos 11.987 \mathrm{t}\right)$

Or $x_{4}(\mathrm{t})=12.08 \cos 40 \mathrm{t}\left(1-\mathrm{e}^{-0.24 \mathrm{t}}(0.172 \cos 11.987 \mathrm{t}+0.02 \sin 11.987 \mathrm{t})\right)$
2.3 Simulation Results
a. The carrier travelling along rail under load $f_{1}$

From the motion equation, the system can be simulated to describe the oscillation and displacement of the robot on time and use Eq. (10) to define displacement of point [10].

The force of load is constant, the resistance force is assumed to be $\mathrm{f}_{1}=423.6(\mathrm{~N})$

From Eq. (12), the system motion is as follows:
$\left\{\begin{array}{l}\mathrm{x}_{1}(\mathrm{t}) \\ \mathrm{x}_{2}(\mathrm{t}) \\ \mathrm{x}_{3}(\mathrm{t})\end{array}\right\}=\left\{\begin{array}{l}0 \\ 9.42\left(1-\mathrm{e}^{-0.02 t}(\cos 1.06 \mathrm{t}+0.02 \sin 1.06 \mathrm{t})\right) \\ 0.023\left(1-\mathrm{e}^{-0.085 t}(\cos 4.25 \mathrm{t}+0.02 \sin 4.25 \mathrm{t})\right)\end{array}\right\}$
$x_{1}(t)=0$, the plot $x_{2}(t)$ and $x_{3}(t)$ is shown in Fig. 2.


Fig. 2. System oscillation under constant with $\omega=1.06(\mathrm{rad} / \mathrm{s})$


Fig. 3. System oscillation under constant load with $\omega=4.25(\mathrm{rad} / \mathrm{s})$

The point's displacement is defined by the principle of superposition.
From Eq. (8), we have

$$
\left\{\begin{array}{l}
u_{1}(t)  \tag{25}\\
u_{2}(t) \\
u_{3}(t)
\end{array}\right\}=\left\{\begin{array}{l}
0.24\left(1-e^{-0.02 t}(\cos 1.06 t+0.02 \sin 1.06 t)\right)+ \\
0.23 \times 10^{-4}\left(1-e^{-0.085 t}(\cos 4.25 t+0.02 \sin 4.25 t)\right) \\
-0.29\left(1-e^{-0.02 t}(\cos 1.06 t+0.02 \sin 1.06 t)\right)- \\
9.43 \times 10^{-4}\left(1-e^{-0.085 t}(\cos 4.25 t+0.02 \sin 4.25 t)\right) \\
-0.3\left(1-e^{-0.02 t}(\cos 1.06 t+0.02 \sin 1.06 t)\right)+ \\
41.63 \times 10^{-4}\left(1-e^{-0.085 t}(\cos 4.25 t+0.02 \sin 4.25 t)\right)
\end{array}\right\}
$$

The point's displacements are given in Table 2.
Table 2. The displacement of points

| Time <br> $\mathrm{t}(\mathrm{s})$ | Displace <br> $-\mathrm{ment} \mathrm{u1} \mathrm{(m)}$ | Displace <br> - ment $\mathrm{u} 2(\mathrm{~m})$ | Displace <br> - ment u3 (m) |
| :--- | :--- | :--- | :--- |
| 0.02 | 4.8178 <br> $\mathrm{x} 10^{-5}$ | -6.1618 <br> $\mathrm{x} 10^{-5}$ | -4.5204 <br> $\mathrm{x} 10^{-5}$ |
| 0.04 | 2.0415 <br> $\mathrm{x} 10^{-4}$ | -2.602 <br> $\mathrm{x} 10^{-4}$ | -1.952 <br> $\mathrm{x} 10^{-4}$ |
| 0.06 | 4.6777 <br> $\mathrm{x} 10^{-4}$ | -5.956 <br> $\mathrm{x} 10^{-4}$ | -4.505 <br> $\mathrm{x} 10^{-4}$ |
| 0.08 | 8.3882 <br> $\mathrm{x} 10^{-4}$ | -0.0011 | -8.112 <br> $\mathrm{x} 10^{-4}$ |
| 0.1 | 0.0013 | -0.0017 | -0.0013 |

Table 3. Point Displacement

| Time <br> $\mathrm{t}(\mathrm{s})$ | Displace <br> $-\mathrm{ment} \mathrm{u1} \mathrm{(m)}$ | Displace <br> $-\mathrm{ment} \mathrm{u} 2(\mathrm{~m})$ | Displace <br> $-\mathrm{ment} \mathrm{u3(m)}$ |
| :--- | :--- | :--- | :--- |
| 0.02 | $3.3624 \times 10^{-5}$ | $-4.3007 \times 10^{-5}$ | $-3.2709 \times 10^{-5}$ |
| 0.04 | $-5.971 \times 10^{-6}$ | $7.6123 \times 10^{-6}$ | $5.9202 \times 10^{-6}$ |
| 0.06 | $-3.455 \times 10^{-4}$ | $4.3995 \times 10^{-4}$ | $3.4483 \times 10^{-4}$ |
| 0.08 | $-8.387 \times 10^{-4}$ | 0.0011 | $8.4055 \times 10^{-4}$ |
| 0.1 | $-8.622 \times 10^{-4}$ | 0.0011 | $8.6695 \times 10^{-4}$ |

With the force of load is periodic, resistance force of the system is assumed to be $f_{1}=423.6 \cos 40 t$

From Eq. (18), we have

$$
\left\{\begin{array}{l}
\mathrm{x}_{1}(\mathrm{t}) \\
\mathrm{x}_{2}(\mathrm{t}) \\
\mathrm{x}_{3}(\mathrm{t})
\end{array}\right\}=\left\{\begin{array}{l}
0 \\
9.42 \cos 40 \mathrm{t}\left(1-\mathrm{e}^{-0.02 \mathrm{t}}(\cos 1.06 \mathrm{t}+0.02 \sin 1.06 \mathrm{t})\right) \\
0.023 \cos 40 \mathrm{t}\left(1-\mathrm{e}^{-0.085 t}(\cos 4.25 \mathrm{t}-0.02 \sin 4.25 \mathrm{t})\right)
\end{array}\right\}
$$

The oscillation plot of $\mathrm{x}_{2}(\mathrm{t})$ and $\mathrm{x}_{3}(\mathrm{t})$ are described in Fig. 4 and Fig. 5.


Fig. 4. System oscillation under periodic load with $\omega=1.06(\mathrm{rad} / \mathrm{s})$


Fig. 5. System oscillation under periodic load with $\omega=4.25(\mathrm{rad} / \mathrm{s})$

The point's displacement is defined by principle of superposition.
From Eq. (8), we have

The point displacement are given in Table 3.

## b. Lifting the table in vertical direction under load $f_{2}$

Resistance force is skipped and $f_{2}=0$. From Eq. (18), the oscillation system is of harmonic motion:

$$
\mathrm{x}_{4}(t)=0.083 \sin 11.989 t+10 \cos 11.989 t
$$

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Fig. 6. Harmonic motion of system with $\omega=11.989(\mathrm{rad} / \mathrm{s})$


Fig. 7. System oscillation under constant load with $\omega=11.978(\mathrm{rad} / \mathrm{s})$

With the force of load is costant, the resistance force is assumed to be $\mathrm{f}_{2}=\mathrm{S}_{\max }=$ 1736.76 N.

With the force of load is periodic, resistance force of the system is assumed to be $f_{2}=\mathbf{1 7 3 6 . 7 6} \cos 40 t$

From Eq. (23), we have
$x_{4}(t)=12.08\left(1-e^{-0.24 t}(0.172 \cos 11.987 t+0.02 \sin 11.987 t)\right)$

From Eq. (24), we have $\mathrm{x}_{4}(\mathrm{t})=12.08 \cos 40 \mathrm{t}\left(1-\mathrm{e}^{-0.24 t}(0.172 \cos 11.987 \mathrm{t}+0.02 \sin 11.987 \mathrm{t})\right)$


Fig. 8. System oscillation under periodic load with $\omega=11.987(\mathrm{rad} / \mathrm{s})$

From the above plots, it can be realized that if we change the vibration frequency or load, the system oscillation and displacement will be change. Alternatively, vibration frequency is depends on lifting body mass, lifting height, stiffness proportionality ...Hence, if we change initial condition design, we will iditificate oscillation modes and kinematic displacement of system.

## 3. CONTROL SYSTEM DEVELOPMENT

There are three computers are used to implement the control logic throughout the factory: host computer, client computer, and station computer. The host computer's function is managing the database of the system, the client computer's function is handling in/out operations, and the station computer's function is monitoring and controlling the AS/RSsystem. The control system architechture is designed to meet the demand of a AS/RS is shown in Fig. 9 .


Fig. 9. System control architecture for AS/RS


Fig. 10. Warehouse Management software Interface

In other words, the control system is composed of two control levels: management control and machine control. The communication between them is via LAN network. As for management control, a server host computer is installed with Warehouse Management software which connect to the warehouse database using Microsoft SQL Server framework. The server host can perform tasks, such as supplier management, customer management, items management, warehouse
structure management. A barcode system is used for the item's identification in warehouse. The interface of Warehouse Management software is shown in Fig. 10.

As for the machine control, a PAC 5010 KW with SCADA system is implemented to control the motion of robot for in/out operations as shown in Fig. 11, and the control panel on AS/RS, Fig. 12. The design has allocated for VIKYNO company's warehouse as shown in Fig. 13.


Fig. 11. SCADA Interface and PAC 5510KW implemented on the AS/RS model


Fig. 12. Control panel of AS/RS model

## 4. CONCLUSION

In this paper, mathematical model of the $\mathrm{AS} / \mathrm{RS}$ is established with several oscillation modes and the kinematic displacement of the system are found respectively. The generalized


Fig. 13. The allocation warehouse of VIKYNO for $\mathrm{AS} / \mathrm{RS}$ implementation.
calculate program was established to verify the behavior of the system and the system identification process. Finally, the development of this system have been done, but the experimental data yet to finish at this time of this writing. It is our works in the future.

## PHÁT TRIỂN HỆ THỐNG LỦU KHO TỬ ĐỘNG

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TÓM TȦT: Bài báo này trình bày mô hình toán học của hệ thống lưu kho tư động (Automated Storage/Retrieval System - $A S / R S$ ). Các chế độ giao động và chuyển vị của hệ thống được khảo sát dựa trên mô hình cơ sở trên. Với mô hình này, việc thiết kế hệ thống robot đura vào lấy ra sẽ hiệu quả hơn truớc khi chế tạo. Ngoài ra, một mô hình hệ thống kho hàng tụ động cùng với hệ thống điều khiển đầy đủ đurợc thiết kế và cài đặt để thấy đurợc sụ hiểu quả của giải pháp đura ra. Nghiên cúu này là một phần dự án nghiên cúu chế tạo thủ̉ nghiệm của Sở khoa học Công nghệ để đáp úng yêu cầu sản xuất kho hàng tự động cho công ty VIKYNO nói riêng, và đáp ưng nhu cầu của các công ty Việt nam nói chung.

Tù khóa: AS/RS, Automated storage/retrieval system.

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