

IMPROVE FAULT TOLERANCE, SHORTEST PATH ROUTING IN HYPER DE BRUIJN ASTER NETWORK

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1.INTRODUCTION

The properties of de Bruijn network has shown it be the next generation after the hypercube for parallel processing applications such as multiprocessor network, VLSI [1,2,4,6,7,8,12]. Hypercube has its own advantages including the degree and diameter are independent [4]. Therefore the combination between de Bruijn graph and Hypercube graph will make an ideal topology for fault tolerance and shortest path routing. In this article, we first present a topology named Hyper de Bruijn Aster and then communication on this topology is investigated.

Routing in de Bruijn graph has been investigated by Samantham, Liu and Mao [1,2,6], but their algorithms cannot achieve shortest path if there is a fault node along the path [7]. Broadcasting in de Bruijn graph has also been investigated by Esfahanian, Ganesan, Ohring [8,10]. However, only working on binary de Bruijn is their drawback. To address the problems of fault tolerance in shortest path routing and broadcasting in high degree de Bruijn network, please refer to the paper [7].

The combination of hypercube and de Bruijn as Hyper de Bruijn has been studied by Elango Ganesan, Dhiraj K Pradhan [3,12]. However, their topology [12] is based on binary de Bruijn graph, and their DeadLock-Free routing algorithm [3] can work in binary de Bruijn network only. Wei Shi and Pradip K Srimani [5] have proposed a very good routing algorithm based on Hyper Butterfly (the combination between Hypercube and Butterfly network). In their article, they criticized that Hyper de Bruijn network (proposed by Ganesan [12]) is not regular, not optimally fault tolerant, and complex routing. All of these criticisms are solved in our article. The Hyper de Bruijn Aster proposed in this article can be used for load balancing and parallel processing. Our fault tolerant properties and shortest path routing algorithm are proved to archive the best performance among routing algorithms in Hyper de Bruijn network. Consequence, Hyper de Bruijn Aster has shown to be the most suitable topology for multiprocessor, VLSI and parallel processing networks.

Section 2 presents some background and properties of Hyper de Bruijn Aster. Section 3 presents shortest path routing algorithm and fault tolerant characteristics of Hyper de Bruijn Aster. And in section 4 comes with the conclusion.

2.HYPER DE BRUIJN ASTER GRAPH

2.1.Hypercube Graph H_n and De Bruijn Graph D_n

Hypercube Graph order m , $H(m)$ includes the set of node Z_2^m . For the two adjacent nodes, address is different in one bit.

Binary de Bruijn Graph(undirected) order n , $D(n)$ includes the set of vertices Z_2^n . Given $\alpha, \beta \in Z_2$ and $x \in Z_2^{n-2}$, each node is presented as $\alpha x \beta$, and linked by:

- $x\beta\alpha$ by shuffle arc
- $x\beta\bar{\alpha}$ by shuffle-exchange arc
- $\beta\alpha x$ by inverse-shuffle arc

- $\bar{\beta}_{\alpha x}$ by inverse-shuffle-exchange arc

Similarly, if we extend $\alpha, \beta \in \mathbb{Z}_d$ and $x \in \mathbb{Z}_d^{n-2}$ ($d \geq 2$, degree higher than 2), then the number of link to each node and the number of adjacent nodes are increased. And hence, it improves fault tolerant in de Bruijn graph.

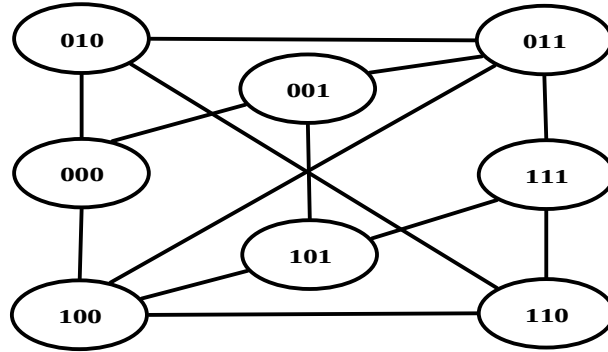


Figure 1. Hypercube graph $H(3)$.

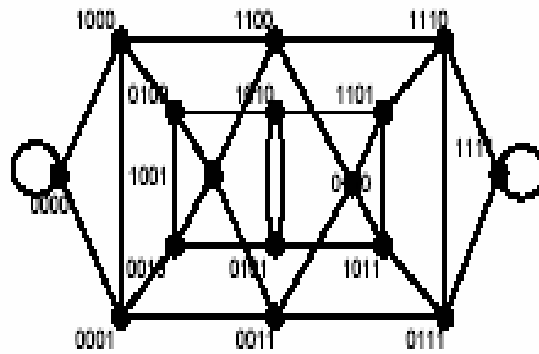


Figure 2. de Bruijn graph $D(2,4)$.

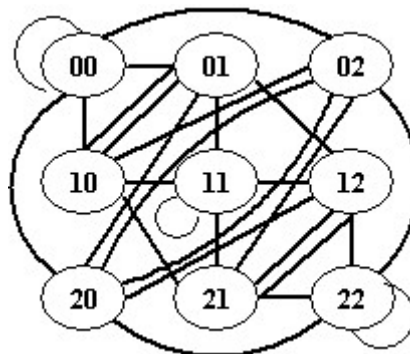


Figure 3. de Bruijn graph, $D(3,2)$.

Corollary 1 $D(d,n)$ is a symmetrical undirected graph, it has degree $2d$, diameter n and total number of nodes dn [7].

Corollary 2 $H(m)$ is a symmetrical undirected graph, it has degree m , diameter m and total number of nodes $2m$ [5].

2.2. Hyper de Bruijn Aster Graph $HD^*(m,d,n)$

Definition: By extending de Bruijn graph to high degree (order) and combining with hypercube, we invent a new topology Hyper de Bruijn Aster order (m,n) symbolized by $HD^*(m,d,n)$, $HD^*(m,d,n)$ is a product of $H(m) \times D(d,n)$.

It is obviously to obtain links from a node $\hat{x}x_{m-1}x_{m-2} \dots x_0, y_{n-1}y_{n-2} \dots y_0$ to the following nodes (in deBruijn part):

$$\hat{x}x_{m-1}x_{m-2} \dots x_0, \alpha y_{n-1}y_{n-2} \dots y_1 \hat{n}$$

$$\hat{x}x_{m-1}x_{m-2} \dots x_0, y_{n-1}y_{n-2} \dots y_1 \alpha \hat{n}, \alpha \hat{I} Z d$$

And link to following node (in Hypercube part):

$$\hat{x}x_{m-1}x_{m-2} \dots \bar{x}_i x_{i-1} \dots x_0, y_{n-1}y_{n-2} \dots y_0 \hat{n}, 0 \leq i \leq m-1$$

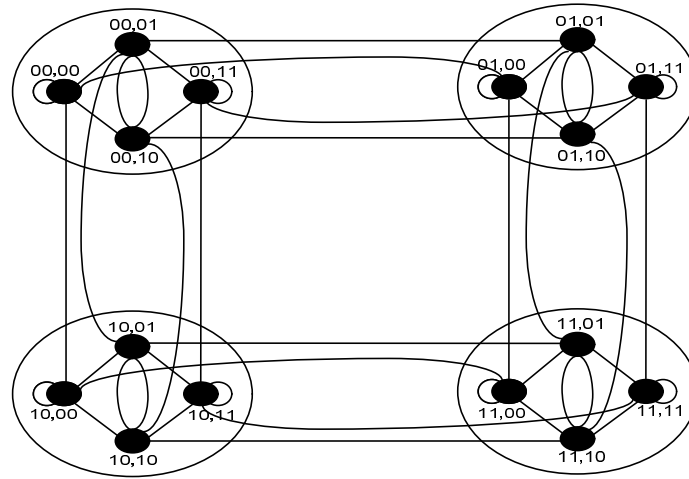


Figure 4. Another presentation of $HD^*(2,2,2)$.

Theorem 1 $HD^*(m,d,n)$ is a symmetrical undirected graph with degree $m+2d$, diameter $m+n$ and total number of node $N(HD^*) = 2mdn$.

Proof:

- Hypercube and de Bruijn graph are symmetrical (corollary 1,2). A Hyper de Bruijn Aster $HD^*(m,d,n) = H(m) \times D(d,n)$, each node of hypercube contains a de Bruijn graph $D(d,n)$. Therefore, $HD^*(m,d,n)$ is symmetrical.
- The total number of node in $HD^*(m,d,n) =$ total number of node in Hypercube $H(m) \times$ total number of node in de Bruijn $D(d,n)$. The total number of node in Hypercube is $2m$ [4], the total number of node in de Bruijn is dn [1,2,7]. So $N(HD^*) = 2mdn$
- Degree is the total number of link connecting to a node [4], because degree of hypercube $H(m)$ is m (corollary 1), degree of de Bruijn $D(d,n)$ is $2d$ (corollary 2) and $HD^*(m,d,n)$ is the combination of $H(m)$ and $D(d,n)$ ($H(m)$ is the base cluster), each node in $HD^*(m,d,n)$ connects to $H(m)$ and $D(d,n)$ network, so the degree of $HD^*(m,d,n)$ is $2d+m$.
- Diameter is the shortest distance between 2 farthest vertices in a graph [4]. Routing in $HD^*(m,d,n)$ from a vertex $v(h,d)$ to a vertex $v(h',d')$ can be done by 2 way: $v(h,d) \hat{a} v(h',d) \hat{a} v(h',d')$ or $v(h,d) \hat{a} v(h,d') \hat{a} v(h',d')$, and diameter of $H(m)$ is m , diameter of $D(d,n)$ is n . Therefore, diameter of $HD^*(m,d,n)$ is $m+n$.

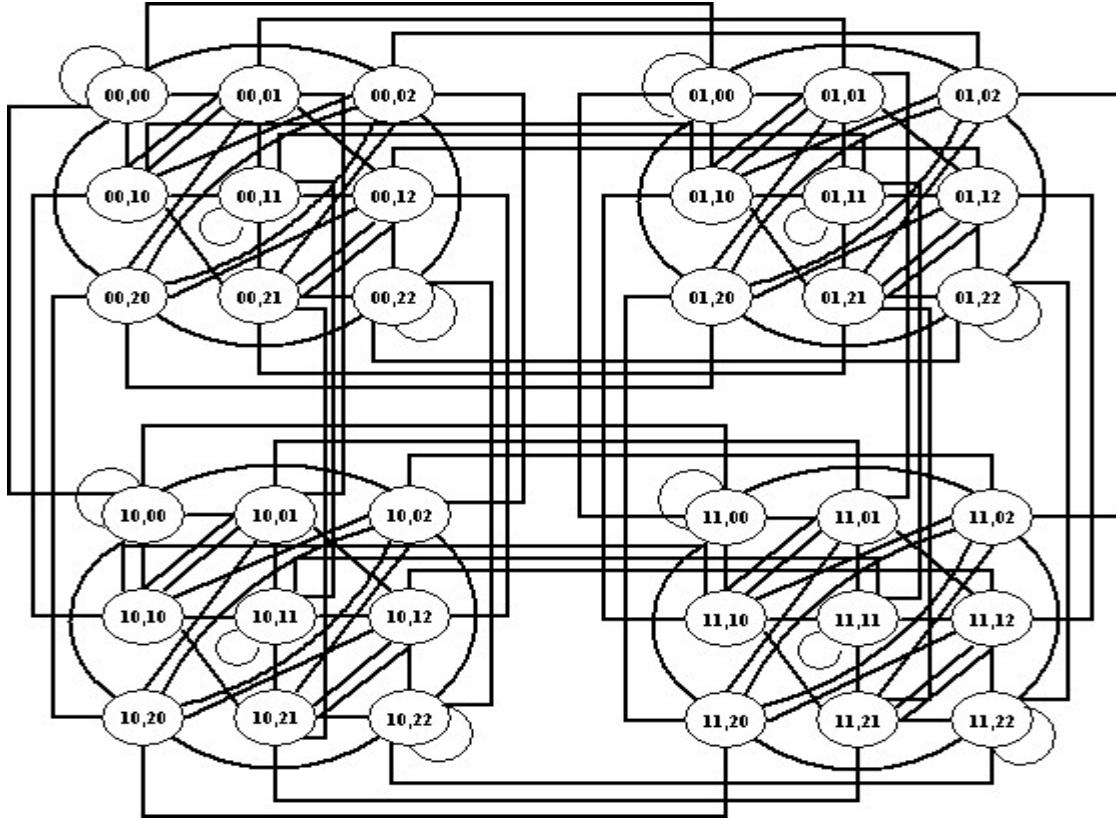


Figure 5. Hyper de Bruijn Aster $HD^*(2,3,2)$.

Corollary 3 Address of a node in HD^* includes 2 parts: the first part is the address of Hypercube graph and the second part is the address of de Bruijn graph.

Proof: this is a corollary directly inferred by definition of Hyper de Bruijn Aster.

3.SHORTEST PATH ROUTING AND FAULT TOLERANCE IN $HD^*(M,D,N)$

3.1.Shortest path routing

Shortest path routing from a vertex $v(h,d)$ to a vertex $v(h',d')$ can be established as follows,

- Shortest path from $v(h,d)$ to $v(h',d)$ can be achieved by shortest path routing scheme in hypercube graph, algorithms and proof are shown in references [4,5,12].
- Shortest path from $v(h',d)$ to $v(h',d')$ can be achieved by shortest path routing scheme in de Bruijn graph, algorithms and proof are shown in references [2,7].
- Or we can do the above steps inversely (routing in de Bruijn first and then routing in Hypercube)

Therefore, we have new theorem,

Theorem 2 $ShortestRouting(HD^*) = ShortestRouting(H) + ShortestRouting(D) (*)$

Or $ShortestRouting(HD^*) = ShortestRouting(D) + ShortestRouting(H)$

By theorem 2, we see that Shortest Path routing in HD^* is its advantage in comparison to Hypercube, de Bruijn, Butterfly... In the next section, we will present a shortest path routing algorithm in Hyper de Bruijn Aster. By applying our algorithm, nodes in the network can perform smoother, faster and especially efficient in Fault Tolerant, Routing and Load Balancing.

The following compare some of properties of Hyper de Bruijn Aster to others,

- Suppose Hyper de Bruijn Aster $HD^*(m_1, d_1, n_1)$, m_1 is hypercube's order(diameter), n_1 is de Bruijn's order(diameter), and d_1 is deBruijn's degree ($d_1 \geq 2$). Hyper Butterfly $HB(m_2, n_2)$, m_2 is hypercube's order, n_2 is Butterfly's order. HD^* and HB have the same total number of node.

- We have, $2^{m_1} d_1^{n_1} = n_2 2^{m_2 + n_2} \Leftrightarrow m_1 + n_1 \log_2 d_1 = m_2 + n_2 + \log_2 n_2 (1)$

Degree of HD^* : $d_{HD^*} = m_1 + 2d_1 \geq m_1 + 4$ because of $d_1 \geq 2$. This makes HD^* network performs better in fault tolerant routing and broadcasting.

Besides, diameter of HD^* : $D_{HD^*} = m_1 + n_1 = m_2 + n_2 + \log_2 n_2 - n_1 \log_2 d_1 + n_1$

Increase d_1 à decrease D_{HD^*} (total number of node is not change) and if $d_1 \geq 2n_2$ then $D_{HD^*} \leq D_{HB}$. It proves that HD^* network is more efficient than HB in Shortest Path routing and broadcasting.

For Fault Tolerance HD^* is proportional to $m_1 + 2d_1 - 2(2)$. Obviously, increasing d_1 will improve Fault-Tolerance of HD^* . While, Fault-Tolerance of HB is proportional to $m_2 + 4$.

By the above discussion, we see that Hyper de Bruijn Aster performs better than Hyper Butterfly in Fault Tolerant routing, Shortest path routing and Broadcasting.

The following table shows comparison among Hyper de Bruijn Aster to others,

Table 1. Hyper de Bruijn Aster in comparison to others.

Graph	Node	Degree	Diameter	Fault-Tolerance
Hypercube $H(m+n)$	$2m+n$	$m+n$	$m+n$	$m+n$
Butterfly $B(m+n)$	$(m+n)2m+n$	4	$3n/2$	4
de Bruijn $D(d, m+n)$	$dm+n$	$2d$	$m+n$	$2d-2$
HyperButterfly $HB(m, n)$	$n2m+n$	$m+4$	$m+3n/2$	$m+4$
Hyper de Bruijn Aster $HD^*(m, d, n)$	$2mdn$	$m+2d$	$m+n$	$m+2d-2$

Explanations for algorithm in figure 6:

- Step 1: use shortest path routing algorithm in de Bruijn graph[2,7] to find all shortest paths from (hS, dS) to (hS, dD) .
- Step 2: check fault tolerant characteristics of these paths from step 1.
- Step 3: if there exist a fault free shortest path from dS to dD , then route from (hS, dS) to (hS, dD) . Otherwise, go to step 7.
- Step 4: use shortest path routing algorithm in hypercube [4,12] to find all shortest paths from (hS, dD) to (hD, dD) .
- Step 5: check fault tolerant characteristics of these paths from step 4.
- Step 6: if there exists a fault free shortest path from hS to hD , then route from (hS, dD) to (hD, dD) , go to END. Otherwise, ignore node, go to step 7.
- Step 7: use shortest path routing algorithm in hypercube [4,12] to find all shortest paths from (hS, dS) to (hD, dS) .
- Step 8: check fault tolerant characteristics of these paths from step 7.
- Step 9: if there exists a fault free shortest path from hS to hD , then route from (hS, dS) to (hD, dS) . Otherwise, go to END.

- Step 10: use shortest path routing algorithm in de Bruijn graph[2,7] to find all shortest paths from (hD,dS) to (hD,dD).

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1  START
2  IF Node(hS,dS) = Node(hS,dD) THEN GOTO END;
3  ELSE
4      Node = Node(hS,dS)
5      Shortest Path Routing in de Bruijn from dS to dD
6  ENDIF
7  i,j belong to the set of nodes of shortest path routing in
Line 5
8  CALL CheckConnect(Node, Node[i,j])
9  IF CheckConnect(Node, Node[i,j]) = True THEN
10     CALL Routing(Node, Node[i,j])
11     i = i+1
12     Node(hS,dS) = Node[i,j]
13     GOTO START
14 ELSE
15     IF (j<maxPath) THEN
16         j=j+1
17         GOTO Line 3
18     ELSE
19         Shortest Path routing in Hypercube
20         k belongs to the set of nodes of shortest path
routing in Line 19
21         CALL CheckConnect(Node, NodeCube[k])
22         IF CheckConnect(Node, NodeCube[k])=True THEN
23             CALL Routing(Node, NodeCube[k])
24             Node(hS,dS)=NodeCube[k]
25             GOTO START
26         ELSE
27             IF (k<maxBit) THEN
28                 k=k+1
29                 GOTO Line 19
30             ELSE
31                 CALL IgnoreNode[i]
32                 RESET i,j
33                 GOTO START
34             ENDIF
35         ENDIF
36     ENDIF
37 ENDIF
38 END

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Figure 6. Shortest Path routing algorithm.

- Step 11: check fault tolerant characteristics of these paths from step 10.
- Step 12: if there exist a fault free shortest path from dS to dD, then route from (hD,dS) to (hD,dD). Otherwise, go to END.
- END.

Corollary 3 from the formula (*), the total number of shortest path cross N nodes (including source and destination node, with the difference between source and destination address is M (in digit)) is $N.M$ path.

Corollary 4 routing in Hypercube and de Bruijn in fault free mode can base on the difference between the source and destination address [4][2].

Corollary 5 Hyper de Bruijn Aster can be mapped following hypercube's edge (as presenting in figure 7). Therefore, if there is a fault when routing from a node to its adjacent node in the same cube then we can move this routing to the neighbor cube. However, we still keep address and routing direction in de Bruijn network

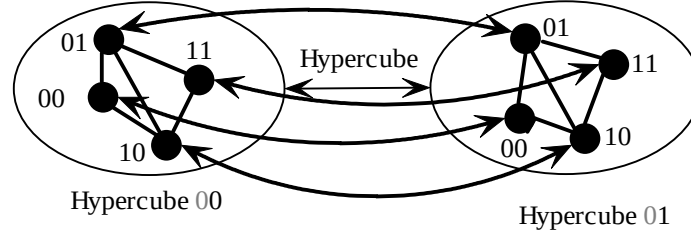


Figure 7. Mapping of each vertex of Hypercube 00 to Hypercube 01.

Example: to route from node (00,01) to node (01,11) in $HD^*(2,2,2)$ (figure 4), we do as follows,

- Use shortest path routing for de Bruijn graph in the same cube, i.e. (00,01) $\rightarrow$ (00,11) $\rightarrow$ (01,11).
- If node (00,11) is fault then it routes (00,01) $\rightarrow$ (01,01) $\rightarrow$ (01,11)

Corollary 6 by routing follows our shortest path routing algorithm, there is a proportion of path length and total number of shortest path, and it belongs to the difference in the number of digit between source and destination address. (Increasing the number of different digit makes longer path and hence increasing the total number of shortest path and fault possibility along the path).

3.2. Some fault tolerant characteristics of HD^*

Fault Tolerance of network is the possibility to continuously work when there are failure nodes in the network.

In HD^* , for a pair of source and destination node, we can find several shortest paths (≥ 1). Therefore, if there is a fault in a shortest path, then we can choose another shortest path to route. This improves fault tolerant. By applying our shortest path routing algorithm in section 3, we can avoid failure node along the path without reinitialize the whole process from the source node. Moreover, combining with “discrete set” concept in [7], we can provide fault free shortest path (optimum shortest path in the case of failure occurred) in HD^* .

In any network, the requirement for a routing algorithm to be successful is that the adjacent nodes of source node and cubes in the middle have to be in good condition (ready for transmission the message). Otherwise, it cannot route the message. Some Shortest path routing algorithms cannot work well in HD^* because of the above circumstances. It leads our shortest path routing algorithm to be optimum for fault tolerance.

Suppose, a node $X = \langle x_0 x_1 x_2 \dots x_{i-1} x_{i+1} \dots x_{n-1} \rangle$, $0 \leq i \leq n-1$

In $HD^*(m, d, n)$ the total adjacent nodes of X , N_{HD^*} is:

$$N_{HD^*} = m + 2d \leftrightarrow \exists x_i \mid x_i \neq x_{i+2}, 0 \leq i \leq n-3 \quad (3)$$

$$N_{HD^*} = m + 2d - 1 \leftrightarrow \left\{ \begin{array}{l} \forall x_i \mid x_i \neq x_{i+2}, 0 \leq i \leq n-3 \\ x_0 \neq x_1 \end{array} \right\} (4)$$

$$N_{HD^*} = m + 2d - 2 \leftrightarrow \forall x_i \mid x_i = x_{i+1}, 0 \leq i \leq n-2 (5)$$

Therefore, fault tolerance of HD* network when applying our shortest path routing algorithm is proportional to NHD* -1. By another mean, fault tolerance and time complexity of algorithm routing between 2 nodes belong to position of the failure nodes in the network.

4.CONCLUSION

Through our study in parallel processing systems, VLSI, multiprocessor networks, we invent a new topology called Hyper de Bruijn Aster HD*(m,d,n). Based on HD*, shortest path routing and fault tolerance are investigated. Our HD* has shown its superior in size (low diameter but large number of node), fault tolerance (because of the extending to high degree) and shortest path routing (our topology can provide more shortest paths than others). Consequently, our HD* is the best candidate for designing network of parallel processing systems, VLSI.

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