IDENTIFICATION OF DAMAGE IN STEEL BEAM BY NATURAL FREQUENCY USING XGB MODEL

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Abstract

Beams have played a significant role in engineering applications and they have been commonly used for modelling civil problems. Different models and methods have been developed to identify the damage to the beams. In this article, the extreme gradient boosting (XGB) model was developed to predict the location, width and depth of the saw-cut of steel beams by the change of natural frequencies. The natural frequencies of a steel beam in different scenarios were identified by the finite element method (FEM). The criterions to evaluate the accuracy of the models were the R squared (RSQ) and the mean square error (MSE). The result indicated that combining the FEM method with XGB would hold significant potential for applications in structural health monitoring.

***Keywords**: Extreme gradient boosting (XGB); prediction; natural frequencies; damage; beams.*

1. Introduction

Beams have played a significant role in engineering applications and they have been commonly used for modelling civil problems. In fact, different models and methods have been developed to identify the damage to the beams. Yang [1] applied the Galerkin's and energy method to identify the crack in vibrating beams. Swamidas [2] used Timoshenko and Euler formulation to determine the cracks in the beam. Gilbert-Rainer Gillich and Zhou Yun-Lai [3-5] detected the damaged crack based on the vibration measurement. Zhou Yun-Lai [6] also studied the forced vibration of the cracked beam. The results of these studies demonstrated a good performance in structural damage detection.

Machine Learning Techniques (MLT), a multidisciplinary field, encompass a variety of methods aimed at gaining new insights. The primary purpose of MLT is prediction, where categorical variables are forecasted through classification, and numerical variables are estimated using regression. Regression involves examining the relationship between one or more independent variables and a dependent variable. As a result, in recent decades, MLT has found successful applications in numerous engineering challenges [7-13].

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DOI: 10.56651/lqdtu.jst.v6.n02.741.sce

In recent years, Artificial Neural Networks (ANNs) are becoming an efficient tool for predicting the damage within the structure. Lee Jong-Won [14] developed a technique to detect location and size of a through-the-thickness crack in straight thin-walled pipe subjected to bending using the modal properties and ANN. Khatir Samir [15] addressed the damage identification problem by means of a Genetic Algorithm (GA) approach based on the change of the natural frequency. B. P. Gowd [16] proposed two algorithms of crack detection one using fuzzy logic (FL) and the other artificial neural networks (ANN). The artificial neural networks (ANN) and adaptive neuro-fuzzy inference systems (ANFIS) were also used to predict the size of the crack and its location based on the natural frequencies and frequency response functions [17]. The natural frequencies used as inputs for ANN were also presented by Nazari and Baghalian [18] and Rao Putti Srinivasa [19].

The use of extended Finite Element (XFEM) and eXtended IsoGeometric Analysis (XIGA) coupled with PSO and Jaya algorithm for predicting crack position and length in plates was presented by Khatir Samir [20]. Khatir Samir also developed a two-stages approach based on the normalized Modal Strain Energy Damage Indicator (nMSEDI) [21]. The result indicated that the Teaching Learning Based Optimization (TLBO) - Artificial Neural Network (ANN) - Particle Swarm Optimization (PSO) combined with IsoGeometric Analysis (IGA) could be used to determine correctly the severity of damage in beam structures. Gillich [22] proposed two machine learning methods (random forest (RF), and ANN) to detect the damage for a prismatic cantilever beam with one crack and ideal and non-ideal boundary conditions by using natural frequency shift and machine learning. The damage identification problem of simply supported uni-directionally reinforced graphite-epoxy beams was addressed with Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) by Khatir [23]. The objective functions used in the optimization process were also based on the dynamic analysis data of the structure, i.e. natural frequencies and mode shape. These above studies all show the ability to apply machine learning methods to predict construction damage with high accuracy. However, these studies have not mentioned the prediction of crack width, depth and position at the same time. The input data of these models were stress intensity factor range, stress ratio etc., these data are difficult quantities to measure in structure causing difficulties in practical application.

In this article, the XGB model is developed to predict the location, width and depth of the saw-cut of steel beams by the change of natural frequencies. The natural frequencies of a steel beam in different scenarios are identified by the FEM model. In order to assess the accuracy of the models, the R-squared (RSQ) and mean square error (RMSE) are utilized as evaluation criteria. By comparing the predicted data with the tested data, relative conclusions can be drawn.

2. Identification of natural frequencies by finite element method (FEM)

A testing structure under consideration is a steel cantilever beam, clamped on one end and free on the other (Fig. 1). The physical parameters of the steel beam are shown in Table 1. A three-dimensional finite element model is constructed in Abaqus, employing elastic beam elements as illustrated in Fig. 1. The beam structure is discretized into 34,080 elements. To replicate damage in the beam, the elements corresponding to the saw-cut are selectively removed, as depicted in Fig. 2. This study encompasses a total of 219 damage scenarios, with 214 scenarios labeled as No. 1 to No. 214 utilized for training and validation in building the ANN model. The remaining five scenarios are exclusively reserved for testing purposes.

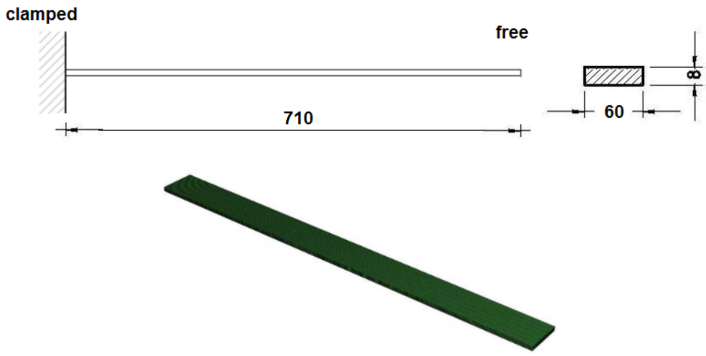

Fig. 1. Cantilever beam and finite element model (unit: mm).

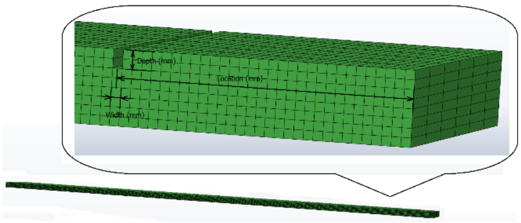

Fig. 2. Finite element model of saw-cut beam.

Table 1. The physical parameters of the steel beam

No.	Parameters	Value	Unit
1	Length	710	mm
2	Density	7850	kg/m^3
3	Modulus of elasticity	2.03×10^5	MPa
4	Width	60	mm
5	Height	8	mm
6	Poisson's ratio	0.28	

Table 2 presents the damage scenario identifications along with their associated natural frequencies, determined through Finite Element Analysis (FEM). Additionally, Table 3 provides essential information about the variable ranges within the database. These ranges are of utmost importance for predictions as they define the boundaries of the models.

Table 2. Database developed by FEM

No.	Location (mm)	Width (mm)	Depth (mm)	Natural frequencies			
				Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)	Mode 4 (Hz)
1	710.5	1	1	12.957	81.141	95.883	227.09
2	700.5	1	1	12.933	81.03	95.828	226.86
3	690.5	1	1	12.936	81.081	95.832	227.08
4	680.5	1	1	12.938	81.129	95.839	227.27
5	670.5	1	1	12.941	81.172	95.846	227.43
…	…	…	…	…	…	…	…
72	710.5	1	2	12.866	80.586	95.569	225.57
73	700.5	1	2	12.783	80.19	95.33	224.76
74	690.5	1	2	12.791	80.355	95.328	225.45
75	680.5	1	2	12.800	80.508	95.349	226.05
76	670.5	1	2	12.808	80.649	95.374	226.56
…	…	…	…	…	…	…	…
143	710	2	1	12.939	81.034	95.844	226.8
144	700	2	1	12.916	80.938	95.781	226.64
145	690	2	1	12.92	81.002	95.785	226.91
146	680	2	1	12.923	81.061	95.794	227.14
147	670	2	1	12.926	81.115	95.803	227.34
…	…	…	…	…	…	…	…
215	75.5	1	1	13.002	81.407	96.043	227.75
216	75.5	1	2	13.005	81.403	96.068	227.57
217	76	2	1	13.005	81.413	96.068	227.74
218	405.5	1	1	12.987	81.245	95.984	227.69
219	602.5	1	1	12.956	81.369	95.892	227.82

Table 3. Ranges of variables in the database

No.	Variable	unit	count	min	max
1	Location	mm	219	0	710.5
2	Width	mm	219	0	2
3	Depth	mm	219	0	2
4	Mode 1	Hz	219	12.783	13.008
5	Mode 2	Hz	219	80.19	81.455
6	Mode 3	Hz	219	95.328	96.084
7	Mode 4	Hz	219	224.76	227.95

3. Overview of extreme gradient boosting

The XGB model, which stands for Extreme Gradient Boosting, represents an enhanced version of the gradient boosting method introduced by [24]. As an advanced tree boosting system, it employs numerous additive functions to predict the outcome as:

$$y_i = y_0 + \eta \cdot \sum_{k=1}^{M} f_k(X_i) \tag{1}$$

where y_i represents the predicted outcome for the i^{th} sample of which the vector of the features is X_i, M refers to the number of estimators, and each estimator f_k (with k from 1 to M) represents an autonomous tree structure, the initial guess y_i^0 corresponds to the average value of the measured outcomes in the training set, η is the learning rate (also known as the shrinkage parameter, aids in enhancing the model's performance by facilitating a smooth improvement when incorporating new trees and preventing overfitting).

The training process is carried out in an additive manner, following the principles outlined in Eq. (2). At each k_{th} step, a new estimator is incorporated into the model, and the k_{th} predicted result, y_i^k, is computed by combining the predicted value from the previous step, y_i^{k-1}, with the estimation provided by the additional k_{th} estimator, f_k, as described below:

$$y_i^k = y_i^{k-1} + \eta \cdot f_k \tag{2}$$

The value of f_k is determined by the leave weights, which are obtained through the minimization of the objective function specific to the k_{th} tree, as expressed by the following equation:

$$obj = \gamma T + \sum_{j=1}^{T} \left[G_j \omega_j + \frac{1}{2}(H_j + \lambda)\omega_j^2 \right] \tag{3}$$

where T represents the number of leaves in the k_{th} tree, while ω_j (with j ranging from 1 to T) corresponds to the weights assigned to each leaf. The regularization parameters, λ and γ, are used to regulate the complexity of the tree structure and prevent overfitting. The terms G_j and H_j refer to the sums of the first and second gradients of the loss function, respectively, over all the samples associated with the j_{th} leaf.

The construction of the k_{th} tree involves iteratively splitting the leaves, beginning with a single leaf. This process is carried out by maximizing the gain parameter, which is defined as follows (4):

$$gain = \frac{1}{2}\left[\frac{G_L^2}{H_L+\lambda} + \frac{G_R^2}{H_R+\lambda} - \frac{(G_L+G_R)^2}{H_L+H_R+\lambda}\right] - \gamma \tag{4}$$

The equation presented above calculates the gain parameter, which is determined by the values G_L, H_L, G_R, and H_R associated with the left and right leaves after the splitting. The splitting is considered valid if the gain parameter is greater than 0. By increasing the regularization parameters λ and γ, the gain parameter is reduced, which helps in maintaining a simpler tree structure by limiting leaf splitting. However, it is important to note that increasing these regularization parameters also decreases the model's capacity to fit the training data accurately.

4. Development of the XGB model

The dataset has been partitioned into two distinct subsets: the training set, utilized for model calibration, and the testing set, employed for model verification. The selection of samples for the testing set is entirely randomized to ensure unbiased evaluation. To ensure equal consideration for all variables during the training process, preprocessing is performed by scaling both the input and output variables within the range of 0.0 to 1.0. The scaling process for each variable is computed as follows:

$$x_n = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \tag{5}$$

where $x_{\max}$ and $x_{\min}$ are the maximum and minimum values of each variable x.

The input variables selected for these experiments were the natural frequencies of four Modes, the output variables chosen were the location, the width and the depth of the saw-cut.

In this study, the XGB model was implemented using Python and the Sklearn library. To ensure optimal performance, we have examined the most critical hyperparameters that affect the model's output. This includes the number of estimators M, the learning rate g, and the regularization parameters λ and γ, which have been carefully analyzed to determine the best possible value. By doing so, default values set in the XGB package are considered for the other parameters. It has been observed from our experiences that the impact of the remaining parameters (excluding the four main parameters) in the XGB package is deemed negligible. The criteria to evaluate the accuracy of the models are the R squared (RSQ) and the mean square error (MSE). A better model is reflected in a higher R-squared, while conversely, a lower MSE signifies an improved model.

4.1. Effect of the number of estimators on model effectiveness

The XGB model undergoes training with a wide array of estimator values, denoted as 'M', spanning from 10 to 1000. The default configurations for the remaining parameters within the XGB package are applied. As depicted in Fig. 3, the optimal combination of RSQ and MSE values is achieved when the number of estimators is set to 100. However, extensive experimentation encompassing estimator values ranging from 200 to 1000 did not yield any noticeable improvements beyond the performance achieved with 100 estimators. Consequently, opting for 100 estimators emerges as the favored choice, striking a balance between predictive accuracy and model simplicity.

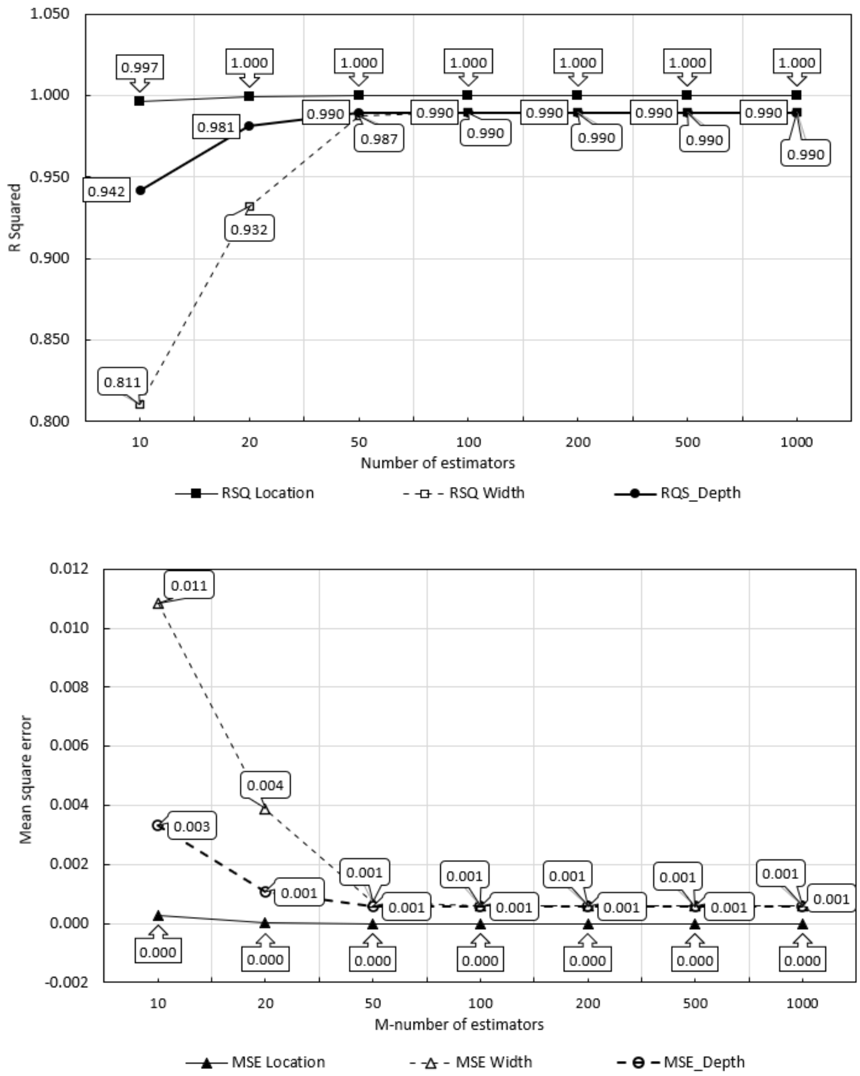

Fig. 3. Effect of the number of estimators on model effectiveness.

4.2. Effect of the learning rate on model effectiveness

The training process of the XGB model involves a range of learning rates, η, extending from 0.03 to 1. The number of estimators is held constant at the optimized value of 100, while default configurations are retained for the remaining parameters. As illustrated in Fig. 4, the most favorable RSQ and MSE values are achieved when the learning rates are configured at 0.3.

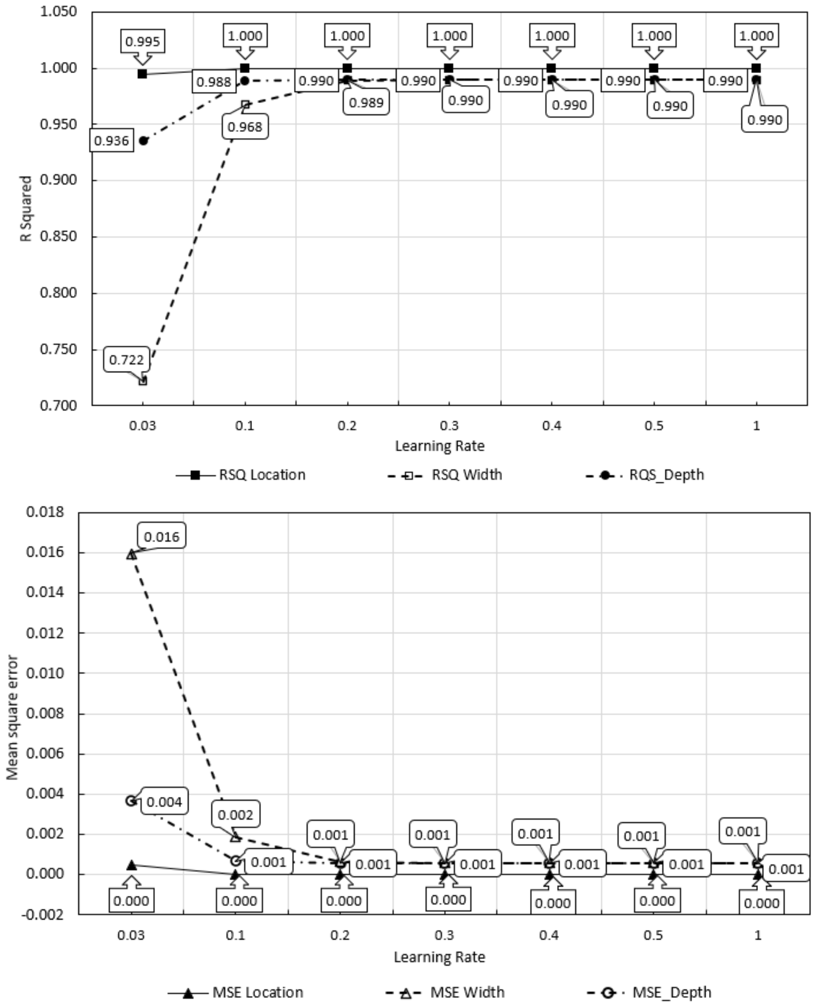

Fig. 4. Effect of the learning rate on model effectiveness.

4.3. Effect the regularization parameter λ on model effectiveness

In order to evaluate the XGB model's responsiveness to changes in the regularization parameter λ, we maintain the number of estimators and learning rate at their optimal settings: M = 100 and η = 0.3, as depicted in Fig. 5. We systematically vary the parameter λ over a spectrum from 0 to 100. The outcomes clearly indicate that selecting λ = 1 leads to superior performance when contrasted with other scenarios.

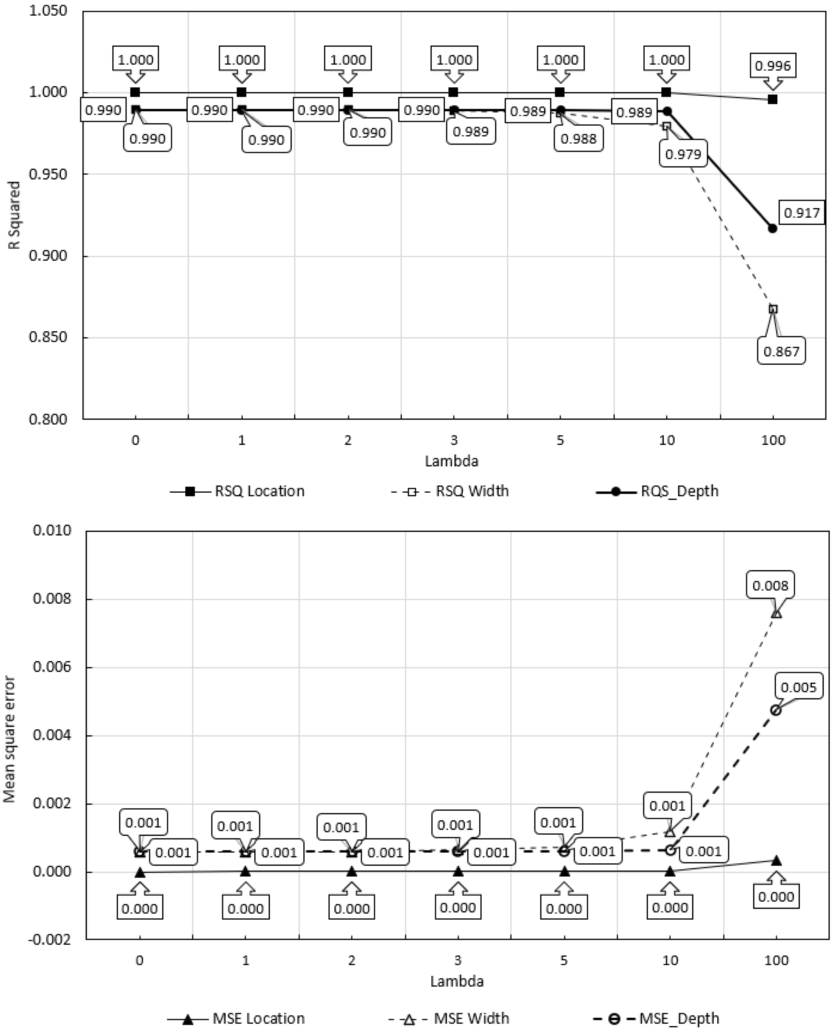

Fig. 5. Effect the regularization parameter λ on model effectiveness.

4.4. Effect the regularization parameter γ on model effectiveness

Similarly, as illustrated in Fig. 6, we examine how the XGB model reacts to variations in the regularization parameter γ. Over the range of 0 to 100, we observed that the influence of γ was relatively negligible, and the model exhibited slightly enhanced performance when γ was configured as 0.

Based on the findings, the ideal configuration for the XGB model included 100 estimators, a learning rate of 0.3, a regularization parameter λ of 1, and a regularization parameter γ of 0. This particular setup demonstrated the highest degree of accuracy and will be chosen for the upcoming model validation and verification phases.

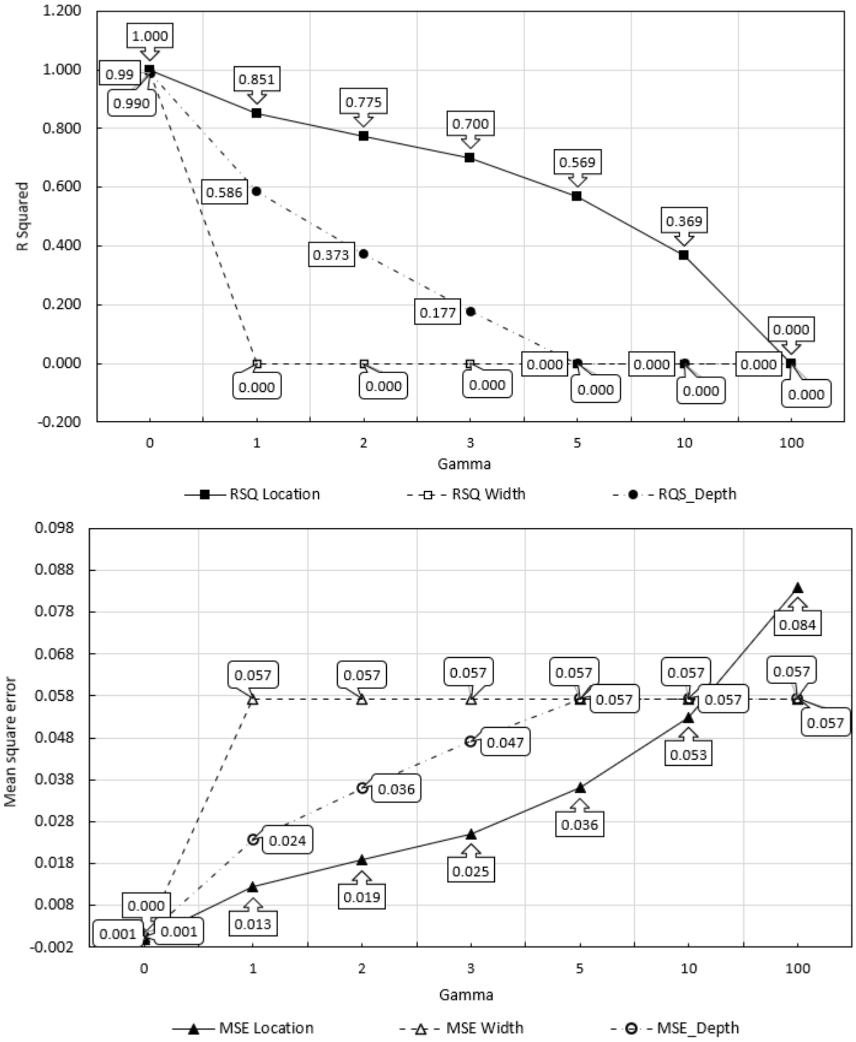

Fig. 6. Effect the regularization parameter γ on model effectiveness.

5. Model validation

Following the training phase, the ANN model undergoes verification using the test dataset. During the testing process, the R-squared value (RSQ) is determined to be 0.952, while the mean squared error (MSE) is calculated as 0.459. This can be attributed to the fact that the test data is entirely randomized and unfamiliar.

Figure 7-9 illustrate the performance of the XGB model with respect to the location, width, and depth of the saw-cut, respectively. Notably, the model exhibits the highest accuracy in predicting the saw-cut's position. The predicted values for the location of the saw-cut demonstrate minimal dispersion around the best-fit line. This could be attributed to the more comprehensive coverage of training data for the location compared to the other parameters.

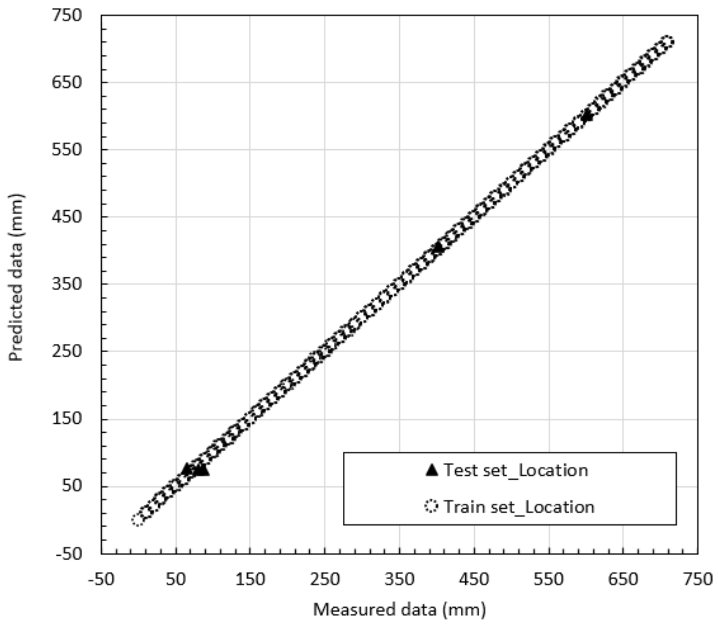

Fig. 7. Scatter plots of predicted versus measured data for the position.

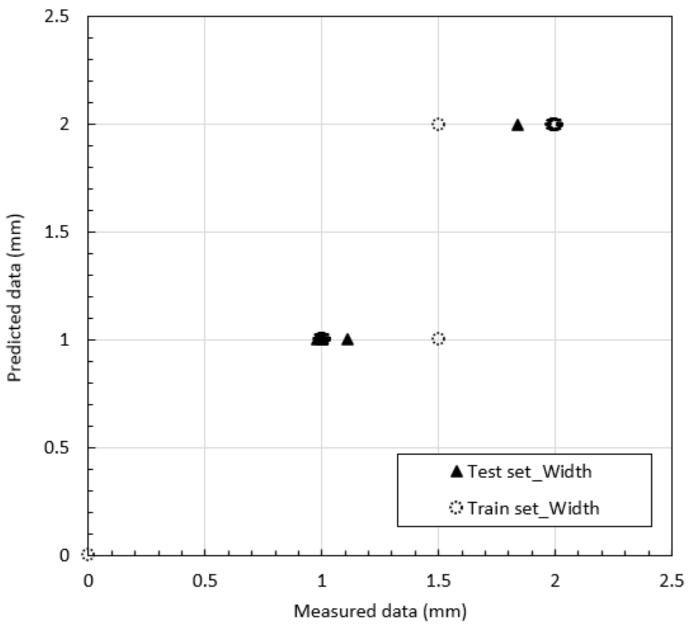

Fig. 8. Scatter plots of predicted versus measured data for the width.

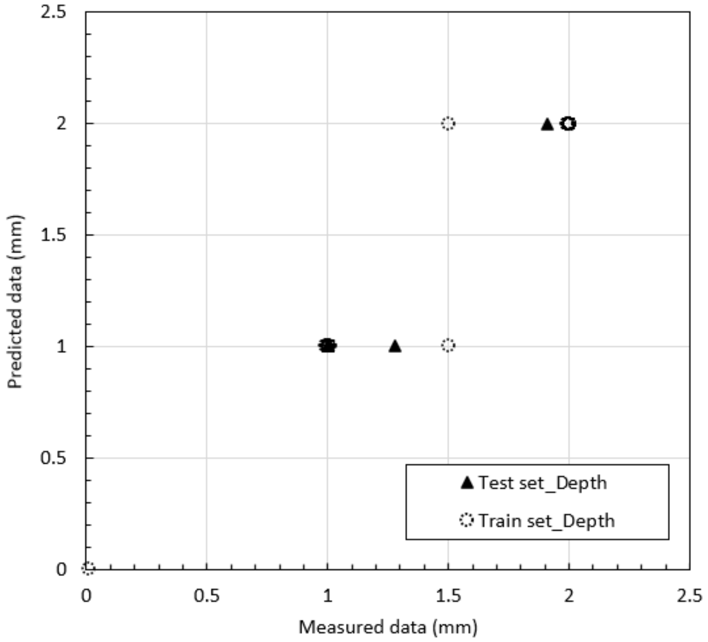

Fig. 9. Scatter plots of predicted versus measured data for the depth.

Table 4. Accuracy in predicting the saw-cut location of the model

Predetermined value (mm)			Predicted value (mm)			Deviation (%)		
Location	**Width**	**Depth**	**Location**	**Width**	**Depth**	**Location**	**Width**	**Depth**
75.5	1	1	81.17	0.98	1.00	7.51	-2.31	0.10
75.5	1	2	86.58	1.11	1.91	14.68	11.06	-4.39
76	2	1	65.26	1.84	1.28	-14.13	-7.99	27.97
405.5	1	1	401.02	1.00	1.00	-1.11	0.22	-0.04
602.5	1	1	600.34	1.00	1.00	-0.36	0.09	0.05

Table 4 presents the prediction accuracy for the saw-cut's location, width, and depth. Notably, the smallest deviation between the measured and predicted values is observed for the saw-cut width, with a maximum deviation of 11.06%. In contrast, the maximum deviations for location and depth are 14.68% and 27.97%, respectively. This trend is further supported by the R-squared values, where the testing set exhibits R-squared values of 0.999 for location and 0.956 for width prediction, both higher than the R-squared value for depth prediction. Nevertheless, all predicted factors maintain a high accuracy level with R-squared values exceeding 0.9. Consequently, utilizing the XGB model proves effective in accurately predicting the saw-cut's position, width, and depth.

6. Conclusion

After using FEM method to generate data and using XGB model to predict the location, the width and of the saw-cut of steel beams by the natural frequency, the main findings of this study were the following:

- The XGB model successfully predicted the location, width, and depth of the saw-cut simultaneously within the beam using natural frequencies. All predicted factors achieved a high level of accuracy with R-squared values exceeding 0.9. Specifically, the model exhibited better accuracy in predicting the saw-cut's location and width compared to its prediction of the saw-cut depth.

- The FEM can be employed to generate a learning dataset, especially when sufficient monitoring data is unavailable initially. Combining the FEM method with XGB and certain natural frequencies identification techniques holds significant potential for applications in structural health monitoring.

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SỬ DỤNG MÔ HÌNH XGB DỰ BÁO HƯ HỎNG CỦA DẦM THÉP THÔNG QUA TẦN SỐ DAO ĐỘNG RIÊNG

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Tóm tắt: Dầm là cấu kiện quan trọng trong kỹ thuật và thường được sử dụng để mô hình hóa cho các bài toán. Đã có nhiều phương pháp, mô hình được phát triển để xác định hư hỏng cũng như khuyết tật của dầm. Trong bài báo này, thuật toán tăng cường độ dốc cấp cao (XGB) được phát triển để dự đoán vị trí, chiều rộng và chiều sâu của vết cắt dầm thép thông qua sự thay đổi tần số dao động riêng. Tần số dao động riêng của dầm thép trong các kịch bản khác nhau được xác định bằng mô hình phần tử hữu hạn (FEM). Các tiêu chí để đánh giá độ chính xác của mô hình là R squared (RSQ) và sai số trung bình bình phương (MSE). Kết quả cho thấy việc kết hợp phương pháp FEM với XGB là rất có tiềm năng và ý nghĩa trong việc quan trắc cảnh báo cho các công trình.

Từ khóa: *Thuật toán tăng cường độ dốc cấp cao (XGB); dự báo; tần số dao động riêng; hư hỏng; dầm.*

Received: 02/10/2023; Revised: 26/12/2023; Accepted for publication: 27/12/2023

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