NONLINEAR STABILITY CONTROL OF INVERTED PENDULUM ON A CART USING LQR-BASED T-S FUZZY CONTROL

Thi Van Anh Nguyen[1], Quy Thinh Dao[1,*]

[1]*School of Electrical and Electronic Engineering,*
Hanoi University of Science and Technology, Hanoi, Vietnam

Abstract

The inverted pendulum on a cart is a challenging and widely studied control problem in the fields of control systems and robotics. To address the need for more effective control strategies, this article presents a novel approach that combines the strengths of two powerful control techniques: Linear-Quadratic Regulator (LQR) and Takagi-Sugeno (T-S) fuzzy control. LQR control is well-established for stabilizing linear systems, but it faces limitations with nonlinear systems like the inverted pendulum. On the other hand, T-S fuzzy control excels in handling nonlinearities by approximating the system's behavior with local linear models. Our proposed approach leverages T-S fuzzy systems to approximate complex nonlinearities, while using LQR control for each local linear subsystem. The combined method's efficacy is substantiated by simulation results, specifically considering criteria such as stability under disturbance conditions, variations in the initial angle of the pendulum, and a comparative analysis between stability-only scenarios and those involving both swing-up and stability control.

Keywords: *Linear-Quadratic Regulator; Takagi-Sugeno fuzzy control; inverted pendulum; stability control.*

1. Introduction

The inverted pendulum on a cart is a classic and challenging control problem that has attracted considerable attention in the field of control systems and robotics [1-3]. Stabilizing such an inherently unstable system is of paramount importance for a wide range of applications, including autonomous vehicles, robotic manipulators, and various balancing mechanisms. Motivated by the need for more effective control strategies for the inverted pendulum on a cart, this article proposes a novel approach that combines the strengths of two powerful control techniques: Linear-Quadratic Regulator (LQR) and Takagi-Sugeno (T-S) fuzzy control.

The Linear-Quadratic Regulator (LQR) control is a well-established technique for stabilizing linear systems [4-7]. It aims to design an optimal control law that minimizes

[*] Email: thinh.daoquy@hust.edu.vn
DOI: 10.56651/lqdtu.jst.v18.n03.693

a quadratic cost function, penalizing both state deviations and control effort. LQR control relies on accurate system modeling and precise state feedback to compute the control input at each time step. While LQR is effective for linear systems, it faces challenges when dealing with nonlinear systems like the inverted pendulum on a cart.

Takagi-Sugeno (T-S) fuzzy control, on the other hand, is a powerful method for handling nonlinear control problems [8-10]. It approximates a complex nonlinear system by dividing the state space into regions and employing local linear models to represent the system’s behavior within each region. The T-S fuzzy control approach uses fuzzy logic to combine the local linear models and generate control actions based on the current state of the system. By using fuzzy membership functions, it can effectively deal with uncertainties and approximate the nonlinear dynamics with a set of simpler local linear models.

Numerous studies have explored the integration of fuzzy control and LQR techniques [11-13]. For instance, in [11], the author applied a combination of the LQR controller and a Mamdani-type fuzzy controller to manage a double inverted pendulum system. However, it’s worth noting that Mamdani fuzzy controllers possess a drawback: they require experiential knowledge for selecting the appropriate fuzzy control rules. In contrast, the incorporation of T-S (Takagi-Sugeno) fuzzy control laws provides a comprehensive representation of the control object’s kinematic model. This approach offers several advantages when compared to the Mamdani method, as it enables a more systematic control strategy.

Our aim is to address the nonlinear nature of the system's dynamics by leveraging T-S fuzzy systems to approximate the complex nonlinearities, while harnessing the control capabilities of LQR for each local linear subsystem. In the proposed approach, we construct a T-S fuzzy system to represent the dynamics of the inverted pendulum. Each fuzzy rule corresponds to a local linear model that approximates the system's behavior in a specific region of the state space. For each local linear subsystem, we design an optimal state feedback gain matrix using LQR control, effectively stabilizing the system around the equilibrium points within each region.

Our contributions in this work are threefold:

Present a comprehensive formulation of the T-S fuzzy system for the inverted pendulum on a cart, enabling the effective representation of its nonlinear dynamics through a set of local linear models.

Apply LQR control to each local linear subsystem, harnessing the advantages of optimal control for stability and performance.

Propose a novel scheme for combining the LQR-based T-S fuzzy control outputs between the local control laws.

2. System modeling

The system model for the inverted pendulum on a cart, as illustrated in Fig. 1, comprises essential parameters that define its dynamics: the mass of the vehicle is denoted by M (kg), the pendulum mass is represented as m (kg), the length of the connecting rod is denoted as l (m), the acceleration due to gravity is given by g (m/s^2), the external force u (N) acts on the cart, the angle of pendulum relative to the vertical is represented as θ (rad), with the positive direction considered as counter-clockwise. In our assumptions, we consider the mass of the connecting rod to be negligible. Therefore, we do not take into account the kinetic energy associated with the connecting rod [14, 15].

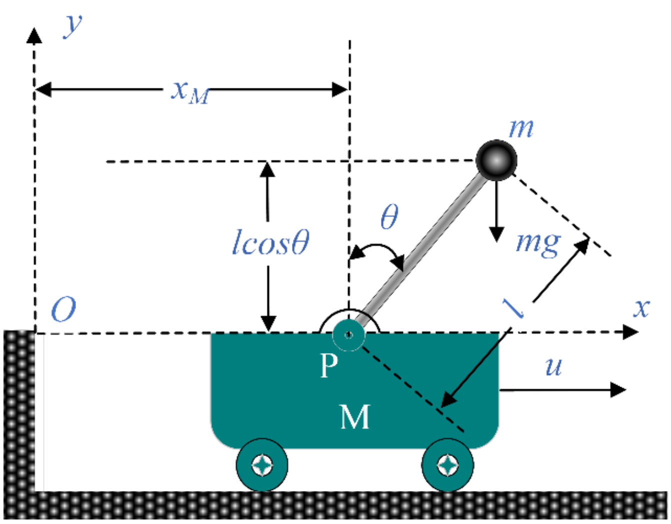


Fig. 1. Inverted pendulum on a cart.

Let the coordinates of the pendulum (x_m, y_m), then:

$$\begin{cases} x_m = x_M + l\sin\theta \\ y_m = l\cos\theta \end{cases} \tag{1}$$

with x_M are the coordinates of the cart.

Taking the time derivative of equation (1), we obtain:

$$\begin{cases} \dot{x}_m = \dot{x}_M + l\dot{\theta}\cos\theta \\ \dot{y}_m = -l\theta\sin\theta \end{cases} \tag{2}$$

Under the assumption that the cart has zero potential energy, the potential energy of the pendulum can be expressed as:

$$V = mgy_m = mgl\cos\theta \tag{3}$$

Kinetic energy of the pendulum:

$$\begin{aligned} T &= \frac{1}{2}M\dot{x}_M^2 + \frac{1}{2}m(\dot{x}_m^2 + \dot{y}_m^2) \\ &= \frac{1}{2}M\dot{x}_M^2 + \frac{1}{2}m[\dot{x}_M^2 + 2l\dot{\theta}\dot{x}_M\cos\theta + l^2\dot{\theta}^2(\cos^2\theta + \sin^2\theta)] \\ &= \frac{1}{2}(M+m)\dot{x}_M^2 + \frac{1}{2}ml^2\dot{\theta}^2 + ml\dot{\theta}\dot{x}_M\cos\theta \end{aligned} \tag{4}$$

The dynamic equation of the inverted pendulum is derived using Lagrange's equation.

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_i}\right) - \frac{\delta L}{\delta q_i} = Q_i \tag{5}$$

with $L = T - V$.

From (5), we obtain:

$$(M+m)\ddot{x}_M + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u \tag{6}$$

$$ml^2\ddot{\theta} + (ml\ddot{x}_M - ml\dot{x}_M\dot{\theta}\sin\theta) - (ml\dot{\theta}\dot{x}_M\sin\theta + mgl\sin\theta) = 0 \tag{7}$$

Writing equation (7) compactly and dividing both sides by ml, we obtain:

$$l\ddot{\theta} + \ddot{x}_M\cos\theta - g\sin\theta = 0 \tag{8}$$

We get:

$$\frac{(M+m)g\sin\theta}{\cos\theta} - \frac{(M+m)l\ddot{\theta}}{\cos\theta} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u \tag{9}$$

or

$$\ddot{\theta} = \frac{(M+m)g\sin\theta}{l[m(1-\cos^2\theta)+M]} + \frac{-ml\dot{\theta}^2\sin\theta\cos\theta}{l[m(1-\cos^2\theta)+M]} + \frac{-\cos\theta u}{l[m(1-\cos^2\theta)+M]} \tag{10}$$

Setting the state variable $x = [\theta\ \dot{\theta}]^T$, rewrite (10) as $\dot{x} = Ax + Bu$:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{(M+m)g\sin\theta}{\theta l[m(1-\cos^2\theta)+M]} & \dfrac{-ml\dot{\theta}\sin\theta\cos\theta}{l[m(1-\cos^2\theta)+M]} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{-\cos\theta}{l[m(1-\cos^2\theta)+M]} \end{bmatrix} u \quad (11)$$

with $A = \begin{bmatrix} 0 & 1 \\ \dfrac{(M+m)g\sin\theta}{\theta l[m(1-\cos^2\theta)+M]} & \dfrac{-ml\dot{\theta}\sin\theta\cos\theta}{l[m(1-\cos^2\theta)+M]} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \dfrac{-\cos\theta}{l[m(1-\cos^2\theta)+M]} \end{bmatrix}$

3. LQR-based T-S Fuzzy Control

The control design for the stability control of the inverted pendulum on a cart using the combined LQR-based T-S fuzzy control approach is presented in the following steps:

Step 1: *Fuzzy Rule Base*

Design the fuzzy rule base to represent the dynamics of the inverted pendulum. Each fuzzy rule corresponds to a local linear model that approximates the system's behavior within a specific region of the state space. Define linguistic variables and membership functions that capture the relevant system states and control actions. Construct the T-S fuzzy system using the designed fuzzy rule base. The T-S fuzzy system approximates the nonlinear dynamics of the inverted pendulum by dividing the state space into regions and utilizing the local linear models defined in the fuzzy rules.

Based on T-S fuzzy model, the equation $\dot{x} = Ax + Bu$ can be presented as [16]:

$$\dot{x} = \sum_{i=1}^{r} h_i (A_i(z_i)x + B_i(z_i)u) \quad (12)$$

with *r* is the number of fuzzy rules, *i-th* rule, h_i is the membership function and z_i is the premisse varialbe. The membership function can be calculated by:

$$h_i = \prod_{i=1}^{r} \omega_i^j \text{ , j = 0,1} \quad (13)$$

$$\omega_i^0 = \frac{z_{i\max} - z_i}{z_{i\max} - z_{i\min}}, \omega_i^1 = 1 - \omega_i^0. \quad (14)$$

From (11), we choose the variable premise as:

$$z_1 = \frac{1}{l\left[m(1-\cos^2\theta)+M\right]},\ z_2 = \frac{\sin\theta}{\theta},\ z_3 = \cos\theta,\ z_4 = \dot{\theta}\sin\theta \tag{15}$$

Then

$$A = \begin{bmatrix} 0 & 1 \\ (M+m)gz_1z_2 & -mlz_1z_3z_4 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -z_1z_3 \end{bmatrix} \tag{16}$$

Table 1. Fuzzy rules

Rules	**z1**	**z2**	**z3**	**z4**
Rule 1	$z_{1\min}$	$z_{2\min}$	$z_{3\min}$	$z_{4\min}$
Rule 2	$z_{1\max}$	$z_{2\min}$	$z_{3\min}$	$z_{4\min}$
Rule 3	$z_{1\min}$	$z_{2\max}$	$z_{3\min}$	$z_{4\min}$
Rule 4	$z_{1\max}$	$z_{2\max}$	$z_{3\min}$	$z_{4\min}$
Rule 5	$z_{1\min}$	$z_{2\min}$	$z_{3\max}$	$z_{4\min}$
Rule 6	$z_{1\max}$	$z_{2\min}$	$z_{3\max}$	$z_{4\min}$
Rule 7	$z_{1\min}$	$z_{2\max}$	$z_{3\max}$	$z_{4\min}$
Rule 8	$z_{1\max}$	$z_{2\max}$	$z_{3\max}$	$z_{4\min}$
Rule 9	$z_{1\min}$	$z_{2\min}$	$z_{3\min}$	$z_{4\max}$
Rule 10	$z_{1\max}$	$z_{2\min}$	$z_{3\min}$	$z_{4\max}$
Rule 11	$z_{1\min}$	$z_{2\max}$	$z_{3\min}$	$z_{4\max}$
Rule 12	$z_{1\max}$	$z_{2\max}$	$z_{3\min}$	$z_{4\max}$
Rule 13	$z_{1\min}$	$z_{2\min}$	$z_{3\max}$	$z_{4\max}$
Rule 14	$z_{1\max}$	$z_{2\min}$	$z_{3\max}$	$z_{4\max}$
Rule 15	$z_{1\min}$	$z_{2\max}$	$z_{3\max}$	$z_{4\max}$
Rule 16	$z_{1\max}$	$z_{2\max}$	$z_{3\max}$	$z_{4\max}$

Step 2: *Local Linear Models*

Within each region defined by the fuzzy rule base, extract local linear models that best represent the system's behavior. Identify the state matrices (A, B, C, D) for each local linear subsystem, which will be used for applying LQR control.

Using four premise variables, a total of 16 fuzzy rules are established in Table 1, where each fuzzy rule corresponds to an attached sub-matrix A and B. Here are some submatrices A and B:

$$A_1 = \begin{bmatrix} 0 & 1 \\ (M+m)gz_{1\min}z_{2\min} & -mlz_{1\min}z_{3\min}z_{4\min} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ -z_{1\min}z_{3\min} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ (M+m)gz_{1\max}z_{2\min} & -mlz_{1\max}z_{3\min}z_{4\min} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -z_{1\max}z_{3\min} \end{bmatrix}$$

$$A_8 = \begin{bmatrix} 0 & 1 \\ (M+m)gz_{1\max}z_{2\max} & -mlz_{1\max}z_{3\max}z_{4\min} \end{bmatrix}, B_8 = \begin{bmatrix} 0 \\ -z_{1\max}z_{3\max} \end{bmatrix}$$

$$A_9 = \begin{bmatrix} 0 & 1 \\ (M+m)gz_{1\min}z_{2\min} & -mlz_{1\min}z_{3\min}z_{4\max} \end{bmatrix}, B_9 = \begin{bmatrix} 0 \\ -z_{1\min}z_{3\min} \end{bmatrix}$$

$$\vdots$$

$$A_{15} = \begin{bmatrix} 0 & 1 \\ (M+m)gz_{1\min}z_{2\max} & -mlz_{1\min}z_{3\max}z_{4\max} \end{bmatrix}, B_{15} = \begin{bmatrix} 0 \\ -z_{1\min}z_{3\max} \end{bmatrix}$$

$$A_{16} = \begin{bmatrix} 0 & 1 \\ (M+m)gz_{1\max}z_{2\max} & -mlz_{1\max}z_{3\max}z_{4\max} \end{bmatrix}, B_{16} = \begin{bmatrix} 0 \\ -z_{1\max}z_{3\max} \end{bmatrix}$$

Step 3: *LQR Control Design*

For each local linear subsystem, design an optimal state feedback gain matrix (K) using the LQR control technique. The LQR control aims to stabilize the system around the equilibrium point within each region, enhancing stability and performance.

The cost function for the LQR control is given as:

$$J = \int_0^\infty (x^T Q x + u_{LQR}{}^T R u_{LQR}) dt \tag{17}$$

where Q is a positive definite matrix weighing the state deviation, and R is a positive definite matrix weighing the control effort. The LQR control design aims to find an optimal state feedback gain matrix K_{LQR} that minimizes the cost function J.

The optimal feedback gain matrix K_{LQR} is computed by solving the algebraic Riccati equation:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{18}$$

where P is a positive definite matrix known as the solution of the Riccati equation. Once the K_{LQR} matrix is obtained, the control input u_{LQR} can be calculated as:

$$u_{LQR} = -K_{LQR}x \tag{19}$$

The LQR control provides a feedback control law that stabilizes the system around the equilibrium point and optimizes the performance based on the chosen weighting matrices Q and R.

Step 4: *Fuzzy Inference and Weighted Control Combination*

During operation, use fuzzy inference to determine the active fuzzy rule (corresponding to the appropriate local linear subsystem) based on the current state of the system. The fuzzy inference process combines information from different linguistic variables and membership functions to identify the active rule.

Some membership functions are given as:

$$h_1 = \omega_1^0 \times \omega_2^0 \times \omega_3^0 \times \omega_4^0$$

$$h_2 = \omega_1^1 \times \omega_2^0 \times \omega_3^0 \times \omega_4^0$$

$$h_8 = \omega_1^1 \times \omega_2^1 \times \omega_3^1 \times \omega_4^0$$

$$h_9 = \omega_1^0 \times \omega_2^0 \times \omega_3^0 \times \omega_4^1$$

$$h_{15} = \omega_1^0 \times \omega_2^1 \times \omega_3^1 \times \omega_4^1$$

$$h_{16} = \omega_1^1 \times \omega_2^1 \times \omega_3^1 \times \omega_4^1$$

Step 5: *Defuzzification and Control*

Combine the control inputs from the active LQR controllers (one for each region) using an appropriate weighting scheme. The weighting scheme considers the degree of membership of the current state in each region. Defuzzification converts the fuzzy controls into a crisp control signal, providing the final control input to stabilize the inverted pendulum on a cart:

$$u = -\sum_{i=1}^{r} h_i K_{LQRi} x \tag{20}$$

Through the integration of LQR-based T-S fuzzy control, this design approach aims to enhance stability, robustness, and control performance, effectively addressing the challenges posed by the nonlinear nature of the inverted pendulum system.

4. Simulation results

The simulation study involves assessing the effectiveness of a proposed controller in stabilizing a pendulum at the vertically upright position, starting from different initial angles. Two scenarios are considered: one with no external disturbances and another with a random disturbance introduced into the system. The simulation parameters are provided, including the pendulum's (M = 1.2 kg), the mass of the cart (m = 0.2 kg), and the length of the pendulum (l = 0.3 m). The initial pendulum angles are set as $\theta_0 = \pi/6; \pi/9; \pi/12$ (rad). The maximum and minimum values of premise variable in (15) are given in Table 2. Some results of linear subsystems are shown in Table 3.

Table 2. The maximum and minimum values of premise variable

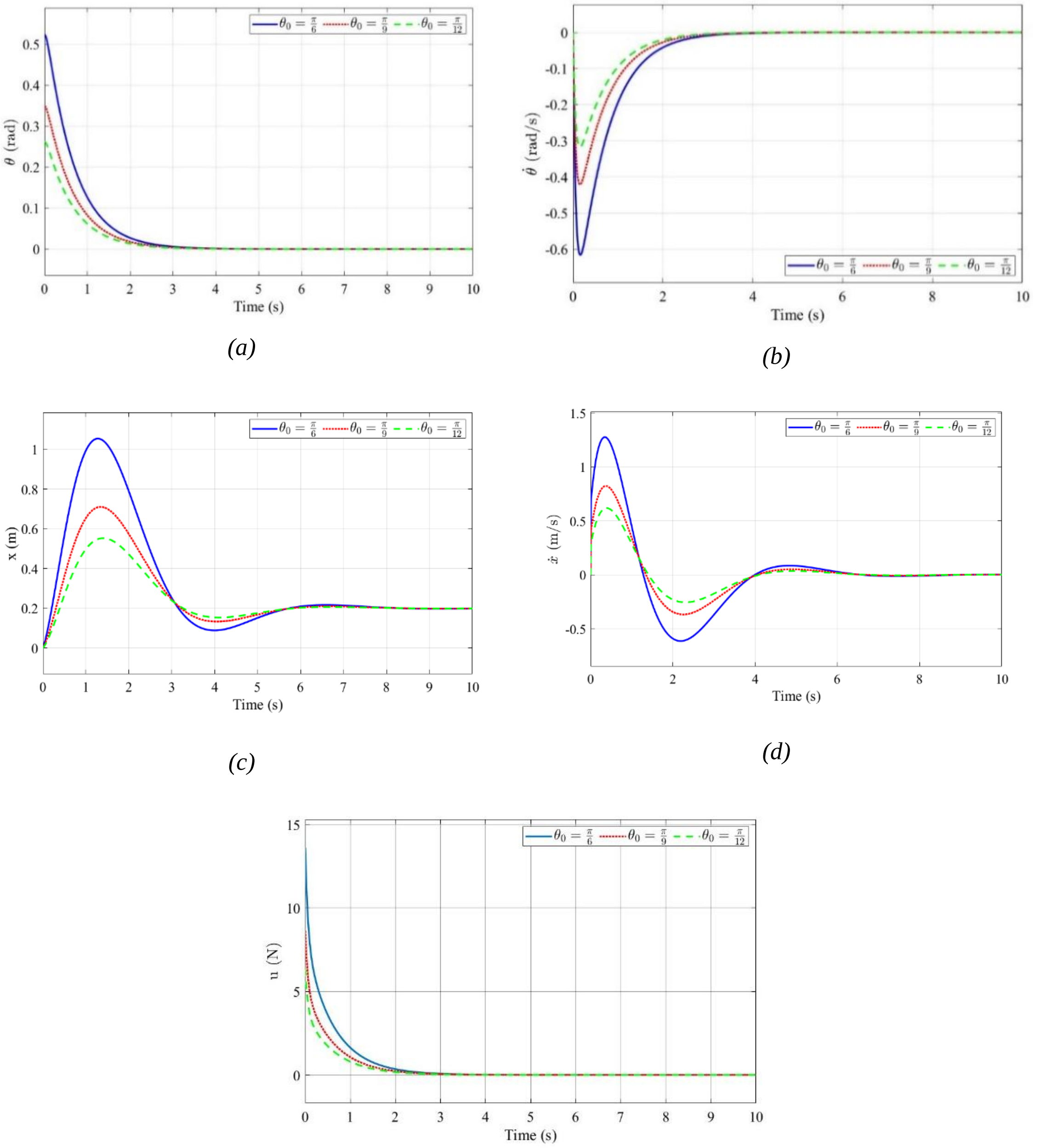
	z_1	z_2	z_3	z_4
max	2.7778	1	1	1.5
min	2.6667	0.9549	0.8660	-1.5

Table 3. Results of linear subsystem

i	A_i	B_i	K_i
1	$\begin{bmatrix} 0 & 1 \\ 34.97 & 0.21 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -2.31 \end{bmatrix}$	$\begin{bmatrix} -31.86 & -5.44 \end{bmatrix}$
2	$\begin{bmatrix} 0 & 1 \\ 36.43 & 0.22 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -2.41 \end{bmatrix}$	$\begin{bmatrix} -31.86 & -5.33 \end{bmatrix}$
8	$\begin{bmatrix} 0 & 1 \\ 38.15 & 0.25 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -2.78 \end{bmatrix}$	$\begin{bmatrix} -29.18 & -4.78 \end{bmatrix}$
9	$\begin{bmatrix} 0 & 1 \\ 34.97 & -0.21 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -2.31 \end{bmatrix}$	$\begin{bmatrix} -31.86 & -5.26 \end{bmatrix}$
15	$\begin{bmatrix} 0 & 1 \\ 36.62 & -0.24 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -2.67 \end{bmatrix}$	$\begin{bmatrix} -29.18 & -4.69 \end{bmatrix}$
16	$\begin{bmatrix} 0 & 1 \\ 38.15 & -0.25 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -2.78 \end{bmatrix}$	$\begin{bmatrix} -29.18 & -4.60 \end{bmatrix}$

In the absence of external disturbances, the proposed controller demonstrates its capability by effectively stabilizing the pendulum at the zero position within approximately 3 seconds for all initial angles. The achieved angular velocity of around 0.6 rad/s is well within the range achievable by typical commercial actuators.

When subjected to an external disturbance in the second scenario, the pendulum's angle experiences slight fluctuations. Despite this disturbance, the proposed controller continues to exhibit its stability-enhancing qualities, successfully restoring the pendulum to the zero position within approximately 3 seconds. This outcome aligns with the results from the first scenario, where no disturbance was present. In summary, the proposed controller showcases its robustness against external disturbances and its efficacy in swiftly stabilizing the pendulum in the desired upright position.



(e)

Fig. 2. Simulation results of the first scenario without disturbance:
The pendulum angle (a) and its angular velocity (b);
The cart position (c) together with its velocity (d) and the control signal (e).

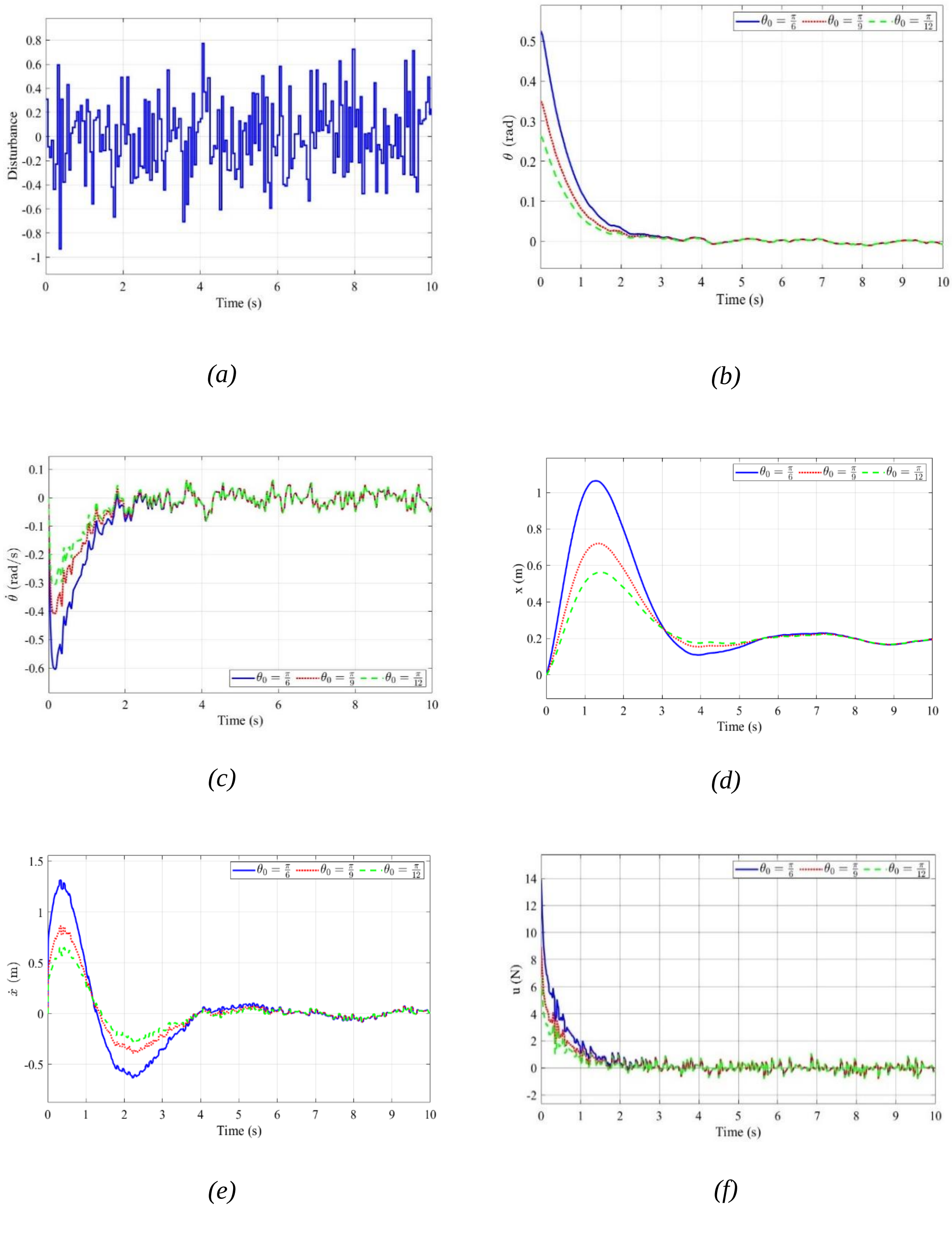
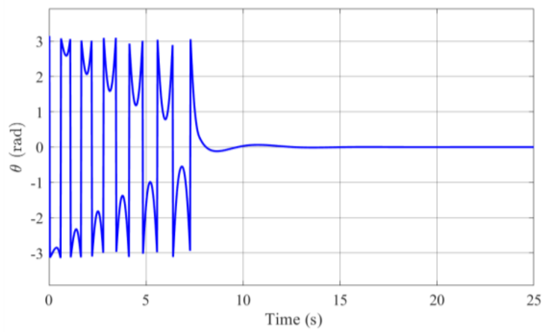
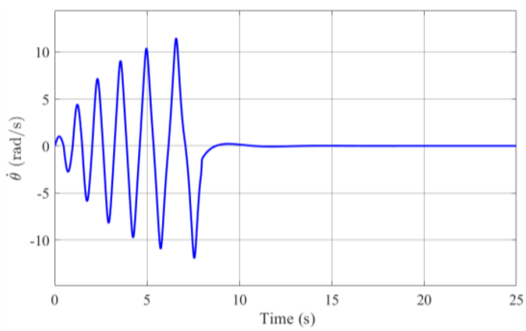


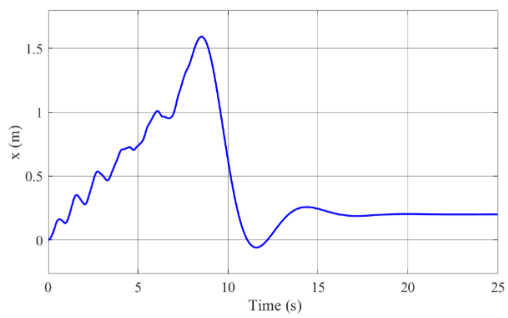
Fig. 3. Simulation results of the second scenario with an external disturbance (a): The pendulum angle (b) and its angular velocity (c); The cart position (d) together with its velocity (e) and the control output signal (f).

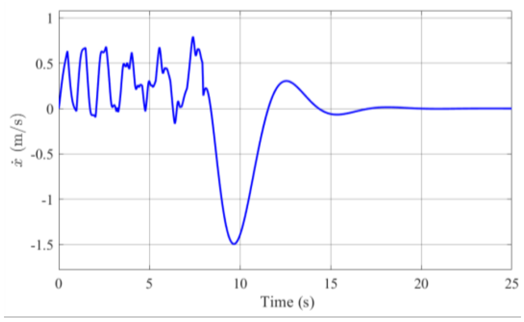
In practice, the pendulum undergoes a real-world scenario where it swings upward from angle π to an upright vertical position, and then aims to maintain its balance. The controller discussed in this article is specifically designed for maintaining stability in the upright position. To initiate the swinging-up motion of the pendulum, the author employs an energy-based method [17]. It is important to note that the swing-up method itself is not detailed in this article. Figure 4 and Fig. 5 in the article depict the results of the pendulum's swing-up and balancing processes, illustrating the performance of combining the Energy controller with the proposed controller. The largest cart distances are about 1.6 m, which are suitable in practice. These results are presented for both scenarios: with and without the presence of external disturbance. The used external disturbance is similar with the previous simulation case. Once again, these findings underscore the efficacy of the proposed controller in stably managing the pendulum system.





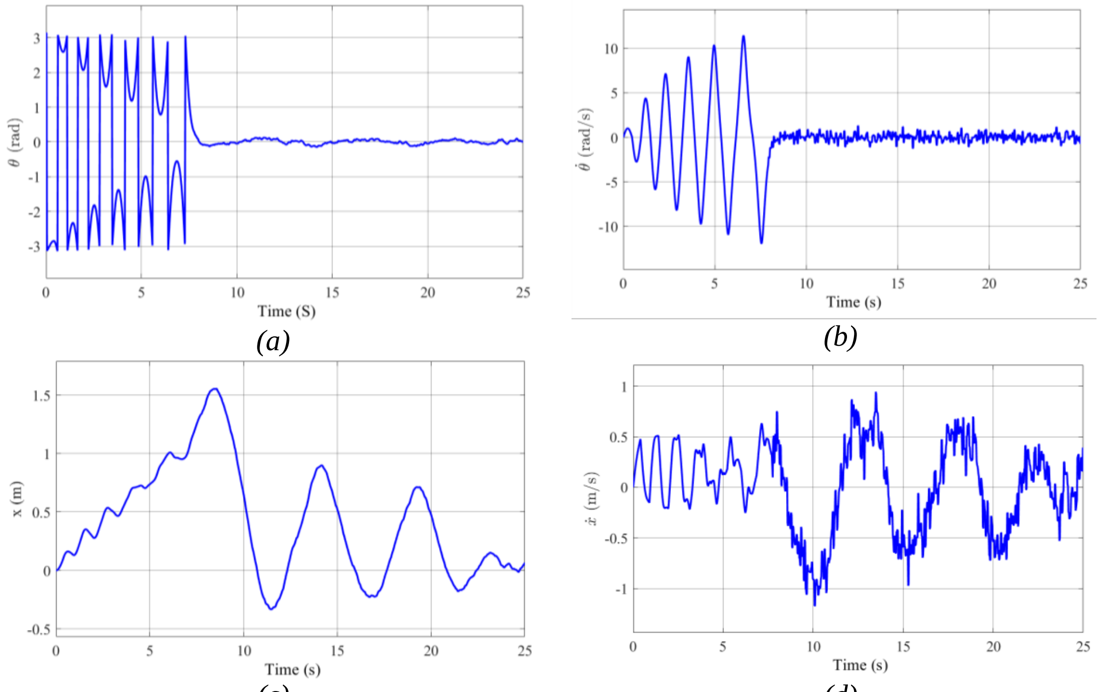
(a) *(b)*





(c) *(d)*

Fig. 4. Simulation results having Energy control for swing-up without disturbance: The pendulum angle (a) and its angular velocity (b); The cart position (c) together with its velocity (d).



(a) *(b)*

(c) *(d)*

Fig. 5. Simulation results having Energy control for swing-up with an external disturbance: The pendulum angle (a) and its angular velocity (b); The cart position (c) together with its velocity (d).

5. Conclusion

In conclusion, this article delved into the intricate realm of nonlinear stability control for an inverted pendulum on a cart, employing an innovative LQR-based T-S fuzzy control approach. Through rigorous simulations and analyses, the effectiveness and robustness of the proposed control scheme were systematically explored across various scenarios.

The study demonstrated that the controller's ability to stabilize the pendulum in the vertically upright position was impressive, even when faced with different initial angles. The meticulous integration of the LQR-based T-S fuzzy control strategy exhibited remarkable adaptability and responsiveness, resulting in the prompt attainment of stability within a brief timeframe. Notably, the angular velocities achieved were well within the realm of feasibility for standard commercial actuators, underscoring the practical viability of the proposed method.

Furthermore, the investigation extended to scenarios involving external disturbances. In these cases, the proposed controller exhibited commendable resilience, as it adeptly counteracted the perturbations and maintained its proficiency in ensuring the pendulum's return to the desired upright position. The consistent outcome across both disturbance-free and disturbed scenarios underscores the controller's stability-enhancing capabilities and its potential for real-world applications.

The findings of this article underscore the merit of the LQR-based T-S fuzzy control approach in nonlinear stability control. The successful application of this method to the inverted pendulum on a cart showcases its potential for addressing intricate control challenges in dynamic systems. The insights gained from this study contribute to the broader understanding of control strategies for similar complex systems, opening avenues for further research and innovation in the field.

References

[1] S. E. Oltean, "Swing-up and stabilization of the rotational inverted pendulum using PD and fuzzy-PD controllers", *Procedia Technology*, Vol. 12, pp. 57-64, 2014. DOI: 10.1016/j.protcy.2013.12.456

[2] S. Kurode, A. Chalanga, and B. Bandyopadhyay, "Swing-up and stabilization of rotary inverted pendulum using sliding modes", *IFAC Proceedings Volumes*, Vol. 44, Iss. 1, pp. 10685-10690, 2011. DOI: 10.3182/20110828-6-IT-1002.02933

[3] H. Qin, W. Shao, and L. Guo, "Research and verification on swing-up control algorithm of rotary inverted pendulum", *The 26th Chinese Control and Decision Conference (2014 CCDC)*, Changsha, China, 2014, pp. 4941-4945. DOI: 10.1109/CCDC.2014.6853058

[4] A. Rajan, A. A. Kumar, and C. S. Kavitha, "Robust control methods for swing-up and stabilization of a rotary inverted pendulum", *2016 International Conference on Emerging Technological Trends (ICETT)*, Kollam, India, 2016, pp. 1-6. DOI: 10.1109/ICETT.2016.7873665

[5] I. Hassanzadeh and S. Mobayen, "Controller design for rotary inverted pendulum system using evolutionary algorithms", *Mathematical Problems in Engineering*, 2011, pp. 1-17, DOI: 10.1155/2011/572424

[6] A. Rahimi, K. Raahemifar, K. Kumar, and H. Alighanbari, "Controller design for rotary inverted pendulum system using particle swarm optimization algorithm", *2013 26th IEEE Canadian Conference on Electrical and Computer Engineering (CCECE)*, Regina, SK, Canada, 2013, pp. 1-5. DOI: 10.1109/CCECE.2013.6567710

[7] Y. F. Chen, A. C. Huang, "Adaptive control of rotary inverted pendulum system with time-varying uncertainties", *Nonlinear Dyn.*, Vol. 76, pp. 95-102, 2014. DOI: 10.1007/s11071-013-1112-4

[8] K. J. Åström and K. Furuta, "Swinging up a pendulum by energy control", *Automatica*, Vol. 36, Iss. 2, pp. 287-295, 2000. DOI: 10.1016/S0005-1098(99)00140-5

[9] Pham, D.-B.; Pham, D.-T.; Dao, Q.-T.; and Nguyen, V.-A., "Takagi-Sugeno fuzzy control for stabilizing nonlinear inverted pendulum", In *Intelligent Systems and Networks: Selected Articles from ICISN 2022, Vietnam*, 2022, pp. 333-341. Singapore: Springer Nature Singapore. DOI: 10.1007/978-981-19-3394-3_38

[10] U. Farooq, J. Gu, M. E. El-Hawary, V. E. Balas, and M. U. Asad, "Experimental study of optimal Takagi Sugeno fuzzy controller for rotary inverted pendulum", In *2015 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2015)*, Istanbul, Turkey, August 2015, pp. 1-7. DOI: 10.1109/FUZZ-IEEE.2015.7337972

[11] Z. B. Hazem, M. J. Fotuhi, and Z. Bingül, "Development of a Fuzzy-LQR and Fuzzy-LQG stability control for a double link rotary inverted pendulum", *Journal of the Franklin Institute*, Vol. 357, Iss. 15, pp. 10529-10556, 2020. DOI: 10.1016/j.jfranklin.2020.08.030

[12] M. K. Habib and S. A. Ayankoso, "Hybrid control of a double linear inverted pendulum using LQR-fuzzy and LQR-PID controllers", In *2022 IEEE International Conference on Mechatronics and Automation (IEEE ICMA)*, August 2022, pp. 1784-1789. DOI: 10.1109/ICMA54519.2022.9856235

[13] B. Bekkar and K. Ferkous, "Design of Online Fuzzy Tuning LQR Controller Applied to Rotary Single Inverted Pendulum: Experimental Validation", *Arabian Journal for Science and Engineering*, Vol. 48, Iss. 5, pp. 6957-6972, 2023. DOI: 10.1007/s13369-022-06921-3

[14] M. Fauziyah, Z. Amalia, I. Siradjuddin, D. Dewatama, R. P. Wicaksono, and E. Yudaningtyas, "Linear quadratic regulator and pole placement for stabilizing a cart inverted pendulum system", *Bulletin of Electrical Engineering and Informatics*, Vol. 9, No. 3, pp. 914-923, 2020. DOI: 10.11591/eei.v9i3.2017

[15] A. Al-Mahturi, F. Santoso, M. A. Garratt, and S. G. Anavatti, "An intelligent control of an inverted pendulum based on an adaptive interval type-2 fuzzy inference system", In *2019 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, June 2019, pp. 1-6. DOI: 10.1109/FUZZ-IEEE.2019.8858948

[16] K. Tanaka and H. O. Wang, *Fuzzy control systems design and analysis: A linear matrix inequality approach*. John Wiley & Sons, 2004.

[17] M. Abdullah, A. A. Amin, S. Iqbal, and K. Mahmood-ul-Hasan, "Swing up and stabilization control of rotary inverted pendulum based on energy balance, fuzzy logic, and LQR controllers", *Measurement and Control*, Vol. 54(9-10), pp. 1356-1370, 2021. DOI: 10.1177/00202940211035406

ĐIỀU KHIỂN ỔN ĐỊNH CHO CON LẮC NGƯỢC SỬ DỤNG BỘ ĐIỀU KHIỂN LQR KẾT HỢP LOGIC MỜ TAKAGI-SUGENO

Nguyễn Thị Vân Anh[a], Đào Quý Thịnh[a]

[a]*Trường Điện - Điện tử, Đại học Bách khoa Hà Nội*

Tóm tắt: Ổn định con lắc ngược đẩy là một bài toán điều khiển đầy thách thức và được nghiên cứu rộng rãi trong lĩnh vực hệ thống điều khiển và rô bốt. Nhằm nâng cao chất lượng điều khiển ổn định con lắc ngược, bài báo trình bày một bộ điều khiển mới kết hợp điểm mạnh của hai sách lược điều khiển hiệu quả: Bộ điều chỉnh tuyến tính-bậc hai (LQR) và điều khiển mờ Takagi-Sugeno (T-S). Bộ điều khiển LQR rất hiệu quả trong việc ổn định các hệ thống tuyến tính, nhưng nó gặp phải những hạn chế với các hệ thống phi tuyến tính như con lắc ngược. Mặt khác, điều khiển mờ T-S vượt trội trong việc xử lý các hiện tượng phi tuyến bằng cách xấp xỉ hành vi của hệ thống với các mô hình tuyến tính cục bộ. Bộ điều khiển được đề xuất tận dụng các hệ mờ T-S để xấp xỉ các tính chất phi tuyến tính phức tạp, trong khi đó điều khiển LQR được sử dụng cho từng hệ thống con tuyến tính cục bộ. Hiệu quả của phương pháp kết hợp được chứng minh bằng các kết quả mô phỏng, xem xét cụ thể các tiêu chí như độ ổn định trong điều kiện có nhiễu tác động, sự thay đổi góc ban đầu của con lắc và phân tích so sánh giữa các kịch bản chỉ có điều khiển ổn định và điều khiển đưa con lắc đến vị trí thẳng đứng và ổn định.

Từ khóa: *Con lắc ngược; logic mờ Takagi-Sugeno; LQR; điều khiển ổn định.*

Received: 15/08/2023; Revised: 12/10/2023; Accepted for publication: 04/12/2023

❑