A NONEXISTENCE RESULT FOR DEGENERATE PARABOLIC EQUATIONS INVOLVING ADVECTION TERMS AND WEIGHTS

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Abstract

In this paper, we are concerned with the following equation

$$v_t - \Delta_\lambda v + \nabla_\lambda w \cdot \nabla_\lambda v = h(x)v^p \ (x,t) \in \mathbb{R}^N \times \mathbb{R}.$$

Here, p is a real number, w is a smooth function, $h \geq 0$ is a weight function which is continuous function satisfying some growth condition at infinity, Δ_λ is a sub-elliptic operator which is defined by

$$\Delta_\lambda = \sum_{i=1}^{N} \partial_{x_i}(\lambda_i^2 \partial_{x_i})$$

and ∇_λ is the corresponding gradient operator associated to Δ_λ. By using a kind of maximum principle and the test function method, we establish the nonexistence of positive supersolutions of the above equation.

Index terms

Nonexistence results, positive supersolutions, degenerate operator, advection terms, Δ_λ-Laplace operator, weight functions.

1. Introduction

In this paper, we prove some nonexistence results for the equation

$$v_t - \Delta_\lambda v + \nabla_\lambda w \cdot \nabla_\lambda v = h(x)v^p \text{ in } \mathbb{R}^N \times \mathbb{R}, \tag{1}$$

where Δ_λ is a sub-elliptic operator given by

$$\Delta_\lambda = \sum_{i=1}^{N} \partial_{x_i}\left(\lambda_i^2 \partial_{x_i}\right).$$

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DOI: 10.56651/lqdtu.jst.v12.n02.748.ict

As in the pioneering work [1], we use in this paper the following hypotheses, see also [2], [3].
(H1) Suppose that there exists a group of dilations $(\delta_t)_{t>0}$

$$\delta_t : \mathbb{R}^N \to \mathbb{R}^N, (x_1, ..., x_N) \mapsto (t^{\varepsilon_1} x_1, ..., t^{\varepsilon_N} x_N)$$

where $1 = \varepsilon_1 \leq \varepsilon_2 \leq ... \leq \varepsilon_N$, such that λ_i is δ_t-homogeneous of degree $(\varepsilon_i - 1)$, that is,

$$\lambda_i(\delta_t(x)) = t^{\varepsilon_i - 1}\lambda_i(x), \text{ for all } x \in \mathbb{R}^N, t > 0, i = 1, 2, ..., N.$$

The homogeneous dimension of $\mathbb{R}^N$ corresponding to $(\delta_t)_{t>0}$ is defined by

$$Q = \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_N. \tag{2}$$

(H2) The function $\lambda_1 = 1$ and for $i = 2, ..., N$, $\lambda_i(x) = \lambda_i(|x_1|, ..., |x_{i-1}|)$. Moreover, the functions λ_i are supposed to be continuous on $\mathbb{R}^N$ and are strictly positive and of class C^1 on $\mathbb{R}^N \setminus \Pi$, where

$$\Pi = \left\{ (x_1, ..., x_N) \in \mathbb{R}^N; \prod_{i=1}^{N} x_i = 0 \right\}.$$

Naturally, the Δ_λ-gradient operator associated to Δ_λ given by:

$$\nabla_\lambda = (\lambda_1 \partial_{x_1}, \lambda_2 \partial_{x_2}, ..., \lambda_N \partial_{x_N}).$$

We easily see that

$$\Delta_\lambda = (\nabla_\lambda)^2.$$

Define the norm associated to the operator Δ_λ by, see [3]

$$|x|_\lambda = \left(\sum_{j=1}^{N} \prod_{i \neq j} \lambda_i^2(x) \epsilon_j^2 |x_j|^2 \right)^{\frac{1}{2\sigma}},$$

with

$$\sigma = 1 + \sum_{j=1}^{N} (\epsilon_j - 1).$$

When v is independent of t, (1) becomes:

$$-\Delta_\lambda v + \nabla_\lambda w \cdot \nabla_\lambda v = h(x) v^p \text{ in } \mathbb{R}^N. \tag{3}$$

In the special case where $\lambda_i = 1$, equation (3) is reduced to

$$-\Delta v + \nabla w \cdot \nabla v = h(x) v^p \text{ in } \mathbb{R}^N. \tag{4}$$

This equation has received a lot of attention in recent years by mathematicians. Concerning to the class of stable solutions, the Liouville properties were shown in [2], [4]–[6] for $p > 1$. Concerning the positive supersolutions of (4) without weight ($h \equiv 1$), some nonexistence result was obtain in [7].

For the parabolic model (1) in the special case $\lambda_i = 1$, $h = 1$ and $w = 0$, we have

$$v_t - \Delta v = v^p \text{ in } \mathbb{R}^N \times \mathbb{R}. \tag{5}$$

The nonexistence of positive solutions is conjectured to be true when the exponent

$$1 < p < \frac{N+2}{N-2}.$$

The answer for this question is confirmed for the low dimension $N \leq 2$, see [8]–[14]. Recently, the existence and nonexistence of positive supersolutions to (5) has been obtained in [15] where the equation has no supersolution provided that

$$p \in (-\infty, 1) \cup \left(1, \frac{N+2}{N}\right].$$

In this paper, we defined a positive supersolution of the equation (1) is a function $v \in C^{2,1}(\mathbb{R}^N \times \mathbb{R})$ such that

$$v_t - \Delta_\lambda v + \nabla_\lambda w \cdot \nabla_\lambda v \geq h(x) v^p.$$

Remark that our model (1) is a natural extension of the models (4) and (5). Therefore, if we can prove some results for (1), we also obtain the corresponding ones for such particular equations. Notice also that the techniques used in [7] and [15] are applied for the Laplace operator only and not for the general operator as Δ_λ. Motivated by recent results on the elliptic and parabolic equations of Lane-Emden type [4], [7], [15], we prove the nonexistence result for the class of positive supersolutions of (1) in the general case of operator Δ_λ. As special cases of our result, we recover some previous results in [7] and [15].

2. Main result

The main result of this paper is as follows:

Theorem 2.1. *Let* (H1), (H2) *hold true. Suppose that h is a weight function which is continuous and satisfies*

$$h(x) \sim |x|_\lambda^\alpha, \alpha \geq 0, \textit{ for } |x|_\lambda \textit{ sufficiently large}$$

and the advection term w is bounded from below and there is a constant $\gamma < 1 + \alpha$ such that

$$|\nabla_\lambda w(x)| \leq C(1 + |x|_\lambda^\gamma) \textit{ in } \mathbb{R}^N.$$

Then, there has no positive supersolution of (1) *provided that*

$$p \in (-\infty, 1) \cup \left(1, \frac{Q+2+\alpha}{Q+2-\min(2, 1-\gamma)}\right]$$

where Q is the homogeneous dimension of $\mathbb{R}^N$.

As mentioned above, our result recovers some results in [7] and [15] due to the general operator Δ_λ. Remark also that our result concerns with a class of advection which is different with that in [2]. In [2], the advection satisfies the free divergence condion, where in our paper, we do not require this. In the super critical case, the existence of positive supersolution of (1) when $p > \frac{Q+2}{Q+2-\min(2,1-\gamma)}$ has not been solved.

We generalize the idea in [7], which is only applied to the Laplace operator, to the general case of Δ_λ. More precisely, to prove Theorem 2.1, we develop the rescaled test-function method in the case

$$1 < p \leq \frac{Q+2+\alpha}{Q+2-\min(2,1-\gamma)}.$$

In the case $p < 1$, we use a kind of the maximum argument, see [15]–[17].

The rest of this paper is to prove Theorem 2.1.

3. Proof of main result

In this section, the proof of Theorem 2.1 is divided into two cases.

Case 1. $p < 1$.

Assume on the contrary that v is a positive supersolution of (1) with $p < 1$. Let us put $z = \frac{1}{v} > 0$, then we obtain the following equation

$$-z_t + \Delta_\lambda z - \nabla_\lambda w \cdot \nabla_\lambda z \geq h(x) z^{2-p} + \frac{2|\nabla_\lambda z|^2}{z}. \tag{6}$$

We use the following notation

$$\mathcal{B}_r = \{(x,t) \in \mathbb{R}^N \times \mathbb{R}; |x_i| < r^{\epsilon_i}, |t| < r^2\}.$$

Let us consider $\phi \in C_c^\infty(\mathbb{R}^N \times \mathbb{R}; [0,1])$ be a cut-off function such that $\phi = 1$ in $\mathcal{B}_1$ and $\phi = 0$ outside $\mathcal{B}_2$. Let m be a positive number chosen later on, for $r > 0$, we construct

$$\phi_r(x,t) = \phi^m\left(\frac{x_1}{r^{\epsilon_1}}, \frac{x_2}{r^{\epsilon_2}}, ..., \frac{x_N}{r^{\epsilon_N}}, \frac{t}{r^2}\right).$$

By the compactness of the support of the function $z\phi_r$, there is a point $(x_r, t_r) \in \mathcal{B}_{2r}$ such that

$$z(x_r,t_r)\phi_r(x_r,t_r) = \max_{\mathbb{R}^N \times \mathbb{R}} z\phi_r.$$

In particular,

$$z(x_r,t_r)\phi_r(x_r,t_r) \to \sup_{\mathbb{R}^N \times \mathbb{R}} z > 0 \text{ as } r \to \infty.$$

It follows from the assumption (H2) on λ_i that

$$(z\phi_r)_t(x_r,t_r) = 0, \nabla_\lambda(z\phi_r)(x_r,t_r) = 0$$

and

$$\Delta_\lambda(z\phi_r)(x_r,t_r) \le 0.$$

Hence, we estimate at (x_r,t_r) as

$$-z_t = \frac{z(\phi_r)_t}{\phi_r},\ \nabla_\lambda z = -\frac{z\nabla_\lambda\phi_r}{\phi_r} \tag{7}$$

and the second derivative

$$\Delta_\lambda z \le -\frac{z\Delta_\lambda\phi_r}{\phi_r} - \frac{2\nabla_\lambda z\cdot\nabla_\lambda\phi_r}{\phi_r}. \tag{8}$$

Inserting (7) into (8), we arrive at (x_r,t_r)

$$\Delta_\lambda z \le -\frac{z\Delta_\lambda\phi_r}{\phi_r} + 2\frac{|\nabla_\lambda z|^2}{z}\phi_r \le 0. \tag{9}$$

Taking into account (6) and (9), we obtain (x_r,t_r)

$$\nabla_\lambda w\cdot\nabla_\lambda z\phi_r + hv^{2-p}\phi_r + z\Delta_\lambda\phi_r \le z(\phi_r)_t. \tag{10}$$

Replacing (7) into (10), we obtain at (x_r,t_r)

$$-\nabla_\lambda w\cdot\nabla_\lambda\phi_r v + hv^{2-p}\phi_r + z\Delta_\lambda\phi_r \le z(\phi_r)_t. \tag{11}$$

Furthermore, the assumptions (H1), (H2) imply that

$$|\Delta_\lambda\phi_r(x_r,t_r)| \le \frac{C}{r^2}\phi_r^{\frac{m-2}{m}}(x_r,t_r),$$

and

$$|\nabla_\lambda\phi_r(x_r,t_r)| \le \frac{C}{r}\phi_r^{\frac{m-1}{m}}(x_r,t_r), |(\phi_r)_t(x_r,t_r)| \le \frac{C}{r^2}\phi_r^{\frac{m-1}{m}}(x_r,t_r).$$

Notice that $|\nabla_\lambda w(x_r)| \le Cr^\gamma$, then (11) follows:

$$h(x_r)z^{2-p}(x_r,t_r)\phi_r(x_r,t_r) \le \frac{C}{r^{\min(2;1-\gamma)}}z((x_r,t_r)\phi_r^{\frac{m-2}{m}}(x_r,t_r).$$

We now choose $m = \frac{2}{1-p}$ which is equivalent to $\frac{m-2}{m} = -p$. Then, using $h(x_r) \ge C|x_r|_\lambda^\alpha$, we obtain:

$$z^{2-p}(x_r,t_r)\phi_r^{2-p}(x_r,t_r) \le \frac{C}{r^{\min(2;1-\gamma)+\alpha}}z(x_r,t_r)\phi_r(x_r,t_r).$$

This shows that

$$z^{1-p}(x_r,t_r)\phi_r^{1-p}(x_r,t_r) \le \frac{C}{r^{\min(2;1-\gamma)+\alpha}}.$$

Since $p<1$ and $\gamma < 1+\alpha$, we have $1-p>0$ and $1+\alpha-\gamma>0$. Letting $r\to\infty$, we obtain $\sup\limits_{\mathbb{R}^N} z \le 0$ which is impossible since $\sup\limits_{\mathbb{R}^N\times\mathbb{R}} z > 0$. $\square$

Case 2. $1 < p \le \dfrac{Q+2+\alpha}{Q+2-\min(2,1-\gamma)}$.

Let us choose a test function $\phi \in C_c^\infty(\mathbb{R}^N \times \mathbb{R}; [0,1])$ with the property that $\phi = 1$ in $\mathcal{B}_1$ and $\phi = 0$ outside $\mathcal{B}_2$. For $r > 0$, let us put

$$\phi_r(x,t) = \phi^m\left(\frac{x_1}{r^{\epsilon_1}}, \frac{x_2}{r^{\epsilon_2}}, ..., \frac{x_N}{r^{\epsilon_N}}, \frac{t}{r^2}\right),$$

where m is a positive parameter chosen later on. Assume that (1) has a positive supersolution v. From (1) with the test function $\phi_r(x,t)\exp(-w)$, one has

$$\int_{\mathcal{B}_{2r}} (v_t\phi_r - \Delta_\lambda v\phi_r + \nabla_\lambda w \cdot \nabla_\lambda v\phi_r)\exp(-w)dxdt \geq \int_{\mathcal{B}_{2r}} hu^p\phi_r \exp(-w)dxdt.$$

Applying an integration by parts, we obtain:

$$\int_{\mathcal{B}_{2r}\setminus\mathcal{B}_r} v((\phi_r)_t - \Delta_\lambda\phi_r) + v\nabla_\lambda w \cdot \nabla_\lambda\phi_r dxdt \geq \int_{\mathcal{B}_{2r}} hu^p\phi_r dxdt. \tag{12}$$

As in the first case, by the assumptions on the functions λ_i, we arrive at

$$|-\Delta_\lambda\phi_r| \leq \frac{C}{r^2}\phi_r^{\frac{m-2}{m}}, \quad |\nabla_\lambda\phi_r| \leq \frac{C}{r}\phi_r^{\frac{m-1}{m}},$$

and

$$|(\phi_r)_t| \leq \frac{C}{r^2}\phi_r^{\frac{m-1}{m}}.$$

In addition, the behavior of the advection terms yields

$$|\nabla_\lambda w(x)| \leq Cr^\gamma \text{ on } \mathcal{B}_{2r} \setminus \mathcal{B}_r.$$

Taking into account these estimates and (12), we receive:

$$\int_{\mathcal{B}_{2r}} hu^p\phi_r dxdt \leq \frac{C}{r^{\min(2,1-\gamma)}} \int_{\mathcal{B}_{2r}\setminus\mathcal{B}_r} v\phi_r^{\frac{m-2}{m}} dxdt. \tag{13}$$

We next apply the Hölder inequality in the right hand side of (13) to obtain:

$$\int_{\mathcal{B}_{2r}\setminus\mathcal{B}_r} v\phi_r^{\frac{m-2}{m}} dxdt \leq \left(\int_{\mathcal{B}_{2r}\setminus\mathcal{B}_r} hu^p\phi_r^{\frac{(m-2)p}{m}} dxdt\right)^{\frac{1}{p}} \left(\int_{\mathcal{B}_{2r}\setminus\mathcal{B}_r} h^{-\frac{1}{p-1}} dxdt\right)^{\frac{p-1}{p}}. \tag{14}$$

We now choose the parameter as $m = \frac{2p}{p-1}$ or equivalently $\frac{(m-2)p}{m} = 1$. Thus, we deduce from (14) and (13) that

$$\int_{\mathcal{B}_{2r}} hu^p\phi_r dxdt \leq Cr^{Q+2-\frac{\alpha}{p-1}-\frac{p\min(2,1-\gamma)}{p-1}}. \tag{15}$$

When $p < \frac{Q+2+\alpha}{Q+2-\min(2,1-\gamma)}$, the exponent in the right hand side of (13) is negative. Thus, we obtain from (15) by letting $r \to \infty$ that

$$\int_{\mathbb{R}^N\times\mathbb{R}} hu^p dxdt = 0.$$

This is impossible since $v > 0$.

We finally consider the borderline case $p = \frac{Q+2+\alpha}{Q+2-\min(2,1-\gamma)}$. Then (15) gives

$$\int_{\mathbb{R}^N \times \mathbb{R}} hu^p dxdt < C$$

which follows that

$$\int_{\mathcal{B}_{2r} \setminus \mathcal{B}_r} hu^p \phi_r dxdt \to 0 \text{ as } r \to \infty.$$

Combining this and (13), we conclude that

$$\int_{\mathcal{B}_{2r}} hu^p \phi_r dxdt \to 0 \text{ as } r \to \infty.$$

which is again impossible since $v > 0$. $\square$

4. Conclusions

In this paper, by using a kind of maximum principle and the test function method, we establish the nonexistence of positive supersolutions of the following equation

$$v_t - \Delta_\lambda v + \nabla_\lambda w \cdot \nabla_\lambda v = h(x) v^p \ (x,t) \in \mathbb{R}^N \times \mathbb{R}.$$

As special cases of our result, we recover some previous results in [7] and [15]. Notice also that the techniques used in [7] and [15] are applied for the Laplace operator only and not for the general operator as Δ_λ. Remark also that our result concerns with a class of advection which is different with that in [2]. In [2], the advection satisfies the free divergence condion, where in our paper, we do not require this. In the super critical case, the existence of positive supersolution of (1) when $p > \frac{Q+2}{Q+2-\min(2,1-\gamma)}$ has not been solved.

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Manuscript received 19-06-2023; Accepted 21-12-2023.

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SỰ KHÔNG TỒN TẠI NGHIỆM CỦA PHƯƠNG TRÌNH PARABOLIC SUY BIẾN CHỨA SỐ HẠNG BÌNH LƯU CÓ TRỌNG

Đào Trọng Quyết, Khuất Quang Thành, Bùi Thị Nga

Tóm tắt

Trong bài báo này, chúng tôi quan tâm đến phương trình

$$v_t - \Delta_\lambda v + \nabla_\lambda w \cdot \nabla_\lambda v = h(x)v^p \ (x,t) \in \mathbb{R}^N \times \mathbb{R}.$$

Ở đây, p là một số thực, w là một hàm trơn, $h \geq 0$ là hàm trọng, liên tục thỏa mãn một số điều kiện tăng trưởng ở vô cực, Δ_λ là toán tử elliptic dưới được định nghĩa bởi

$$\Delta_\lambda = \sum_{i=1}^{N} \partial_{x_i}(\lambda_i^2 \partial_{x_i})$$

và ∇_λ là gradient tương ứng với toán tử Δ_λ. Sử dụng nguyên lý cực đại và phương pháp hàm thử, chúng tôi thiết lập kết quả về sự không tồn tại nghiệm trên dương của bài toán trên.

Từ khóa

Kết quả về sự không tồn tại, nghiệm trên dương, toán tử suy biến, số hạng bình lưu, toán tử Δ_λ-Laplace, hàm trọng.