AN IMPROVED POLARIZATION WEIGHTS CONSTRUCTION FOR POLAR CODES

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Abstract

Polar codes proposed by Arikan based on channel polarization have been proven to achieve Symmetric Binary-Discrete Input Memoryless Channels capacity. 3GPP has also chosen the Polar codes as the error correction code in the fifth mobile communication system (5G New Radio - 5G NR) with a maximum codeword length of 1024 bits. The design of the code based on the reliability of the synthetic channels has been researched with many different methods to satisfy the requirements demanded in the application of 5G NR. The Polar codes design requirements are ease of implementation, low decoding complexity, and high performance in error correction. This paper has proposed a method of designing Polar codes by determining a synthetic channels reliability index sequence based on the Polarization Weights (PW) method called the Improved Polarization Weights (IPW) method. The simulation results show that the error correction performance of the code designed according to IPW is better than the code structure selected for 5G NR and designed according to the original PW method.

Index terms

Polar codes, channel coding, FEC for 5G NR, RS, PW, IPW.

1. Introduction

Polar codes, along with the Successive Cancellation (SC) decoding algorithm proposed by Arikan in 2009, are grounded in the principle of channel polarization [1]. They have been both theoretically proven and empirically substantiated through simulations as the first practical codes capable of achieving the capacity of Symmetric Binary-Input Discrete Memoryless Channels (B-DMC) while maintaining low encoding and decoding complexity. The process of channel polarization is recursive, wherein channel W_n is decomposed into virtual aggregated channels $W_{n,1}$ and $W_{n,2}$. Arikan demonstrated that as the number of recursions becomes sufficiently large, these aggregated channels "polarize" into "reliable" channels that are nearly noise-free, and "unreliable" channels that exhibit noise interference. The fraction of reliable channels among the total aggregated channels approaches the capacity of the B-DMC.

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The encoding process involves arranging data bits to be transmitted through reliable channels, while unreliable channels do not transmit information but only convey uniform "frozen bits" that are predetermined and consistent between the transmitter and receiver, often set to '0'. With a code length of N, the sequence of aggregated channel indices is sorted in ascending order of channel reliability, referred to as the Reliability Sequence (RS). Then, K channels with higher reliability are selected to transmit data bits, while frozen bits are transmitted over the remaining N-K channels. The complexity lies in the recursive computation of the reliability of aggregated channels when using Binary-Erasure Channels (BEC), which is linear, whereas for other channels, the computational complexity is $O(NlogN)$.

Research in encoding and decoding for Polar codes, in general, and specifically, simpler methods for computing the RS, has become essential as Polar codes have been adopted for encoding control information in both the uplink and downlink of 5G New Radio (5G NR) systems [2], [3]. These codes are utilized for Enhanced Mobile Broadband (eMBB), Ultra-Reliable Low-Latency Communications (URLLC), and Massive Machine-Type Communications (MMTC). While calculating the reliability ordering for bits in long codes can be complex, suitable algorithms exist. However, within the context of 5G NR applications, where various block lengths and encoding rates must be supported, the challenge is to design Polar codes that are easy to implement, have low decoding complexity, and maintain good error correction performance [4]. To meet the design requirements for Polar codes outlined above, one crucial issue is the rapid, simple, and memory-efficient generation of the RS. Because the order of channel reliability depends on the physical channel conditions and code length, this non-uniform property presents challenges for realizing Polar codes when a wide range of code lengths and encoding rates are required. The remaining sections of the paper are structured as follows. Section 2 summarizes some of the previously researched methods for computing the RS. Next, in section 3, the paper presents a code design method using PW. Section 4 introduces a new solution in code design based on the PW method, referred to as IPW, with mathematical foundations and supporting simulation results. Finally, the conclusions of the paper are presented in section 5.

2. Several methods for computing the reliability sequence

In his renowned research [1], Arikan introduced the use of the Bhattacharyya upper bound method, as well as Monte-Carlo simulations, to calculate the reliability of aggregated channels in Polar codes. The Density Evolution (DE) method was initially proposed in [5] and later refined in [6], ensuring accuracy through theory but at the cost of high complexity. The development of this method, known as the Gaussian Approximation of Density Evolution (GA/DE) [7], allowed for accurate computation of channel reliability with limited complexity. However, these on-the-fly methods introduced significant encoding delays, making them unsuitable for the requirements of 5G NR. Recent studies have applied Partial ordering theory to arrange the reliability order of aggregated channels [8], [9], offering new opportunities to create a universal RS independent of physical channel attributes. In [8], the authors identified the existence of a partial ordering relationship in the reliabilities of aggregated channels in Polar codes. This discovery significantly reduced the complexity of code construction to below linear complexity. Furthermore, in [10], it was demonstrated that by utilizing the partial ordering property, Polar codes of length N can be efficiently constructed by computing the reliability of a subset of aggregated channels, only a fraction of which is $log_{3/2}N - 1$. Additionally, [11] introduced a formula and algorithm known as PW to characterize the reliability of aggregated channels. In [9], number theory was applied to systematize the theory of the PW algorithm, referred to as β-expansion. It was combined with the nesting property of partial ordering to demonstrate that Polar codes could be efficiently constructed by applying β-expansion and selecting suitable values of B for each iteration of nested construction. These research results, along with extensive simulations, enabled the standardization group for 5G NR to define a universal RS for 1024 aggregated channels. This sequence serves as the foundation for determining specific RS for each Polar code within the 5G NR standard. In practice, it is possible to extract from this universal RS sub-sequences for shorter codes without relying on physical channel conditions, thereby reducing memory space requirements. This nested structure of RS represents a significant breakthrough in Polar codes design and is considered a major achievement of the 5G NR standardization process.

3. Polar code design method using polarization weights

In [9], [11], PW are described as a method to create a sequence of channel indices sorted by the reliability of the channels. Independent of the Signal-to-Noise Power Ratio (SNR), PW is computed as the reliability of aggregated channels, arranged in ascending order, and the index sequence $Q = \{q, 1 \leq q \leq N\}$ is stored for use in Polar codes with a block length of N. The notation $\left(\bar{b}_{k,n-1}, \cdots, \bar{b}_{k,1}, \bar{b}_{k,0}\right)$ represents a binary vector of length $n = \log_2 N$ bit, representing the decimal value $k-1$, with the leftmost bit being the most significant bit (MSB). At this point, the PW value of the k-th aggregated channel, denoted as x_k, is defined as a weighted sum:

$$x_k = \sum_{i=0}^{n-1} b_{k,i}\beta^i \qquad (1)$$

With β playing the role of the base for the total weight, similar to how 2 serves as the base in the decimal number q's binary expansion, and formula (1) is referred to as the β-expansion, and the value of β needs to be carefully chosen as it determines the quality of the PW method. Several suggestions have been made for the value or range of values for B, depending on the block length N, and fundamentally, $\beta = 1.1892 \approx 2^{1/4}$ is recommended based on the Partial ordering theory [9-11].

The author's group conducted research to enhance the accuracy of the RS of aggregated channels using the PW method. They achieved this by comparing the RS generated by the original PW method with the RS adjusted using the DE/GA method and extensive simulation results during the 5G NR standardization process. The sequence of channel indices Q is divided into two sets: F, which contains the indices of frozen bits, and I, which contains the indices of information bits. A Polar code is defined given the block length N and the set F. The sequence Q contains natural numbers from 1 to N, so an index, if not present in F, will be part of I. Notations $F_A(I_A)$ and $F_B(I_B)$ represent the sets of indices for frozen bits (information bits) of Polar codes constructed using methods A and B, respectively. We have some observations as follows:

Observation 1: Two Polar codes with the same block length N, the same number of information bits K, and the same set of frozen bit indices F have the same quality regardless of the order of channel index arrangement.

Table 1 Comparison of the sequence of index numbers of frozen bits in the 5G NR standard with the PW method, $R = 1/2$

SN	Codeword lenght	The indexes are in F_{PW} but not in F_{5G}	The indexes are in F_{5G} but not in F_{PW}
1	16	7	10
2	32	8	25
3	64	...	...
4	128	...	...
5	258	48	164
6	512	64, 284	180, 229
7	1024	222, 316, 336, 365, 544	603, 689, 709, 793, 803

Observation 2: Suppose $1 \leq a \leq N, a \notin F_s$ but $A \in F_B$ and $1 \leq b \leq N, b \notin F_b$ but $B \in F_A$. Then swapping the two numbers $a \in I_A$ and $b \in F_A$ is equivalent to switching from the Polar code construction method B to A, and conversely, swapping the two numbers $b \in I_B$ and $a \in F_B$ is equivalent to switching from the Polar code construction method A to B.

Observation 3: To switch from the Polar code construction method A to B with the aim of improving the quality of Polar codes, it is essentially necessary to consider a limited number of channel index orderings around the boundary that separates the RS into sets of frozen bit indices and information bit indices.

Therefore, to compare the two methods of generating channel index sequences, it is sufficient to compare the two sets of frozen bit indices. The comparison results of the set F_{5G} in the 5G standard with the set F_{PW} for block length $N = 8, 16, ..., 1024$ are shown in Table 1 and Table 2, respectively, for the encoding rate $R = K/N = 1/2, 1/4$.

By simply rearranging a few numbers in the index sequence Q near the boundary that divides Q into sets F_{PW} and I_{PW} to obtain the necessary adjustments for the recommended 1024 index sequence of reliability for the 5G NR standard, the working

Table 2 Comparison of the sequence of index numbers of frozen bits in the 5G NR standard with the PW method, $R = 1/4$

SN	Codeword lenght	The indexes are in F_{PW} but not in F_{5G}	The indexes are in F_{5G} but not in F_{PW}
1	16	...	...
2	32	...	...
3	64	...	...
4	128	56	107
5	258	64	123
6	512	344	481
7	1024	382, 624, 820	699, 726, 869

groups of the 3GPP Project relied on the PW method, but it required a lot of effort and computation time, as noted in [3], because essentially the adjustments involve swapping from one set to another for some index numbers near the boundary separating set F_{PW} and I_{PW}.

4. Proposal for designing a weight-based polarization code

The need for adjusting the RS using the Polar Weighting (PW) method is necessary and reasonable because this method relies on the principle of partially ordered sets (POSET). This means that there are some, even many, elements of the set that are non-comparable. Additionally, PW is based entirely on discrete mathematics theory, which may not take into account the physical properties of the encoding problem in communication. Polar codes themselves are linear codes. Each Polar code of (N, K) is defined by a pair of sets of reliability order indices, F and I, and a transformation matrix created by the Kronecker product of n times $G_N = G_2^{\otimes n}$ with $n = \log_2^N$ and :

$$G_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{2}$$

is the basic polarization kernel. The input vector $u = (u_0, u_1, \cdots, u_{N-1})$, where $u_i = 0, i \in F$ represents the frozen bits, and the rest are information bits. The resulting code is denoted as $c = (c_0, c_1, \cdots, c_{N-1}) = uG_N$. Assuming the frozen bits are set to 0, the generator matrix for the Polar code consists of K rows of the matrix with indices from set I. The parity-check matrix for the Polar code is created by transposing the matrix of columns from the matrix G_N with indices from set F. The output bits from the encoding are sent through a B-DMC. In the described encoding process, it's evident that the index (and) the k-th column of G_N determine which B-DMC channel is used in the k-th transmission of the information from the input bits u_i. On the other hand, the index (and) the k-th row of G_N provides information about which input bit u_k is transmitted during the various uses of the B-DMC channel. We denote:

$$G_N = \{a_{i,j}, 1 \leq i \leq N, 1 \leq j \leq N, a_{i,j} = \{0, 1\}\} \tag{3}$$

We have the following premises:

Premise 1: For all $1 \leq k \leq N$, if we let $\left(b_{k,n-1}, \cdots, b_{k,1}, b_{k,0}\right)$ be the binary representation of the number $(k-1)$, then we have:

$$\sum_{j=1}^{N} a_{k,j} = 2^{\sum\limits_{s=0}^{n-1} b_{k,s}} \tag{4}$$

Demonstration:

With $N = 2, n = 1$, we have $G_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

The formula (4) becomes $\sum\limits_{j=1}^{2} a_{k,j} = 2^{b_k}$

With $k = 1$, then $a_{11} + a_{12} = 1 + 0 = 2^0 = 1 = x$

With $k = 2$, then $a_{21} + a_{22} = 1 + 1 = 2^1 = 2 = y$

With N = 4, n =2, we have

$$G_4 = \begin{bmatrix} G_2 & 0 \\ G_2 & G_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

In that case, the set of values for $k-1$ is $[0, 1, 2, 3]$, which will have binary representations as $[00, 01, 10, 11]$. Therefore, equation (4) can be expanded to:

With $k = 1$, then $a_{11} + a_{12} + a_{13} + a_{14} = 1 + 0 + 0 + 0 = 2^{0+0} = 1 = x$

With $k = 2$, then $a_{21} + a_{22} + a_{23} + a_{24} = 1 + 1 + 0 + 0 = 2^{0+1} = 2 = y$

With $k = 3$, then $a_{31} + a_{32} + a_{33} + a_{34} = 1 + 0 + 1 + 0 = 2^{1+0} = 2 = 2x$

With $k = 4$, then $a_{11} + a_{42} + a_{43} + a_{44} = 1 + 1 + 1 + 1 = 2^{1+1} = 4 = 2y$

Consider the general case:

When $1 \leq k \leq N/2$, then $\sum\limits_{j=1}^{N/2} a_{k,j} = 2^{\sum\limits_{s=0}^{n-2} b_{k,s}}$. (*)

When $N/2 + 1 \leq k \leq N$, then $\sum\limits_{j=(N/2)+1}^{N} a_{k,j} = 2 \times 2^{\sum\limits_{s=0}^{n-2} b_{k,s}}$. (**)

Adding both sides (*) and (**) we get: $\sum\limits_{j=1}^{N} a_{k,j} = 2^{\sum\limits_{s=0}^{n-1} b_{k,s}}$

Premise 2: For all $1 \leq k \leq N$, the product of the number of '1' in column k with the number of '1' in row k of the matrix G_N is equal to N, specifically:

$$\sum_{i=1}^{N} a_{i,k} \times \sum_{j=1}^{N} a_{k,j} = N \tag{5}$$

Premise 3: For all $1 \leq k \leq N$, the number of '1' in row k of matrix G_N is equal to the number of '1' in column $N-k+1$, specifically:

$$\sum_{j=1}^{N} a_{k,j} = \sum_{i=1}^{N} a_{i,N-k+1} \tag{6}$$

Premises 2 and 3 are proven similarly using the induction method.

If $a_{i,j} = 1$, then the information of bit i-th is transmitted in the j-th transmission using the B-DMC channel. From premise 1, we can determine the number of '1' in row k of the matrix G_N, which represents the number of times the B-DMC channel is used to transmit information for bit u_k. Simultaneously, we can also calculate the number of '1' in column k of the matrix G_N, which represents the k-th use of the B-DMC channel to transmit the information of how many bits u_i. With a sufficiently large signal-to-noise ratio (corresponding to a bit error rate low than 10^{-5}), it is evident that the information of a bit is sent more reliably through channels that are used more frequently, and channels with fewer bits participating provide higher reliability for those bits. In other words, the reliability when transmitting the Polar code bits is directly proportional to the sum along the rows and inversely proportional to the sum along the columns of the channel transformation matrix G_N. From premise 2, we have:

$$\frac{1}{\sum_{i=1}^{N} a_{i,k}} = \frac{\sum_{j=1}^{N} a_{k,j}}{N} \tag{7}$$

So:

$$\frac{1}{\sum_{i=1}^{N} a_{i,k}} + \frac{\sum_{j=1}^{N} a_{k,j}}{N} = 2 \times \frac{\sum_{j=1}^{N} a_{k,j}}{N} = 2 \times 2^{\sum_{s=0}^{n=1} b_{k,s}} \tag{8}$$

In this paper, we use the value $2 \times 2^{\sum_{s=0}^{n=1} b_{k,s}}$ to represent the relationship between the reliability of transmitting Polar code bits with index k. Up to this point, we have discussed the number of times a channel is used to carry information for a bit or the number of bits transmitted in a single channel use. However, we've only been considering the B-DMC so far, without taking into account that the row index of the transformation matrix corresponds to a cumulative channel index, associated with the polarization effect of the channel. If we solely use the total number of '1' in rows or columns of the transformation matrix, it's insufficient to determine the reliability of the cumulative channel. But when combined with an ordering of the cumulative channel reliabilities, it can improve the accuracy of sorting these reliabilities. This study is based on a sequence of reliability ordering indices for cumulative channels using the PW method to determine a new sequence of reliability ordering indices for cumulative channels, referred to as IPW.

From the use of the PW method to create a sequence of reliability ordering indices and the challenges that this method faces, we propose an improved solution for PW, called IPW, based on the study of the encoding process of Polar codes using the channel transformation matrix. For a natural number $n > 2$, a channel index $k, 1 \leq k \leq N, N = 2^n$, and $(b_{k,n-1}, \cdots, b_{k,1}, b_{k,0})$ denoting the binary representation of the number $k-1$, the reliability value of the k-th cumulative channel is calculated as follows:

$$x_k = \left(\sum_{s=0}^{n-1} b_{k,s}\beta^s\right) \times g^{\frac{2^{\sum_{s=0}^{n-1} b_{k,s}}}{N}} \tag{9}$$

with $\beta = 1.1892, g = \left(\left(1+\sqrt{5}\right)/2\right)^{\nu}$.

The value of ν depends on the approach used to determine the reliability ordering index sequence for the cumulative channel Q, from which the set of frozen bit indices Q and information bit indices I are extracted. For every application of Polar codes for transmission channels in the 5G NR standard, the first step is to determine the original code length N, followed by selecting a rate matching scheme. By definition, the code length N of Polar codes must be a power of 2, while the number of information bits can vary. However, for common applications in 5G NR, the number of information bits is fixed, and the code length needs to be adjusted to achieve the required data rate for the application. Although Polar codes are linear codes, the rate matching translates into the adjustment of code length using classical methods such as puncturing, shortening, and extending. However, it might be necessary to recalculate the reliability values of cumulative channels to determine the positions of frozen bits, which increases the complexity.

Once the value of N is determined based on the specific requirements of each 5G NR application, you can calculate the reliability for N cumulative channels using a predefined algorithm (such as PW or IPW) and sort them in increasing order of reliability. The second approach involves calculating and sorting the order for a certain N_{max}, storing it in memory, and then extracting it for a specific N. The first approach falls into the on-the-fly category, offering better accuracy at the cost of increased processing latency. The second approach ensures low latency but may not achieve the same level of accuracy for certain specific code lengths. In both approaches, the quality depends on the code rate, meaning it depends on the number of information bits K.

In the IPW method, we choose:

$$\nu = 8\frac{N_{\max}}{N}\left(\frac{K}{N}\right)^2 \tag{10}$$

for all $0 < K < N < N_{\max}$.

The symbols F_{5G} and F_{IPW} represent the sets of frozen bit indices for the 5G NR standard and the IPW method, respectively. The comparison results of these sets for different code lengths $N = 6, 16, ..., 1024$ are presented in table 3 and table 4, corresponding to the code rate $R = K/N = 1/2, 1/4$.

Table 3 Comparison of the sequence of frozen bit indices between the 5G NR standard and the IPW method, $R = 1/2$

SN	Codeword lenght	The indexes are in F_{PW} but not in F_{5G}	The indexes are in F_{5G} but not in F_{PW}
1	16	7	10
2	32	...	...
3	64	...	...
4	128	99	30
5	258	...	...
6	512	313, 339, 391, 394, 449	119, 122, 174, 180, 200
7	1024	365, 423, 660, 776	191, 220, 232, 603

Table 4 Comparison of the sequence of frozen bit indices between the 5G NR standard and the IPW method, $R = 1/4$

SN	Codeword lenght	The indexes are in F_{PW} but not in F_{5G}	The indexes are in F_{5G} but not in F_{PW}
1	16	...	...
2	32	...	...
3	64	...	...
4	128	...	...
5	258	64	123
6	512	454	236
7	1024	820	699

Table 5 compares the sets of reliability ordering indices around the boundary between information and frozen bits using various methods such as PW, 5G NR, and IPW. The colored indices in the table are those located near the "boundary", where the yellow indices are close to the end of the reliability ordering sequence for the set of frozen bits, while the green indices are near the beginning of the reliability ordering sequence for the set of information bits. As mentioned earlier, to obtain the RS of 5G NR, it requires adjusting several indices near the boundary between the set of frozen and information bits, which involves substantial effort and simulations when using PW based on DE/GA. However, if IPW is used, the number of adjusted indices may be higher, but it only requires the mentioned computations.

With the corresponding sets of information and frozen bit indices, we conducted simulations to assess the quality of the code constructed using these indices and compared it with previously published sets of indices. The simulations were performed on an AWGN channel model using BPSK modulation, with code lengths ranging from 16 to 2048 bits and code rates of 1/2 and 1/4. Figures 1, 2, and 3 depict the simulation results comparing the performance of Polar codes with a code rate $R = 1/2$, constructed using the PW, IPW, and 5G NR standard methods, for code lengths ranging from 16 to 2048 bits.

Table 5 Comparison of the reliability ordering index sets generated by various methods around the boundary between information and frozen bits

Codeword length	Methode	Some index values in the frozen bit set correspond to the index values of the message bits	Some index values in the message bit set are adjacent to the index values of the frozen bits
16	PW	1 2 3 5 9 4 6 7	10 11 13 8 12 14 15 16
	5G NR	1 2 3 5 9 4 6 10	7 11 13 8 12 14 15 16
	IPW	1 2 3 5 9 4 6 7	10 11 13 8 12 14 15 16
32	PW	1 2 3 5 9 17 4 6 7 10 11 18 13 19 21 8	10 12 14 20 15 22 23 26 27 29 16 24 28 30 31 32
	5G NR	1 2 3 5 9 17 4 6 10 7 18 11 19 13 21 25	12 12 20 14 15 22 23 26 27 29 16 24 28 30 31 32
	IPW	1 2 3 5 9 17 4 6 7 10 11 18 13 19 21 25	8 12 14 20 15 22 23 26 27 29 16 24 28 30 31 32
128	PW	68 42 29 97 70 43 16 50 71 45 74 51 24 75 53 82 28 77 40 83 57 30 98 85	44 31 99 72 46 89 52 101 47 76 54 105 78 55 84 58 79 113 86 59 32 100 87 61
	5G NR	97 68 42 29 70 43 50 75 71 45 82 51 74 16 53 24 77 83 57 28 98 40 85 30	44 99 89 31 72 46 101 52 47 76 105 54 78 55 84 58 113 79 86 59 100 87 61 90
	IPW	68 97 42 29 70 43 50 71 45 74 51 16 75 53 82 77 24 83 57 98 28 85 40 99	30 89 101 44 31 72 46 105 52 47 76 113 54 78 55 84 58 79 86 59 100 87 61 90
256	PW	113 140 86 59 32 195 100 169 87 142 61 90 148 197 102 143 48 91 177 150 103 201 106 93	164 151 56 154 107 80 166 209 114 155 60 109 167 196 115 170 88 157 62 225 198 171 117 144
	5G NR	79 195 86 59 169 140 100 87 61 90 197 142 102 148 177 143 32 201 91 150 103 106 164 93	48 209 151 154 166 107 56 114 155 80 109 225 167 196 60 170 115 157 88 198 117 171 62 178
	IPW	79 195 169 140 86 59 197 100 87 142 61 90 177 148 102 201 143 91 32 150 103 106 93 164	209 151 154 107 48 166 114 155 109 225 167 196 115 170 157 56 198 171 80 117 178 60 199 173
512	PW	111 330 388 307 226 280 213 172 118 159 64 331 417 227 390 309 200 119 338 174 284 217 122 333	180 391 296 229 339 175 394 313 204 123 286 96 182 449 354 395 341 300 233 287 206 125 183 402
	5G NR	158 330 111 118 213 172 331 227 388 309 217 417 272 280 159 338 119 333 390 174 122 200 180 229	339 313 391 175 394 284 123 449 354 204 64 341 395 182 296 286 233 125 206 183 287 300 355 212
	IPW	110 417 213 168 116 331 158 227 390 111 309 338 217 280 172 118 333 159 391 229 339 449 394 313	200 119 174 284 122 354 180 395 341 233 296 175 402 204 123 355 286 397 182 345 403 300 241 64
1024	PW	625 407 312 245 652 598 571 191 544 410 220 363 336 897 802 707 612 422 789 232 681 599 465 370 654 573 411 316 249 602 222 365	660 803 423 776 709 614 452 371 655 560 426 344 793 236 603 413 318 223 128 689 662 805 615 481 834 454 427 373 780 713 618 400
	5G NR	648 349 420 407 465 681 802 363 591 410 571 789 598 573 220 312 709 599 602 652 422 793 803 612 603 411 232 689 654 249 370 191	365 655 660 336 481 316 222 371 614 423 426 452 615 544 236 413 344 373 776 318 223 427 454 238 560 834 805 713 835 662 809 780
	IPW	407 188 245 789 652 681 598 571 465 304 410 363 803 612 216 422 709 599 190 370 654 573 411 249 793 602 365 660 689 423 312 776	805 614 452 481 191 371 834 655 544 426 220 713 603 413 336 662 232 615 835 454 427 373 316 809 780 618 222 605 676 663 434 721

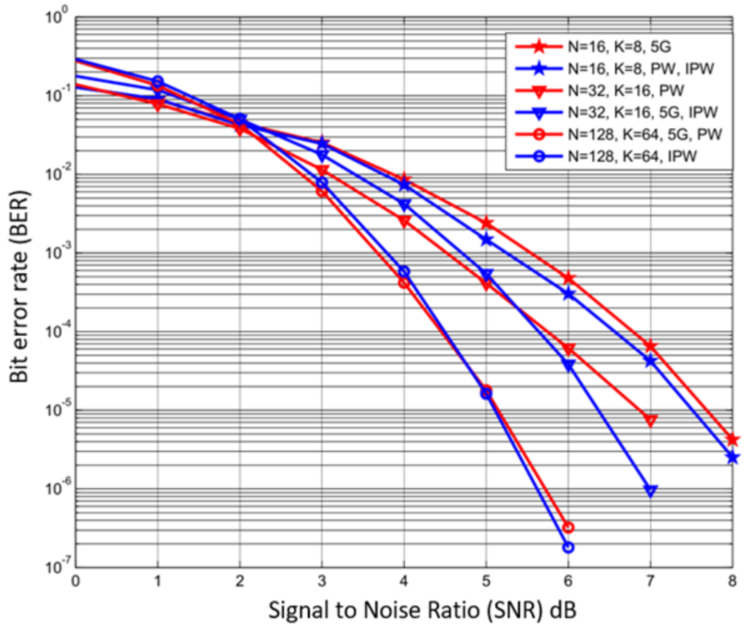

Figure 1. Comparison of the quality of Polar codes with a code rate of 1/2 constructed using the PW, IPW, and 5G NR standard methods for code lengths N = 16, N = 32, N = 128.

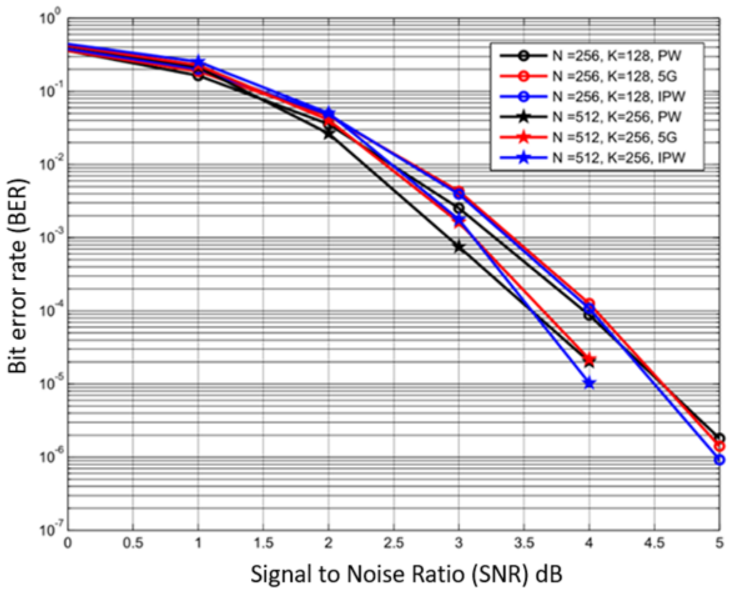

Figure 2. Comparison of the quality of Polar codes with a code rate of 1/2 constructed using the PW, IPW, and 5G NR standard methods for code lengths N = 256, N = 512.

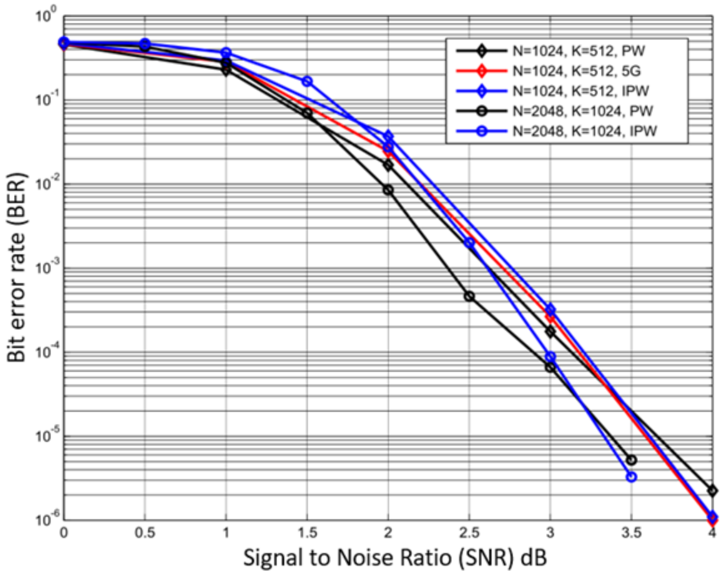

Figure 3. Comparison of the quality of Polar codes with a code rate of 1/2 constructed using the PW, IPW, and 5G NR standard methods for code lengths N = 1024, and the quality of Polar codes constructed using PW and IPW for code length N = 2048.

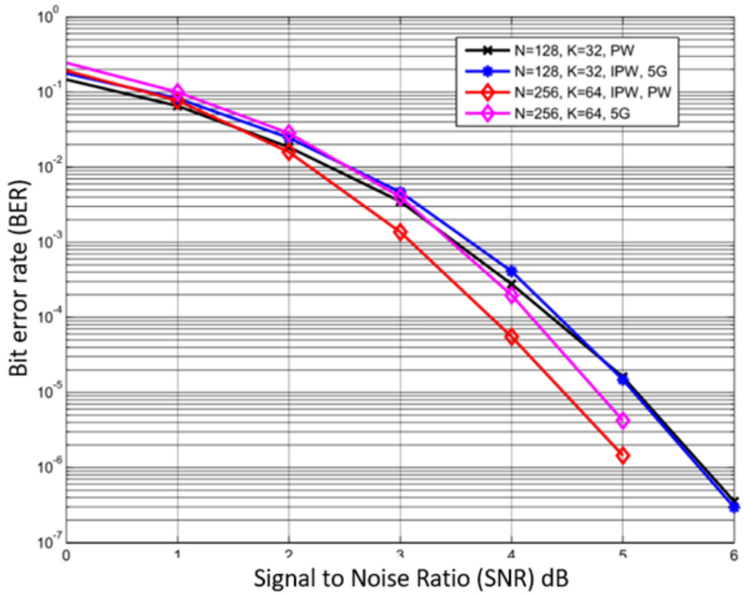

Figure 4. Comparison of the quality of Polar codes with a code rate of 1/4 constructed using the PW, IPW, and 5G NR standard methods for code lengths N = 128 and N = 256.

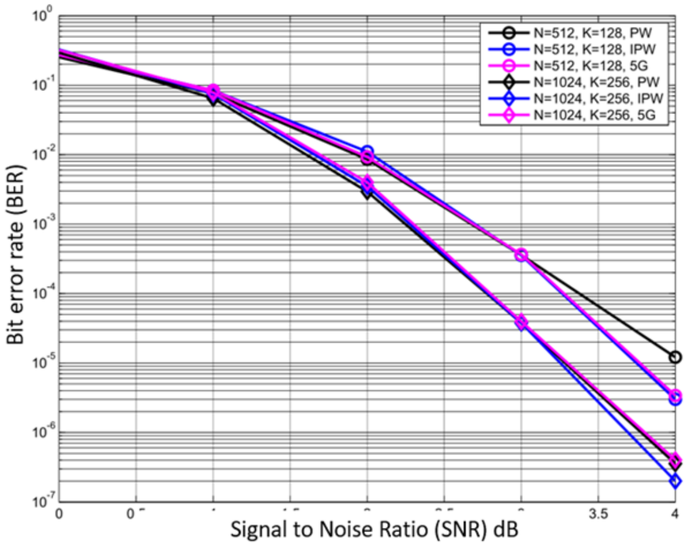

Figure 5. Comparison of the quality of Polar codes with a code rate of 1/4 constructed using the PW, IPW, and 5G NR standard methods for code lengths N = 512 and N = 1024.

Figure 4. and Figure 5. represent the simulation results comparing the quality of Polar codes with code rates $R = 1/4$, constructed using the PW, IPW, and 5G NR standard methods, for code lengths ranging from 128 to 1024 bits.

We can observe that the quality of Polar codes according to the 5G NR standard is superior (lower bit error rate BER at the same signal-to-noise ratio SNR) compared to the PW method, indicating that the standardization efforts for 5G NR have yielded excellent results. Meanwhile, IPW provides better quality than both PW and the 5G NR standard at the region with a sufficiently low BER (less than 10^{-5}), which is a more critical area for communication. It's worth noting that the comparison of Polar code quality for a length of 2048 bits was only conducted for the PW and IPW methods because the 5G NR standard currently only provides reliability ordering sequences up to a code length of 1024 bits. Please note that this study used SC (Successive Canceling) decoding, and these conclusions hold for all improvements of SC because, as mentioned earlier, the code quality with specific decoding is determined solely by the selection of the set of frozen and information bits.

5. Conclusion

The paper has analyzed and proposed an improvement to the PW method for computing and arranging the RS of Polar codes, referred to as the IPW method. The IPW method maintains the computational complexity similar to PW but provides better performance in terms of low bit error rate, making it suitable for communication applications in the emerging fifth-generation (5G NR) wireless communication systems. Furthermore, IPW can be utilized to generate reliable codewords with lengths greater than 1024 bits, specifically 2048 bits.

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Manuscript received 09-07-2023; Accepted 21-12-2023.

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GIẢI PHÁP TÍNH TOÁN TRỌNG SỐ PHÂN CỰC MỚI TRONG THIẾT KẾ MÃ PHÂN CỰC

Phạm Xuân Nghĩa, Đàm Đức Thuận, Lại Tiến Đệ

Tóm tắt

Mã phân cực được Arikan đề xuất năm 2009 dựa vào nguyên lý phân cực kênh đã được chứng minh đạt dung lượng kênh đầu vào nhị phân đối xứng rời rạc. Mã phân cực cũng đã được 3GPP chọn làm mã sửa lỗi trong hệ thống thông tin di động thứ 5 (5G New Radio - 5G NR) với độ dài từ mã tối đa 1024 bit. Việc thiết kế mã dựa vào độ tin cậy của các kênh tổng hợp đã được nghiên cứu với nhiều phương pháp khác nhau với mục đích thỏa mãn những yêu cầu trong ứng dụng của 5G NR. Xuất phát từ yêu cầu với thiết kế mã phân cực là dễ thực thi, độ phức tạp giải mã thấp và duy trì hiệu năng sửa lỗi tốt, bài báo đã đề xuất một phương pháp thiết kế mã phân cực bằng cách xác định chuỗi chỉ số tin cậy (Reliability Sequence - RS) kênh tổng hợp dựa trên phương pháp trọng số phân cực mới (Improved Polarization Weights - IPW). Kết quả mô phỏng cho thấy hiệu suất sửa lỗi của mã thiết kế theo IPW tốt hơn các cấu trúc mã thiết kế theo (Polarization Weights - PW) và cấu trúc mã của 5G NR tại vùng tỉ lệ lỗi bit nhỏ, đặc biệt phương pháp này còn xây dựng chuỗi RS cho từ mã dài 2048 bit trong khi mới chỉ có chuỗi RS cho bộ mã dài 1024 bit cho chuẩn 5G NR.

Từ khóa

Mã phân cực, mã kênh, FEC cho 5G NR, RS, PW, IPW.