SECOND-ORDER SLIDING MODE CONTROL FOR DOUBLE-PENDULUM THREE-DIMENSIONAL OVERHEAD CRANES

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Abstract:

This research proposes a second order sliding mode controller for a 3D crane system with a double pendulum effect and constant rope length. The 3D crane system is an underactuated system, and the control requirement is to ensure that the trolley reaches the desired position while keeping payload oscillations within acceptable limits. Therefore, the SMC algorithm is designed in two stages to meet these demands. First, the sliding surface is selected, and then the SMC scheme is designed to push all system states to the reference points on the sliding surface. The controller parameters are selected based on Lyapunov criterion and input-to-state stability (ISS). The simulation results show that the proposed controller ensures the 3D crane system effectively meets the desired requirements.

Keywords:

Lyapunov function, Double-pendulum three-dimensional Overhead cranes, Second-order Sliding Mode Controller

Tóm tắt:

Nghiên cứu này đề xuất bộ điều khiển chế độ trượt bậc hai cho hệ thống cầu trục 3D với hiệu ứng con lắc kép và chiều dài dây không đổi. Hệ thống cầu trục 3D là hệ thống thiếu cơ cấu chấp hành và yêu cầu đặt ra trong quá trình điều khiển là đảm bảo đưa xe con đến vị trí mong muốn, đồng thời sự dao động của tải trọng phải nằm trong mức cho phép. Do đó, thuật toán SMC được thiết kế gồm hai giai đoạn để đáp ứng yêu cầu trên. Đầu tiên, mặt trượt được lựa chọn và sau đó sơ đồ SMC được thiết kế để đưa tất cả các trạng thái của hệ thống tới mặt trượt. Các tham số của bộ điều khiển được lựa chọn dựa trên tiêu chuẩn Lyapunov và điều kiện ổn định đầu vào đến trạng thái (ISS). Các kết quả mô phỏng cho thấy bộ điều khiển đề xuất đảm bảo hệ thống đáp ứng tốt các yêu cầu mong muốn.

Từ khóa: Hàm Lyapunov, Cầu trục 3D với hiệu ứng con lắc kép, Bộ điều khiển trượt bậc hai

1. INTRODUCTION

For decades, overhead cranes have played a vital role in transporting goods. They are particularly common in shipyards, construction sites, warehouses, and factories due to their high load-bearing capacity. Studies have shown that overhead cranes are underactuated systems, requiring appropriate methods to design controllers that ensure the trolley reaches the desired position while keeping load oscillations within acceptable limits. However, the trolley's acceleration often induces unwanted oscillations, which can lead to load damage or even accidents. Therefore, various control strategies for overhead cranes have been proposed and refined to address these challenges effectively.

To date, various control approaches have been proposed for single-pendulum systems, including PID and PD controllers [1], the input shaping method [2], enhanced coupling controllers [3], the Lyapunov-based control approach [4], model predictive control methods [5], and the two-degree-of-freedom control approach [6], among others. However, in many cases, the load sway exhibits double-pendulum effects due to a large hook mass or irregularly-shaped loads, complicating system analysis and controller design. As a result, research on this system has attracted increasing attention in recent years. Specifically, Masoud et al. proposed a hybrid input shaping method for double-pendulum overhead cranes [7]. Optimal control [8] focuses on minimizing travel time and energy consumption but requires precise modeling and does not prioritize improving control quality, such as reducing oscillations or overshooting. While intelligent controllers are adaptive and do not require an exact model of the controlled object [9], their design has not yet fully accounted for constraints on control inputs and state variables. Therefore, a Lyapunov-based Model Predictive Control (LMPC) approach was introduced in [10], which allows for establishing limits on state variables for the double-pendulum crane system. This method offers good real-time performance but is less robust to reference path deviations and positioning errors, while also imposing a large computational burden on the system, making practical operation challenging.

Sliding Mode Control (SMC) is a well-known robust technique widely used in nonlinear systems, offering advantages such as simple implementation, fast response, and strong robustness against system parameter variations [11]. A second-order sliding mode controller was proposed by LA Tuan et al. [12] for a 3D single-pendulum crane system. An APD-SMC method was proposed for a 2D double-pendulum crane system in [13], achieving efficient positioning and rapid suppression of payload swings. In this study, we examine the dynamic model of a 3D double-pendulum crane system with a constant cable length and propose a second order sliding mode controller to enhance system performance. The controller ensures that the trolley follows the desired trajectory while minimizing load oscillation during movement.

The study is structured as follows: The dynamic model of the crane system is analyzed in Section 2. Section 3 explores the second-order sliding mode controller. Simulation results are presented in Section 4 to demonstrate the effectiveness of the approach. Conclusions and directions for future research are provided in Section 5.

2. MATHEMATICAL MODEL

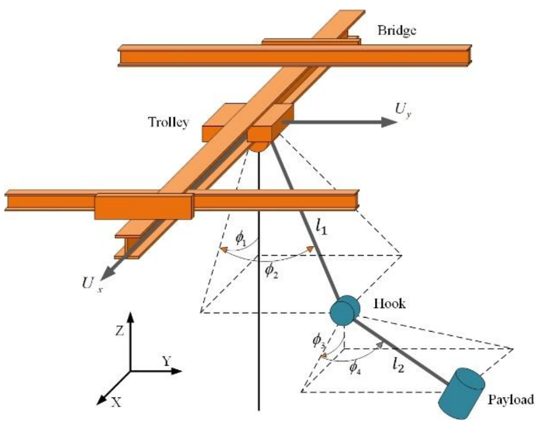

Figure 1. System dynamic model

Figure 1 provides the basis for analyzing and calculating the mathematical model of a 3D overhead crane with a double-pendulum system. x, y denotes the position of the trolley along x and y axis; ϕ_1, ϕ_2, ϕ_3, ϕ_4 represent the swing angles of the hook and the load along the x and y axis, respectively. M_1 is the mass of the trolley, and M_2 is the sum of the mass of the trolley and the bridge. m_1 and m_2 are the masses of the hook and payload, respectively. l_1 is the length of the cable and l_2 is the distance from the hook to the payload. b_{rx}, b_{ry} are frictions of the trolley and bridge motions, respectively. The driving forces along the x-axis and the y-axis are U_x, U_y, respectively. Defining the forces vector affecting the systems as $\mathbf{U} = \left[U_x, U_y, 0, 0, 0, 0\right]^T$, the state vector $\mathbf{q} = \left[x, y, \phi_1, \phi_2, \phi_3, \phi_4\right]^T$. The dynamic equation of the system can be formulated in the matrix as same [14] as:

$$\mathbf{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = U} \quad (1)$$

where $\mathbf{M(q)} \in \mathbf{R}^{6\times6}$ denotes a symmetric mass matrix, $\mathbf{C(q,\dot{q})} \in \mathbf{R}^{6\times6}$ is a Coriolis and centrifugal terms, and $\mathbf{G(q)} \in \mathbf{R}^{6\times1}$ denotes a gravity vector. The coefficients of $\mathbf{M(q)}, \mathbf{C(q,\dot{q})}$ and $\mathbf{G(q)}$ are defined similarly in [14].

3. THE SECOND-ORDER SLIDING MODE CONTROLLER (SO-SMC)

3.1. Decoupling

In this section, the force acting on the system is defined: $\mathbf{U}_a = \left[U_x, U_y\right]^T$. The 3D crane double-pendulum system is an underactuated system, so to control it, the states variables are separated into two terms: the actuated states $\mathbf{q}_a = \left[x, y\right]^T$ and unactuated states $\mathbf{q}_u = \left[\phi_1, \phi_2, \phi_3, \phi_4\right]^T$. The dynamic (1) can be rewritten into two equations:

$$\begin{cases} \mathbf{M}_{m1}(\mathbf{q})\ddot{\mathbf{q}}_a + \mathbf{M}_{n1}(\mathbf{q})\ddot{\mathbf{q}}_u + \mathbf{C}_{m1}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_a \\ +\mathbf{C}_{n1}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_u + \mathbf{G}_{m1}(\mathbf{q}) = \mathbf{U}_a \\ \mathbf{M}_{m2}(\mathbf{q})\ddot{\mathbf{q}}_a + \mathbf{M}_{n2}(\mathbf{q})\ddot{\mathbf{q}}_u + \mathbf{C}_{m2}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_a \\ +\mathbf{C}_{n2}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_u + \mathbf{G}_{n1}(\mathbf{q}) = \mathbf{0} \end{cases} \quad (2)$$

The two equations in (2) are written in terms of state variables with complete actuator mechanism, which can be expressed in the following form :

$$\begin{aligned}&\tilde{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}}_a+\tilde{\mathbf{C}}_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_a\\&+\tilde{\mathbf{C}}_2(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_u+\tilde{\mathbf{G}}(\mathbf{q})=\mathbf{U}_a\end{aligned} \quad (3)$$

where :

$$\tilde{\mathbf{M}}(\mathbf{q})=\mathbf{M}_{m1}(\mathbf{q})-\mathbf{M}_{n1}(\mathbf{q})\mathbf{M}_{n2}^{-1}(\mathbf{q})\mathbf{M}_{m2}(\mathbf{q})$$

$$\tilde{\mathbf{C}}_1(\mathbf{q},\dot{\mathbf{q}})=\mathbf{C}_{m1}(\mathbf{q},\dot{\mathbf{q}})-\mathbf{M}_{n1}(\mathbf{q})\mathbf{M}_{n2}^{-1}(\mathbf{q})\mathbf{C}_{m2}(\mathbf{q},\dot{\mathbf{q}})$$

$$\tilde{\mathbf{C}}_2(\mathbf{q},\dot{\mathbf{q}})=\mathbf{C}_{n1}(\mathbf{q},\dot{\mathbf{q}})-\mathbf{M}_{n1}(\mathbf{q})\mathbf{M}_{n2}^{-1}(\mathbf{q})\mathbf{C}_{n2}(\mathbf{q},\dot{\mathbf{q}})$$

$$\tilde{\mathbf{G}}(\mathbf{q})=\mathbf{G}_{m1}(\mathbf{q})-\mathbf{M}_{n1}(\mathbf{q})\mathbf{M}_{n2}^{-1}(\mathbf{q})\mathbf{G}_{n1}(\mathbf{q})$$

The equation (3) is used to design the controller for the system.

3.2. Control scheme design

A second order sliding mode controller is proposed for the crane system to ensure that the trolley and bridge follow the reference trajectory, while preventing payload oscillation during movement. All state variables of the system are assumed to be measurable. Define: $\mathbf{q}_{ar}=[x_r,y_r]^T$ and $\mathbf{q}_{ur}=[\phi_{1r},\phi_{2r},\phi_{3r},\phi_{4r}]^T=[0,0,0,0]^T$ are reference positions and swing angles of system.

The sliding surface is defined as follows:

$$\begin{aligned}\mathbf{s}_t&=\dot{\mathbf{e}}_a+\boldsymbol{\alpha}\mathbf{e}_a+\boldsymbol{\beta}\mathbf{e}_u\\&=(\dot{\mathbf{q}}_a-\dot{\mathbf{q}}_{ar})+\boldsymbol{\alpha}(\mathbf{q}_a-\mathbf{q}_{ar})+\boldsymbol{\beta}(\mathbf{q}_u-\mathbf{q}_{ur})\end{aligned} \quad (4)$$

where $\mathbf{e}_a=\mathbf{q}_a-\mathbf{q}_{ar}$ and $\mathbf{e}_u=\mathbf{q}_u-\mathbf{q}_{ur}$ are tracking error vectors. $\boldsymbol{\alpha}=\mathrm{diag}(\alpha_1,\alpha_2)$ and $\boldsymbol{\beta}=\begin{bmatrix}\beta_1 & 0 & \beta_3 & 0\\ 0 & \beta_2 & 0 & \beta_4\end{bmatrix}$ are control parameters with $\alpha_1,\alpha_2>0;\beta_i\in\mathbb{R}$.

Derivative of the sliding surface with respect to time:

$$\begin{aligned}\dot{\mathbf{s}}_t&=\ddot{\mathbf{e}}_a+\boldsymbol{\alpha}\dot{\mathbf{e}}_a+\boldsymbol{\beta}\dot{\mathbf{e}}_u\\&=(\ddot{\mathbf{q}}_a-\ddot{\mathbf{q}}_{ar})+\boldsymbol{\alpha}(\dot{\mathbf{q}}_a-\dot{\mathbf{q}}_{ar})+\boldsymbol{\beta}\dot{\mathbf{q}}_u\end{aligned} \quad (5)$$

The dynamic equation for the sliding surface is constructed as follows:

$$\dot{\mathbf{s}}_t+\boldsymbol{\alpha}\mathbf{s}_t=0 \quad (6)$$

Substituting (4) and (5) into (6), we get:

$$\begin{aligned}&(\ddot{\mathbf{q}}_a-\ddot{\mathbf{q}}_{ar})+\boldsymbol{\alpha}(\dot{\mathbf{q}}_a-\dot{\mathbf{q}}_{ar})+\boldsymbol{\beta}\dot{\mathbf{q}}_u\\&+\boldsymbol{\alpha}\left(\dot{\mathbf{q}}_a-\dot{\mathbf{q}}_{ar}+\boldsymbol{\alpha}\mathbf{e}_a+\boldsymbol{\beta}\mathbf{q}_u\right)=0\end{aligned} \quad (7)$$

Combining the two equation (3) and (7), we obtain the following equivalent control input:

$$\begin{aligned}\mathbf{U}_{eq}&=\tilde{\mathbf{C}}_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_a+\tilde{\mathbf{C}}_2(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_u+\tilde{\mathbf{G}}(\mathbf{q})\\&-\tilde{\mathbf{M}}(\mathbf{q})\begin{Bmatrix}2\boldsymbol{\alpha}\left(\dot{\mathbf{q}}_a-\dot{\mathbf{q}}_{ar}\right)\\+\boldsymbol{\beta}\dot{\mathbf{q}}_u+\boldsymbol{\alpha}^2\mathbf{e}_a+\boldsymbol{\alpha\beta}\mathbf{q}_u-\ddot{\mathbf{q}}_{ar}\end{Bmatrix}\end{aligned} \quad (8)$$

To consistently maintain the system's trajectories on the sliding surface, we introduce a switching action, expressed as follows: $\mathbf{U}_{sw}=-\mathbf{K}\,\mathbf{sign}(\mathbf{s}_t)$

with $\mathbf{K}=\mathrm{diag}(k_1,k_2)$, in which k_1,k_2 are positive real constants. Thus, the control signal vector of the crane system is determined as follows:

$$\begin{aligned}\mathbf{U}_a&=\tilde{\mathbf{C}}_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_a+\tilde{\mathbf{C}}_2(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_u+\tilde{\mathbf{G}}(\mathbf{q})\\&-\tilde{\mathbf{M}}(\mathbf{q})\begin{Bmatrix}2\boldsymbol{\alpha}\left(\dot{\mathbf{q}}_a-\dot{\mathbf{q}}_{ar}\right)\\+\boldsymbol{\beta}\dot{\mathbf{q}}_u+\boldsymbol{\alpha}^2\mathbf{e}_a+\boldsymbol{\alpha\beta}\mathbf{q}_u-\ddot{\mathbf{q}}_{ar}\end{Bmatrix}\\&-\mathbf{K}\,\mathbf{sign}(\mathbf{s}_t)\end{aligned} \quad (9)$$

To reduce chattering, $\mathbf{sign}(\mathbf{s}_t)$ function should be replaced by a saturation function as follows: $\mathbf{sat}(\mathbf{s}_t) = \begin{cases} 1 & ,\text{if } |\mathbf{s}_t/\varepsilon| > 1 \\ \mathbf{s}_t/\varepsilon & ,\text{if } |\mathbf{s}_t/\varepsilon| < 1 \end{cases}$, where ε is a constant indicating the thickness of the boundary layer. With control law (9), the dynamic equation for sliding surface become:

$$\dot{\mathbf{s}}_t + \boldsymbol{\alpha}\mathbf{s}_t = -\mathbf{K}\mathbf{sat}(\mathbf{s}_t) \qquad (10)$$

3.3. Stability analysis

To analyze the stability of the system, the Lyapunov function is chosen as follows:

$$V = \frac{1}{2}\mathbf{s}_t{}^T\mathbf{s}_t \geq 0 \qquad (11)$$

The time derivative of the Lyapunov function is determined as follows:

$$\begin{aligned} \dot{V} &= \mathbf{s}_t{}^T\dot{\mathbf{s}}_t \\ &= -\mathbf{s}_t{}^T\boldsymbol{\alpha}\mathbf{s}_t - \mathbf{s}_t{}^T\tilde{\mathbf{M}}^{-1}(\mathbf{q})\mathbf{K}\mathbf{sat}(\mathbf{s}_t) \end{aligned} \qquad (12)$$

Since $\tilde{\mathbf{M}}(\mathbf{q})$ is a square, invertible, positive definite symmetric matrix and $\mathbf{K}, \boldsymbol{\alpha}$ are diagonal matrices with positive coefficients, therefore $\dot{V} \leq 0$, lead to $V(t) \leq V(0)$. By applying Barbalat's lemma, it follows that $\lim_{t\to\infty} V = 0$, which lead to $\lim_{t\to\infty} \mathbf{s}_t = 0$, we have asymptotically stable sliding surface $\mathbf{s}_t$. This finding implies that all state trajectories converge towards the aforementioned surface.

From equation (4), it is apparent that $\lim_{t\to\infty} \mathbf{s}_t = 0$ alone is insufficient to draw a conclusion about the convergence of the system outputs. Therefore, clear mathematical proof is needed. The errors vector is defined as:

$$\mathbf{z} = [\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \mathbf{z}_4]^T = [\mathbf{e}_a, \mathbf{e}_u, \dot{\mathbf{e}}_a, \dot{\mathbf{e}}_u]^T$$

We have the derivation of $\mathbf{z}_3$ and $\mathbf{z}_4$:

$$\begin{aligned} \dot{\mathbf{z}}_3 &= -(\boldsymbol{\alpha}^2\mathbf{e}_a + \boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{e}_u + 2\boldsymbol{\alpha}\dot{\mathbf{e}}_a + \boldsymbol{\beta}\dot{\mathbf{e}}_u) \\ &= -(\boldsymbol{\alpha}^2\mathbf{z}_1 + \boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{z}_2 + 2\boldsymbol{\alpha}\mathbf{z}_3 + \boldsymbol{\beta}\mathbf{z}_4) \end{aligned} \qquad (13)$$

$$\begin{aligned} \dot{\mathbf{z}}_4 &= \ddot{\mathbf{e}}_u = \ddot{\mathbf{q}}_u = -\mathbf{M}_{n2}^{-1}(\mathbf{M}_{m2}\ddot{\mathbf{q}}_a \\ &+ \mathbf{B}_{n1}\dot{\mathbf{q}}_a + \mathbf{C}_{m2}\dot{\mathbf{q}}_a + \mathbf{C}_{n2}\dot{\mathbf{q}}_u + \mathbf{G}_{n1}) \end{aligned} \qquad (14)$$

After some calculation, $\dot{\mathbf{z}}_4$ can be represented as:

$$\dot{\mathbf{z}}_4 = \mathbf{P}_1\mathbf{z}_1 + \mathbf{P}_2\mathbf{z}_2 + \mathbf{P}_3\mathbf{z}_3 + \mathbf{P}_4\mathbf{z}_4 + \mathbf{P}_5 \qquad (15)$$

with $\mathbf{P}_1 = \mathbf{M}_{n2}^{-1}\mathbf{M}_{m2}\boldsymbol{\alpha}^2$; $\mathbf{P}_2 = \mathbf{M}_{n2}^{-1}\mathbf{M}_{m2}\boldsymbol{\alpha}\boldsymbol{\beta}$; $\mathbf{P}_3 = 2\mathbf{M}_{n2}^{-1}\mathbf{M}_{m2}\boldsymbol{\alpha} - \mathbf{M}_{n2}^{-1}\mathbf{C}_{m2}$; $\mathbf{P}_4 = \mathbf{M}_{n2}^{-1}\mathbf{M}_{m2}\boldsymbol{\beta} - \mathbf{M}_{n2}^{-1}\mathbf{C}_{n2}$; $\mathbf{P}_5 = -\mathbf{M}_{n2}^{-1}\left(\mathbf{M}_{m2}\ddot{\mathbf{q}}_{ar} + \mathbf{C}_{m2}\dot{\mathbf{q}}_{ar} + \mathbf{G}_{n1}\right)$.

The equation (14) and (15) can be written:

$$\begin{bmatrix} \dot{\mathbf{z}}_1 \\ \dot{\mathbf{z}}_2 \\ \dot{\mathbf{z}}_3 \\ \dot{\mathbf{z}}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{2\times2} & \mathbf{0}_{2\times4} & \mathbf{I}_{2\times2} & \mathbf{0}_{2\times4} \\ \mathbf{0}_{4\times2} & \mathbf{0}_{4\times4} & \mathbf{0}_{4\times2} & \mathbf{I}_{4\times4} \\ -\boldsymbol{\alpha}^2 & -\boldsymbol{\alpha}\boldsymbol{\beta} & -2\boldsymbol{\alpha} & -\boldsymbol{\beta} \\ \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 & \mathbf{P}_4 \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \mathbf{z}_3 \\ \mathbf{z}_4 \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{2\times1} \\ \mathbf{0}_{4\times1} \\ \mathbf{0}_{2\times1} \\ \mathbf{P}_5 \end{bmatrix}$$

$$\text{or} \qquad \dot{\mathbf{z}} = \mathbf{P}\mathbf{z} + \mathbf{Q} \qquad (16)$$

This system is stable if $\mathbf{P}_{|\mathbf{z}=\mathbf{z}_r}$ is a Hurwitz matrix, so if the design parameters satisfy:

$$\begin{cases} \alpha_1, \alpha_2 > 0 \\ \beta_1 < \alpha_1/2, \beta_2 < \alpha_2/2 \end{cases} \qquad (17)$$

the system response will approach the set value $\mathbf{q}_a \to \mathbf{q}_{ar}$, $\mathbf{q}_u \to \mathbf{q}_{ur}$ when $t \to \infty$.

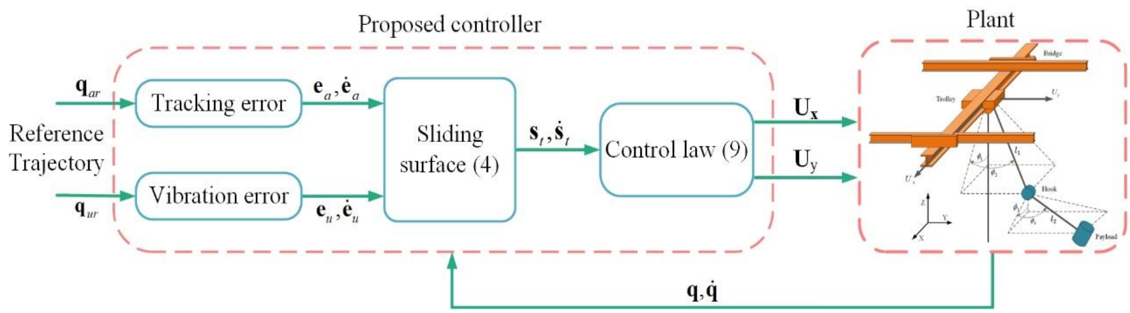

Figure 2. Closed-loop control system diagram

4. SIMULATION

This section refers to the utilization of 3D double-pendulum crane models with specific parameter values: $M_1 = 20\,\text{kg}$, $M_2 = 30\,\text{kg}, m_1 = 5\,\text{kg}, m_2 = 5\,\text{kg}, l_1 = 1\,\text{m}$, $l_2 = 0.5\,\text{m}, g = 9.8\,\text{m/s}^2$ and $b_{rx} = 20\,\text{Nm/s}$, $b_{ry} = 30\,\text{Nm/s}$. The parameters of the controller is chosen: $\mathbf{K} = \text{diag}(0.005;0.1)$; $\boldsymbol{\alpha} = \text{diag}(0.225;19.8)$;

$$\boldsymbol{\beta} = \begin{bmatrix} -3.6 & 0 & 0 & 0 \\ 0 & -7.2 & 0 & 0 \end{bmatrix} \text{ and } \varepsilon = 0.09.$$

Simulation was conducted on MATLAB/Simulink software. The trolley is driven to move from its initial position to the desired 3-meter displacement along the x-axis. The trajectory of the bridge along the y-axis is designed as an S-shaped curve. The simulation results are verified in the two cases outlined below.

Case 1: The crane parameters used in this case are outlined above.

Case 2: System with uncertainties. The system's responses are investigated when the payload mass increases by 200% $(m_2 = 15\text{kg})$ and when the damping coefficients increase by 10% $\left(b_{rx} = 22 \text{ Nm/s},\ b_{ry} = 33 \text{ Nm/s}\right)$.

Figure 3 shows the movement of the trolley reaching the desired position after a significantly short period, and the tracking error of the trolley quickly converges to zero in both cases. Figure 4 displays the tracking trajectories of the bridge. It demonstrates that the tracking control results of the proposed controller are effective, closely following the reference trajectory with relatively small tracking errors, ensuring convergence to zero even in the presence of system uncertainties. The swing angles of the hook and payload are shown in Figure 5 and Figure 6, indicating that the payload is lifted with relatively small oscillations. Although system uncertainties cause an increase in the swing angles, the overshoot does not exceed 0.02 rad. Finally, Figure 7 shows that the control input signals are not excessive, and the overshoot does not exceed 10 N.

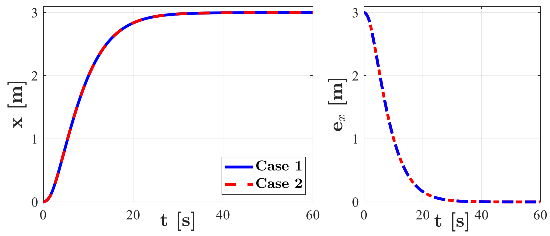

Figure 3. Trolley motion and tracking error along x axis

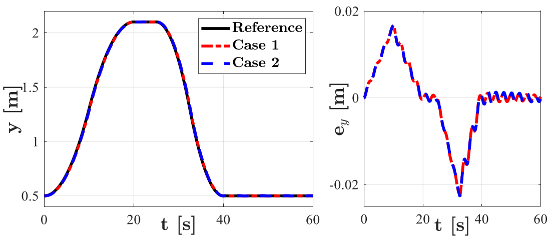

Figure 4. Trolley motion and tracking error along y axis

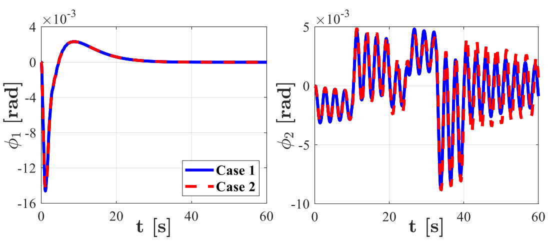

Figure 5. Hook swing angles

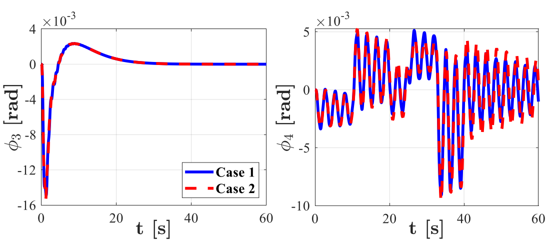

Figure 6. Payload swing angles

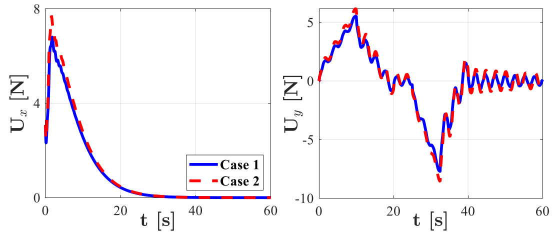

Figure 7. Input control signals $\mathbf{U}_x, \mathbf{U}_y$

5. CONCLUSION

In this study, a second-order sliding mode controller is proposed to control a 3D crane system with a double-pendulum effect. The simulation results show that the system asymptotically stabilizes towards the equilibrium point. Both the trolley and bridge movements closely approach the reference input values, with no significant overshoot. During operation, the swing angles of the hook and payload are minimized and fully eliminated after a certain period. At the same time, the proposed controller ensures the robustness of the system in the event of system parameter changes. In the future, experiments will be conducted on a laboratory overhead crane to validate the controller's accuracy and practical applicability. At the same time, the observer will be studied to estimate the state variables and external disturbances affecting the system.

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