

## HÀM MỤC TIÊU CHO BÀI TOÁN ƯỚC LƯỢNG TRẠNG THÁI HỆ THỐNG ĐIỆN KHI SỬ DỤNG THUẬT TOÁN TỐI ƯU BẦY ĐÀN

### OBJECTIVE FUNCTION FOR POWER SYSTEM STATE ESTIMATION WITH PARTICLE SWARM OPTIMIZATION

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#### Tóm tắt:

Bài toán ước lượng trạng thái có mục tiêu là xác định trạng thái gần giống nhất của hệ thống dựa trên tập các giá trị đo đang có, giúp người vận hành đánh giá hệ thống và đưa ra những quyết định phù hợp. Để giải quyết bài toán, bài báo trình bày nghiên cứu sáu kết hợp giữa hai thuật toán (tối ưu bầy đàn, tối ưu bầy đàn với quá trình tách biến) và ba dạng hàm mục tiêu (bình phương cực tiểu có trọng số, cực tiểu trị tuyệt đối, cực tiểu trị tuyệt đối có trọng số). Bên cạnh đó, việc đánh giá giá trị các biến trạng thái được thực hiện trong thuật toán thay vì đưa hàm phạt vào hàm mục tiêu như các nghiên cứu trước đây. Các kết hợp được thực hiện mô phỏng cho lưới điện IEEE 14 nút và IEEE 30 nút với trường hợp giả sử dữ liệu đo từ thiết bị đo thông thường hoặc từ thiết bị PMU. Các kết quả mô phỏng cho thấy các thuật toán kết hợp với hàm bình phương cực tiểu có trọng số cho kết quả ước lượng tốt nhất trong các trường hợp nghiên cứu.

#### Từ khóa:

Ước lượng trạng thái hệ thống điện, thuật toán tối ưu bầy đàn, tách biến, WLS, LAV, WLAV

#### Abstract:

The state estimation problem aims to determine the likelihood state of the power system based on the available measurement values. This helps operators to analyze and evaluate the systems so they can make appropriate control decisions. This paper examines six combinations of two algorithms (particle swarm optimization and particle swarm optimization with decoupled variables) and three objective functions (weighted least squares, least absolute values, and weighted least absolute values) to solve the power system state estimation. In addition, rather than employing a penalty function within the objective function as in previous studies, this work use a procedure within the algorithm to verify whether the state variable values remain within the prescribed boundaries. These combinations are simulated for 14-bus and 30-bus IEEE power systems, assuming that input data comes from conventional measuring devices or phasor measurement units. The estimation results show that using the weighted least square function gives the best estimation results.

#### Keywords:

Power system state estimation, particle swarm optimization, decoupled variable, WLS, LAV, WLAV

## 1. INTRODUCTION

Nowadays, the power system scale is expanding according to the growth of the load and the addition connecting of renewable energy sources. Therefore, the modern power system is developing towards automation and intelligence to ensure it operates in a safe, reliable, and efficient mode. This target requires much support from computer programs with input data from the Supervisory Control and Data Acquisition system (SCADA) and/or Phasor Measurement Units (PMU). Currently, measuring devices are not often placed on all buses, so we can not acquire all the state parameters. Consequently, we must solve the power system state estimation (PSSE) problem to obtain unknown parameters. This problem target is to determine the likelihood state of the power system based on the available measurement values. The input parameters include system topology, lines, transformers, compensation devices, and measured values such as bus voltage magnitudes, phase angles, active powers, reactive powers, branch currents, etc. The outputs are estimated values of bus voltage magnitude and phase angle. These values will help the operators in system analysis and evaluation. Then, they can make appropriate control decisions.

We can describe the PSSE problem using the Weighted Least Squares (WLS) objective function, which assumes that the measurement errors are known, independent, and randomly distributed

according to a Gaussian distribution [1]. However, there may be other parameters that could affect the estimation results, whether or not they contain errors. In [2], the author introduced the concept of M-estimators, which aim to minimize a function  $\rho(r)$  that quantifies the deviation between measured and estimated values. Alternatively, various forms of the function  $\rho(r)$  have been proposed in [3] and [4]. If  $p(r)$  is an absolute value function, it will correspond to the Least Absolute Value (LAV) model. Articles [5] and [6] add a weighting factor to the LAV function, so  $p(r)$  corresponds to the Weighted Least Absolute Value (WLAV) function. In [4], the authors propose the function  $p(r)$  with a tuning factor  $\alpha$  and show that the proposed function behaves like LAV when the tuning factor is small and like WLS when the tuning factor is high.

The problem of estimating the state variable's value with these objective functions mentioned above can be solved iteratively by algorithms like Newton. However, when using the LAV estimator, the weighting matrix will be zero, so it must be used other methods, such as integer programming. In addition, this problem can also be solved by heuristic search algorithms, as in some published papers.

This paper presents the use of Particle Swarm Optimization (PSO), a heuristic search algorithm, to address the PSSE problem. In this algorithm, solutions are

primarily derived from the information and orientation of the objective function value. An advantage of utilizing PSO, in contrast to traditional like Newton's method, is that it does not require the calculation of derivative matrices, making it a more efficient and accessible option. Additionally, the algorithm is not impacted by zero matrix issues when the LAV formula is used. However, selecting an appropriate objective function is critical for achieving optimal results with the PSO algorithm. Consequently, this work conducts a thorough analysis and evaluation of the PSO algorithm as it pertains to various types of objective functions.

The authors in the paper [8-11] performed state estimation using the PSO algorithm combined with WLS or WLAV objective function. In these studies, both WLS and WLAV incorporated a penalty function that assigns an infinitely positive value if the state variable falls outside the defined search space. In the paper [12], the PSO algorithm was also applied to the PSSE problem, and the authors provided insight regarding the impact of measurements' type and location on the estimation results. In [13], PSO and the genetic algorithm with decoupled variables are presented. The objective function used in articles [12] and [13] is also WLS with a penalty function. By studying the PSO algorithm, various function combinations can be employed. For instance, instead of implementing a penalty function in the

objective function, a particle evaluation can be performed through a position update process, ensuring that each one is within the allowed limits.

The analysis above indicates that few studies have focused on algorithms that omit the use of a penalty function within the objective function. Therefore, further evaluation from this perspective is necessary, as the results can serve as basis for choosing a more accurate objective function. Moreover, in all previous studies [8-10] and [12], the application of the PSO algorithm did not incorporate decoupled variable techniques. Accordingly, the following sections of this paper will focus on the PSO algorithm without and with the decoupled variables when combined with the WLS, LAV, and WLAV objective functions without the penalty function. The evaluation and proposal of the objective function rely on simulation results with IEEE 14-bus and IEEE 30-bus power systems.

## 2. OBJECTIVE FUNCTIONS FOR POWER SYSTEM STATE ESTIMATION

The estimation problem of power system state variables is formulated using input data, including system connection details, line parameters, transformers, measured values, etc.

Suppose there is a set of  $m$  measurements  $z_i$ , and the function  $h_i(x_1, x_2, \dots, x_n)$  corresponds to the measurement type of  $z_i$ . Then we have:

$$z = h(x) + e \quad (1)$$

The constraints of the state variable  $x_i$ :

$$x_i^{\min} \leq x_i \leq x_i^{\max} \quad (2)$$

where :

- $x$  is the state variable vector which includes the value of bus voltage magnitude and phase angle ;
- $e$  is the error of measurement ;
- $h_i(x)$  is reactive power function (or active power function, or branch current, or voltage, or phase angle) depending on the type of  $z_i$ ;

### 2.1. Weighted Least Squares

To determine the state variables  $x_1, x_2, \dots, x_n$  corresponding to the set of measured values  $z$ , the weighted least squares method minimizes the following objective function [1]:

$$J(x) = w_i^2 (z_i - h_i(x))^2 \\ = \sum_{i=1}^m \frac{1}{\sigma_i^2} e_i^2 \quad (3)$$

where:

- $w_i = \frac{1}{\sigma_i^2}$ , with  $\sigma_i$  is the standard deviation of the  $i$ -th measurement, representing the expected accuracy of the measuring device.
- $m$  is number of measurement ;

### 2.2. Least Absolute Value

The Least Absolute Value objective function is described as finding the minimum value of the function  $F(x)$  as in

equation (4):

$$F(x) = \sum_{i=1}^m |z_i - h_i(x)| \quad (4)$$

### 2.2. Weighted Least Absolute Value

The Weighted Least Absolute Value objective function is described as finding the minimum value of the function  $F(x)$  as in equation (5) [1][5][6]:

$$F(x) = \sum_{i=1}^m \frac{1}{\sigma_i} |z_i - h_i(x)| \quad (5)$$

## 3. THE ALGORITHM

When employing the Particle Swarm Optimization (PSO) algorithm to estimate power system state variables, several terms within the algorithm are defined as follows:

- A “particle” denotes a specific set of values that represent the state variables associated with bus voltage magnitudes and/or phase angles.
- The “particle position” refers to the specific values of the state variables contained within a single particle.
- “Particle velocity” signifies the amount added to the current state variable values to generate new ones. This adjustment expands the search space and enhances the likelihood of finding the global optimal solution.
- A “swarm” represents the collective group of multiple particles.

### 3.1. Particle Swarm Optimization

Particle Swarm Optimization is an algorithm that seeks the optimal solution

by leveraging information from a swarm of particles. In the context of a power grid with  $N$  nodes, each particle represents both the voltage magnitude and phase angle of the buses, resulting in a total of  $(2N-1)$  state variables. The algorithm starts by randomly initializing a specified number of particles. Each particle has the capability to remember the best position it has achieved thus far, as well as the best position found by the entire swarm. The particle's movement is then influenced by its velocity, which is calculated using a specific formula (referred to as formula (6) in reference [14]). Subsequently, the particle's new position is updated according to formula (7).

$$V_{id} = \chi \cdot (V_{id} + c_1 \cdot r_1 \cdot (p_{ib} - x_{id}) + c_2 \cdot r_2 \cdot (p_{gb} - x_{id})) \quad (6)$$

$$x_{id} = x_{id} + V_{id} \quad (7)$$

where:

- $\chi = 0,729$ ;  $c_1 = c_2 = 2,05$ ;
- $r_1$  and  $r_2$  are random values in the range  $(0,1)$ ;
- $p_{gb}$  is the global best, referred to the overall best solution found by the swarm;
- $p_{ib}$  is the personal best position of  $i$ -th particle;

The state variable within the search space has defined limits, specifically a minimum value ( $x_{min}$ ) and a maximum value ( $x_{max}$ ). After updating an individual's position, it is important to ensure that the state variable stays within these boundaries. To achieve this, the algorithm checks the value of the state variable and makes necessary adjustments if needed. If the

revised value,  $x_{id}$ , falls below  $x_{i}^{min}$  or exceeds  $x_{i}^{max}$ , the algorithm will set  $x_{id}$  to  $p_{ib}$ . The process of estimating the state variable value using PSO algorithm is illustrated in the block diagram shown in Figure 1.

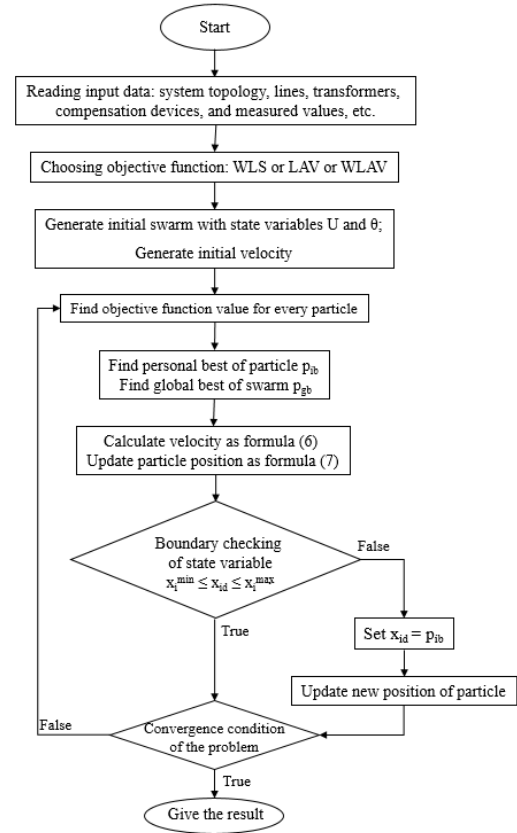


Figure 1. Diagram of the PSO algorithm

### 3.2. Separate V-θ estimation using Particle Swarm Optimization (SPSO)

In this section, we address the problem of estimating the state variable values using the SPSO (Separate U-θ estimation process) algorithm based on the Particle Swarm Optimization technique. The distinguishing feature of the SPSO algorithm is its methodical approach to estimating state variables separately. In

one N-bus power grid, two particle types are involved:  $\theta$ , which consists of (N-1) voltage phase angle values, and U, which consists of N voltage magnitude values. The estimation of voltage magnitudes and phase angles is performed separately and iteratively. When estimating the voltage phase angle values, the voltage magnitude values are held constant based on the best individuals identified in the previous result. Conversely, the same principle applies when estimating voltage magnitude values. The overall process of the SPSO algorithm is depicted in the block diagram shown in Figure 2.

Convergence for both PSO and SPSO is considered achieved when either the maximum number of iterations is reached or the objective function value remains constant over m consecutive iterations.

#### 4. SIMULATION

The simulation data, including branch data, bus data, and measurement values are obtained from [15], [16], and [17]. Two algorithms simulate the IEEE 14-bus and IEEE 30-bus systems under two distinct case studies, excluding bad data impacts.

- Case 1: assuming measurement data from conventional measuring devices, including the measure values of bus injection powers and one voltage magnitude. The total number of input measurements used for IEEE 14-bus and 30-bus grids is 29 and 31, respectively.

- Case 2: In this case study, the measurement data is assumed to be

collected from Phasor Measurement Units placed in the system.

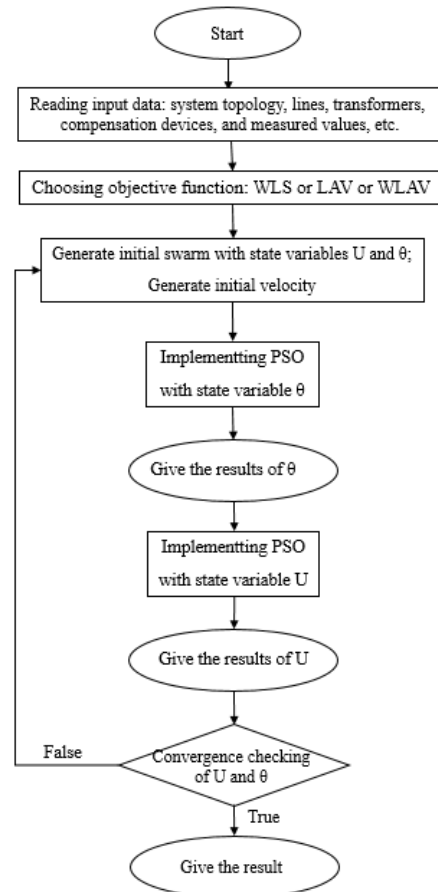


Figure 2. Diagram of the SPSO algorithm

The data includes the bus voltage magnitude, the bus voltage phase angle, the branch current, and the branch current phase angle. The locations for PMU placement are determined as suggested in reference [18]. For the IEEE 14-bus system, the PMUs are installed at buses 2, 6, 7, and 9. In the IEEE 30-bus system, the PMUs are placed at buses 2, 4, 6, 9, 10, 12, 15, 18, 25, and 27. The total number of input measurements utilized for these systems is 38 and 104, respectively.

In both cases, this paper examines six combinations of the PSO and SPSO algorithms with the WLS, LAV, and WLAV functions: PSO-WLS, PSO-LAV, PSO-WLAV, SPSO-WLS, SPSO-LAV, and SPSO-WLAV. To evaluate the estimation accuracy of the state variables, the estimated values are compared with their corresponding reference values. Specifically, the estimated voltage magnitude and phase angle are compared with their respective reference values  $U_{ref}$  and  $\theta_{ref}$ . Accordingly, the percentage error of each state variable is determined as follows:

$$\%e_i = \frac{x_{est\ i} - x_{ref\ i}}{x_{ref\ i}} \cdot 100 \quad (8)$$

Where:

- $\%e_i$  is the percentage error in the estimation of state variable  $i$ ;
- $x_{est\ i}$  is the estimated value of state variable  $i$ ;
- $x_{ref\ i}$  is the reference value of state variable  $i$ ;

The algorithm presented in section 3 is tested in each system with the following parameters:

- Maximum iteration: 60000 PSO; 100 for outer loop and 3000 for inner loop of SPSO;
- Population size: 40 for IEEE 14-bus, 100 for 30-bus;

For case 1, the results are presented in Figures 3 and 5 for the IEEE 14-bus, and in Figures 7 and 9 for the IEEE 30-bus. The estimated results for case 2 are shown in Figures 4, 6, 8, and 10. The maximum error

value of the estimated  $U-\theta$  results is listed in Table 1.

#### 4.1. Simulation of case study 1

The results of estimating the bus voltage magnitude for the IEEE 14-bus grid indicate that the simulations yield values that are close to the reference value,  $U_{ref}$ . The combinations of SPSO-LAV and PSO-WLAV show two large deviations, but errors just below 1,8%. When estimating the voltage phase angle, the combination of SPSO-LAV and PSO-LAV exhibits significant discrepancies from the reference value,  $\theta_{ref}$ , with the largest errors reaching 15,12% and 18,97%. In contrast, the SPSO-WLS combination provides the closest estimation to  $\theta_{ref}$ , followed by PSO-WLS, with errors of 0,93% and 1,42%, respectively.

In the simulation of the IEEE 30-bus grid, the estimation results differ notably. For voltage magnitude, the two combinations with the best estimation values are SPSO-WLS and PSO-WLS, while the PSO-LAV and PSO-WLAV combinations yield the poorest results. Regarding voltage phase angle results, both combinations with the WLAV and LAV function result in high errors, reaching up to 48%.

#### 4.2. Simulation of case study 2

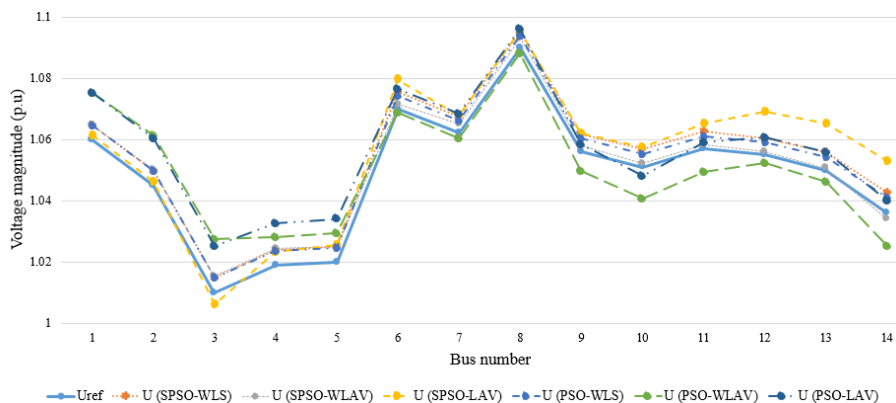
A comparison of the results shown in Figures 4, 6, 8, and 10 with those in Figures 3, 5, 7, and 9 indicates that data from Phasor Measurement Units provides more accurate estimations than data from

conventional measuring devices. Specifically, for the IEEE 14-bus grid, in comparison to case 1, the error in the  $\theta$  estimation using the LAV function is reduced from 18,97% to less than 10.4%. Similarly, the  $\theta$  estimation error for the IEEE 30-bus grid is below 10%. In the 14-bus system, five combinations yield good estimation results, excluding SPSO-LAV.

For 30-bus, only two combinations (SPSO-WLS and PSO-WLS) achieve  $\theta$  results with errors under 1%, while the other four combinations have  $\theta$  estimation errors ranging from 7% to 8%. Additionally, the estimated voltage magnitude values for both grids in this case study have an error of less than 4%.

**Table 1. The highest estimation error**

Grid	Combination	Case 1		Case 2	
		% error U	% error $\theta$	% error U	% error $\theta$
IEEE 14-bus	PSO-WLS	0,50	1,42	0,37	0,81
	PSO-LAV	1,49	15,12	1,21	2,82
	PSO-WLAV	1,73	3,23	1,38	2,02
	SPSO-WLS	0,65	0,93	0,04	0,11
	SPSO-LAV	1,63	18,97	2,97	10,33
	SPSO-WLAV	0,54	7,03	3,31	3,93
IEEE 30-bus	PSO-WLS	0,54	4,06	1,30	0,83
	PSO-LAV	6,02	48,12	3,97	8,04
	PSO-WLAV	5,16	37,83	3,44	6,69
	SPSO-WLS	0,78	1,13	0,09	0,60
	SPSO-LAV	4,89	39,66	1,91	7,30
	SPSO-WLAV	3,82	40,71	1,95	7,20



**Figure 3. The estimated voltage magnitude in IEEE 14-bus – Case 1**



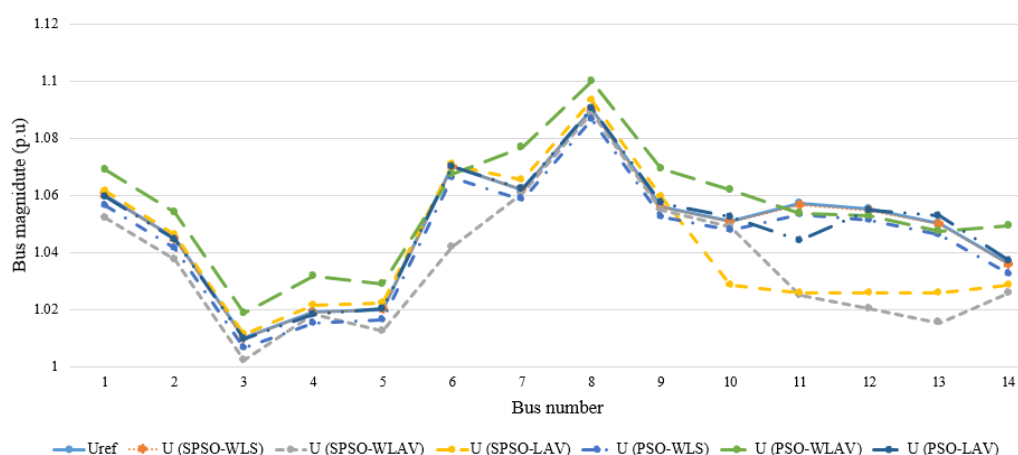


Figure 4. The estimated voltage magnitude in IEEE 14-bus – Case 2

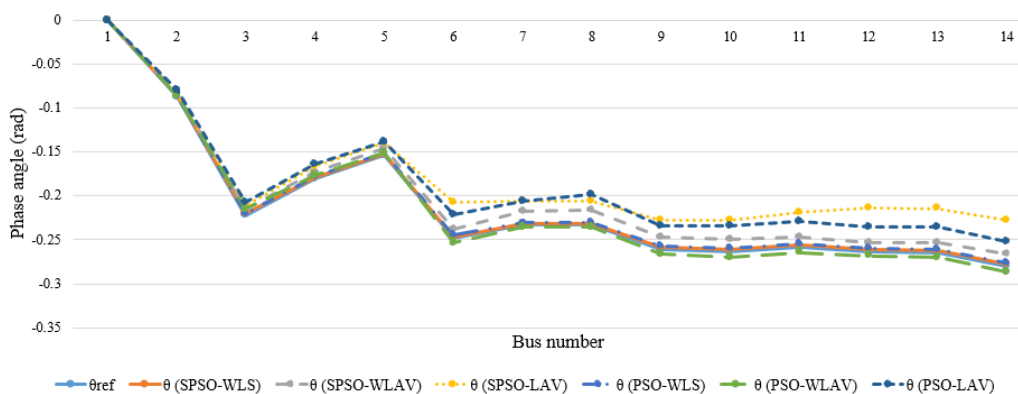


Figure 5. The estimated voltage phase angle in IEEE 14-bus – Case 1

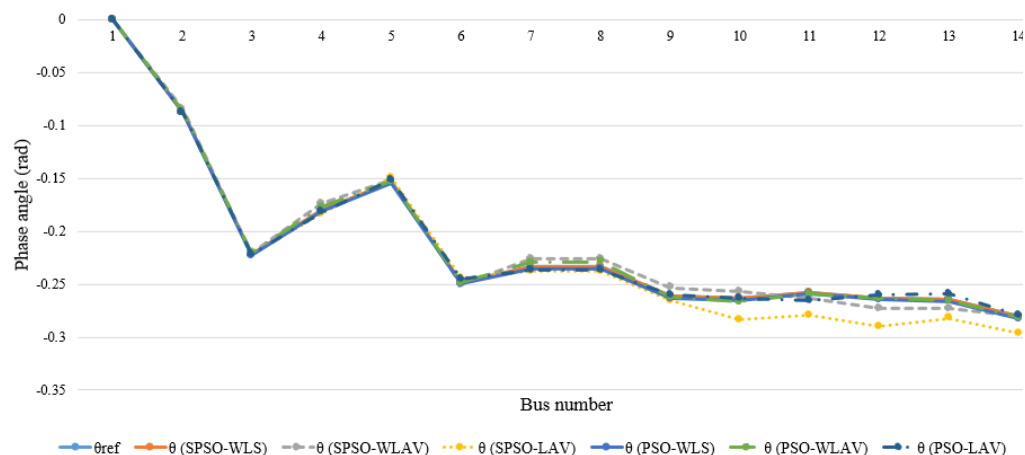


Figure 6. The estimated voltage phase angle in IEEE 14-bus – Case 2

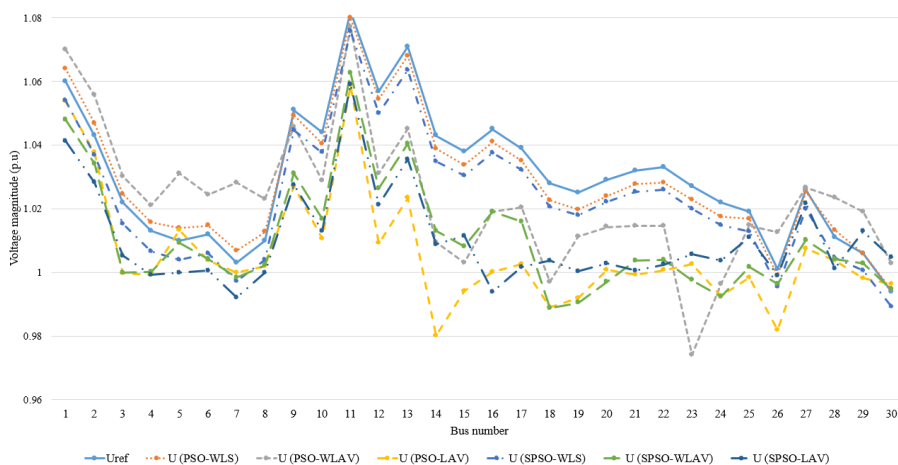


Figure 7. The estimated voltage magnitude in IEEE 30-bus – Case 1

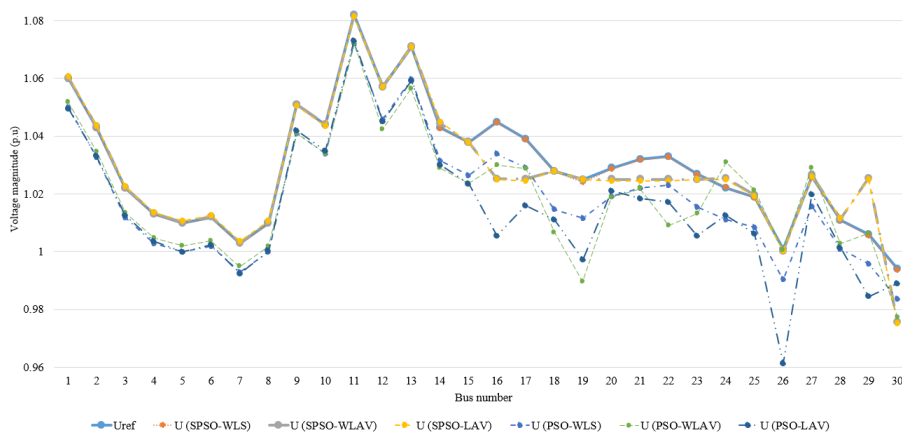


Figure 8. The estimated voltage magnitude in IEEE 30-bus – Case 2

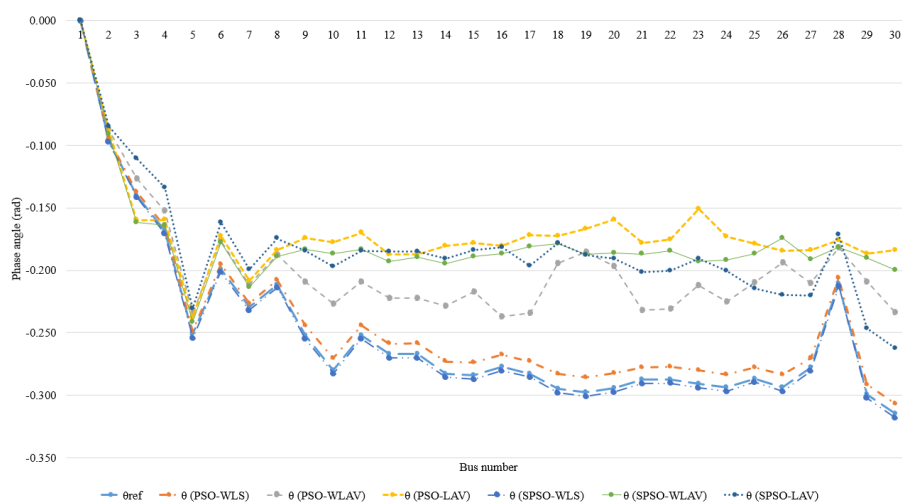


Figure 9. The estimated voltage phase angle in IEEE 30-bus – Case 1

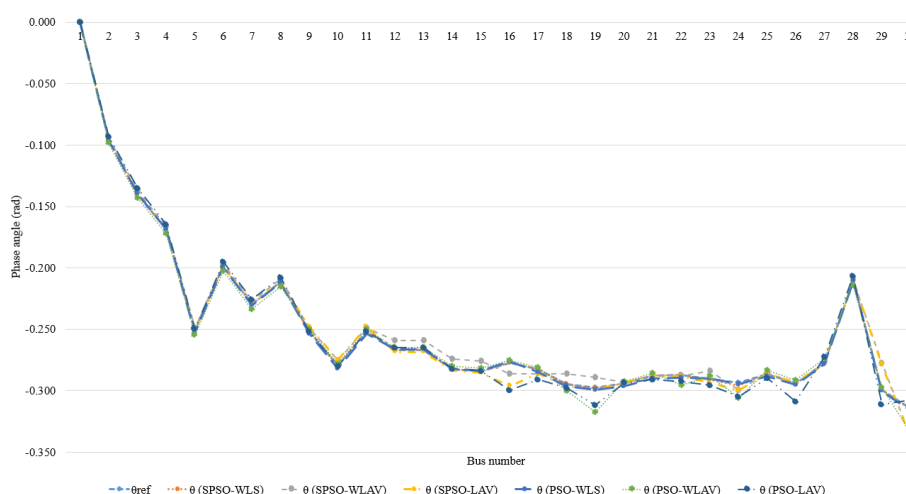


Figure 10. The estimated voltage phase angle in IEEE 30-bus – Case 2

## 5. CONCLUSION

The paper presents a study on estimating the value of state variables in power systems using the swarm optimization algorithm, both with and without the variable separation process. This study utilizes three objective functions (Weighted Least Squares, Least Absolute Values, and Weighted Least Absolute Values), with input data derived from conventional measuring devices or Phasor Measurement Units. The simulation results, which encompass 24 cases of six combinations across two IEEE benchmark grids, indicate

that the use of PSO or SPSO algorithms with the WLS function yields the best and most stable estimation for both power grids in all scenarios. Based on the results and analysis presented in the paper, the study recommends choosing the WLS objective function when applying the PSO or SPSO algorithms to address the estimation of state variable values in power systems.

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