

RESEARCH ARTICLE

A COMPARISON OF LYAPUNOV AND FUZZY APPROACHES TO TRACKING CONTROLLER DESIGN

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ARTICLE DETAILS

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ABSTRACT

There are numerous types of locomotion of mobile robots. Therein, the most widespread type of locomotion is motion using wheels. The task of robot is transport themselves from place to place. And tracking control is always an important problem to apply robots in practice. The robot has to reach the final goal by following a referenced trajectory. The paper proposes two methods based on the lyapunov stability standard and fuzzy law. Then, we simulate the algorithms to evaluate the results.

KEYWORDS

tracking controller, mobile robot, lyapunov, fuzzy

1. INTRODUCTION

The field of automation in industry has been growing significantly for decades. Because it provides a competitive advantage for businesses by increasing productivity, improving product quality, reducing production risks. One of numerous tasks of industrial automation is to move products between locations in the factory for the import and export process. This can be achieved by various solutions, such as conveyor belts or human employees. Employees are expensive and error prone. Conveyor belts are static, take a lot of space and are not flexible when the arrangement is revised. Therefore, another automatic solution that is currently prevailing is mobile robot system. Seaports use robots to transport containers, the factory uses robots to transport raw materials and products between zones. The warehouse uses robots to move shelves. Today, robots are also widely used in the service industry, restaurants use robots to deliver food to customers, or with recently covid pandemic, many robots are used to deliver food, medicine to patients.

Based on the structure, mobile robots are divided into three main types (Russo and Ceccarelli, 2020). First, robots operating under water use motors with propellers, helping them to dive, float, forward, reverse, or spin in water. Some typical water-based robots are ROV (Remotely operated vehicle) and AUV (Autonomous underwater vehicle). Second, the robot flies in the air, popularly known as Quadcopter. Third is the type of mobile robots moving on the ground, which can be walking or using wheels.

Related to mobile robots, there are many topics of interest such as mechanical design, localization, perception of environment, motion control, navigation, obstacle avoidance (Iris, 2006). Besides, some new topics can be mentioned such as building cognitive abilities or intelligence for robots (Primatesta et al., 2016).

There are lots of algorithms to move robots along a defined path. The algorithm of line tracking based on Lyapunov standard was introduced with high stability (Wang and Wang, 2012). Fuzzy Logic algorithm has also been applied to the motion control problem for mobile robots (Benbouabdallah et al., 2012). In addition, another popular algorithm, PID, has also been modified for application (Lee et al., 2013). In general, the algorithms are processed on a MIMO multi-input system. The paper gives two approaches of line tracking controllers based on Lyapunov stability standard and fuzzy law.

The paper is divided into 5 parts. Part 1 gives an introduction to Mobile robot and the line tracking problem. Part 2 describes a kinematic model of the robot using driving wheels. Part 3 describes two algorithms. Part 4 shows the simulations. Finally, there is the conclusion and future work.

2. WHEELED MOBILE ROBOT

2.1 Kinematical Model

The mobile robot platform used in this research is described in Figure 1, which is a "nonholonomic" system. It consists of 2 driving wheels mounted on the same shaft and 2 passive wheels. The robot's movement is created by 2 independent motors that provide torque to the two driving wheels. The center of gravity of the robot is $P_c(x_c, y_c)$, P_0 is the midpoint of the segment connecting the two driving wheels. The length and width of the robot frame are respectively a and $2b$. The radius of each wheel is R_w .

The robot is represented by a vector as follows:

$$q = [x_c, y_c, \varphi, \theta_1, \theta_2]^T \quad (1)$$

where θ_1, θ_2 : is the angle of 2 driving wheels of robot.

x_c, y_c : are the coordinates of center of gravity.

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φ : is the direction of the robot compared with the direction X.

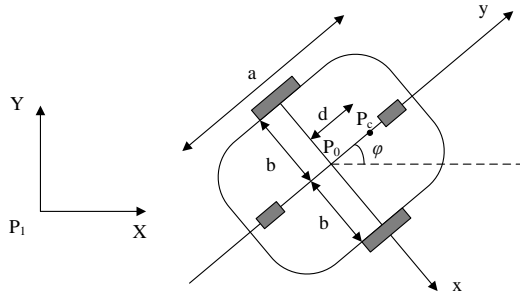


Figure 1: Two-wheel nonholonomic mobile robot system

With wheel rolls without slipping, the “nonholonomic” constraints are represented as follows.:

$$R_a \dot{\theta}_1 = \dot{x}_c \cos \varphi + \dot{y}_c \sin \varphi + b \dot{\varphi}$$

$$R_a \dot{\theta}_2 = \dot{x}_c \cos \varphi + \dot{y}_c \sin \varphi - b \dot{\varphi} \quad (2)$$

$$0 = \dot{y}_c \cos \varphi - \dot{x}_c \sin \varphi - d \dot{\varphi} \quad (3)$$

Where (2) are the two constraints of rolling without slipping when the vertical velocity of the robot is determined by the angular velocity of the two driving wheels, (3) is the constraint that the horizontal velocity of the robot is always zero.

The forward velocity v and angular velocity ω of the robot are calculated as follows:

$$v = \frac{R_a(\dot{\theta}_1 + \dot{\theta}_2)}{2} \quad (4)$$

$$\omega = \frac{R_a(\dot{\theta}_1 - \dot{\theta}_2)}{2b}$$

The robot kinematic model are shown as follows:

$$\begin{cases} \dot{x}_c = \Theta \cos \varphi \\ \dot{y}_c = \Theta \sin \varphi \\ \dot{\varphi} = \omega \end{cases} \quad (5)$$

2.2 Modelling of error

Coordinate of tracking point $C(x_c, y_c)$.

Coordinate of reference point $R(x_R, y_R)$.

e_1 is the distance error from the tracking point to the reference point in the x-direction of the vehicle.

e_2 is the distance error from the tracking point to the reference point in the y-direction of the vehicle.

e_3 is the error of angle between the direction of the vehicle and the direction of reference point.

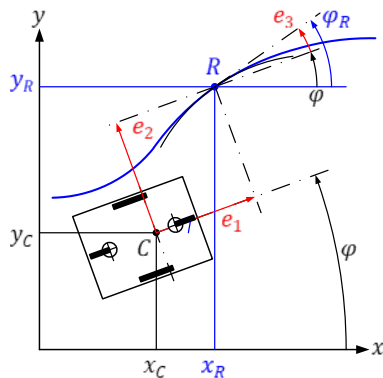


Figure 2: Model of error

At that time, we have the following tracking errors:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_R - x_c \\ y_R - y_c \\ \varphi_R - \varphi \end{bmatrix} \quad (6)$$

Get the derivative over time:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} v_R \cos e_3 \\ v_R \sin e_3 \\ \omega_R \end{bmatrix} + \begin{bmatrix} -1 & e_2 \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (7)$$

3. TRACKING CONTROLLERS DESIGN

3.1 Lyapunov Approach

Select the Lyapunov function:

$$V = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 + \frac{1 - \cos e_3}{k_2} \geq 0$$

Get the derivative \dot{V} :

$$\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + \frac{\sin e_3 \dot{e}_3}{k_2}$$

$$\dot{V} = e_1 (v_R \cos e_3 - v + \omega e_2) + e_2 (v_R \sin e_3 - \omega e_1) + \frac{\sin e_3 (\omega_R - \omega)}{k_2}$$

$$\dot{V} = e_1 (v_R \cos e_3 - v) + \frac{\sin e_3 (k_2 v_R e_2 + \omega_R - \omega)}{k_2}$$

Put:

$$v_R \cos e_3 - v = -k_1 e_1$$

$$k_2 v_R e_2 + \omega_R - \omega = -k_3 \sin e_3$$

Then:

$$\dot{V} = -k_1 e_1^2 - k_3 (\sin e_3)^2 \leq 0$$

Lyapunov based stability is satisfied.

Thus, the control law is shown as follow:

$$\begin{cases} v = v_R \cos e_3 + k_1 e_1 \\ \omega = k_2 v_R e_2 + \omega_R + k_3 \sin e_3 \end{cases}$$

3.2 Fuzzy logic

Mô hình hóa

Error e_1 with state variables: N, Z, P.

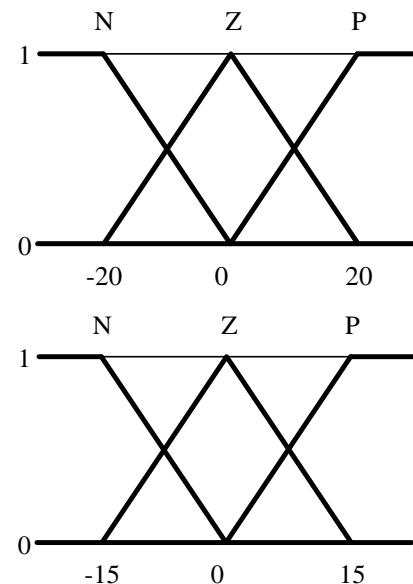
Error e_2 with state variables: N, Z, P.

Error e_3 with state variables: N, Z, P.

Forward speed v with state variables: Z, PS, PB.

Angular velocity ω with state variables: N, Z, P.

Membership Function for input (Fig. 3)



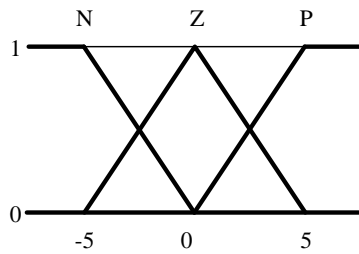


Figure 3: Membership Function for input

Membership Function for output (Figure 4)

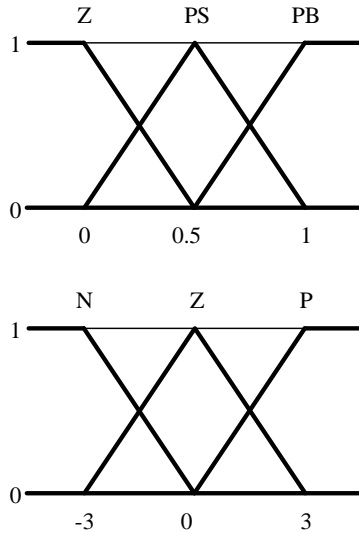


Figure 4: Membership Function for output

Then, Fuzzy rules are given in the following tables:

Table 1: Fuzzy rule with $e_1 = N$				
$e_3 \backslash e_2$	N	Z	P	
N	PS/N	PS/N	PS/P	
Z	PS/N	PS/Z	PS/P	
P	PS/N	PS/P	PS/P	

Table 2: Fuzzy rule with $e_1 = Z$ và $e_1 = P$				
$e_3 \backslash e_2$	N	Z	P	
N	PS/N	PS/N	PS/P	
Z	PS/N	PS/Z	PS/P	
P	PS/N	PS/P	PS/P	

Finally, the fuzzy solution used is CA (center-average).

4. SIMULATION

Algorithms based on Lyapunov stability and Fuzzy Logic Rule were simulated using Matlab software. The results of simulation consist of line tracking, tracking error and wheel velocity, which is shown respectively in Figure 5, Figure 6 and Figure 7.

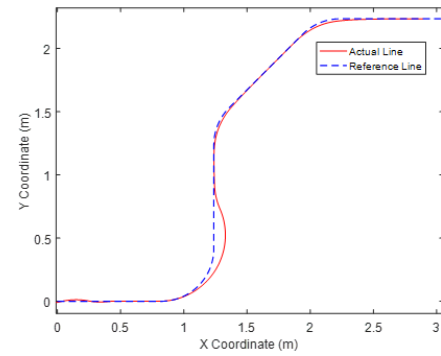


Figure 5: Line Tracking

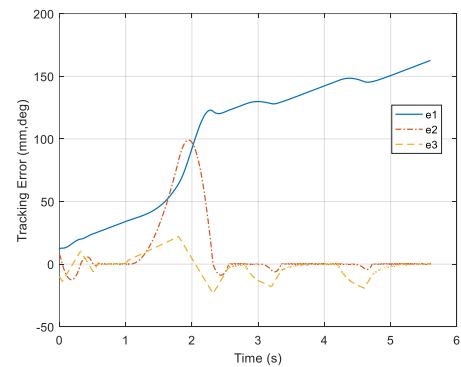
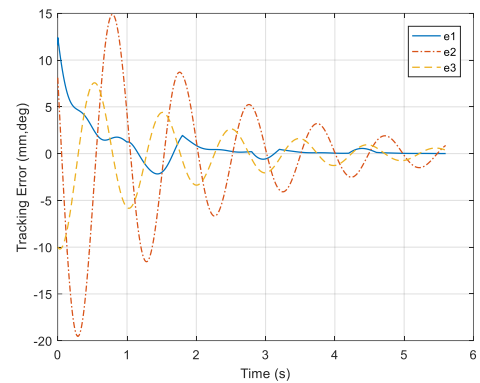


Figure 6: Tracking error

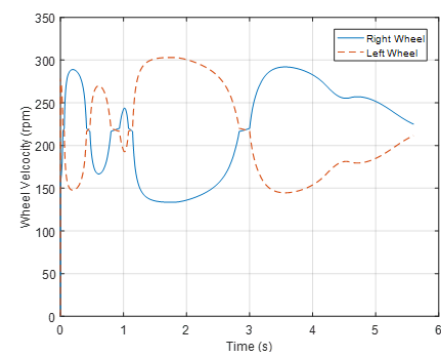
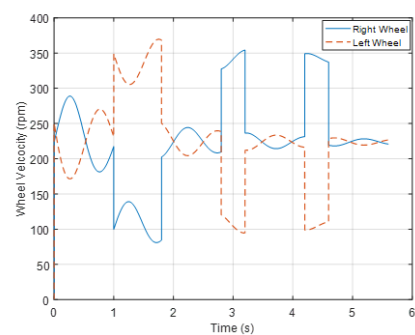
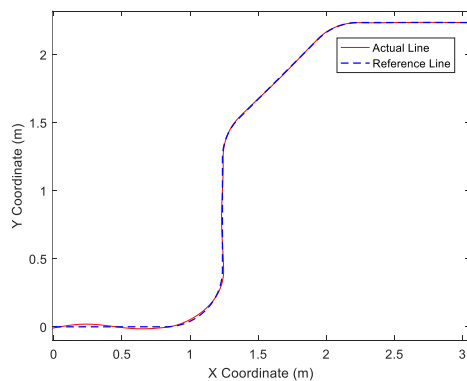


Figure 7: Wheel Velocity



5. CONCLUSION

The paper introduced the problem of line tracking for mobile robots. From there, we propose two approaches based on Lyapunov stability and Fuzzy logic rule to control the movement of the robot along the defined path. Then, we proceed to simulations. The results showed that the two algorithms both achieved the goals. To improve the performance of line tracking controllers, some other algorithms based on adaptive standards will be studied in the near future. In addition, we will conduct the experiments to evaluate the practical applicability of the algorithms.

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