Communications in Physics, Vol. 14, No. 1 (2004), pp. 1 – 6

GAIN AND NOISE IN ERBIUM-DOPED FIBER AMPLIFIER (EDFA) - A RATE EQUATION APPROACH (REA)

PHUNG QUOC BAO AND LE HONG SON

College of Natural Sciences, Hanoi National University

Abstract. *We present a rate equation approach (REA) based on the propagation equations in single-mode Erbium-Doped Fiber Amplifiers (EDFAs). Special attention is paid to the gain and the amplified spontaneous emission (ASE) noise as functions of position including the effects of some main parameters such as pump power, signal power.*

I. INTRODUCTION

EDFA are currently attracting increased interest, especially for use in optical communication wavelength-division multiplexed (WDM) systems. The main reason is the possiblility of compensating for optical fiber losses in broad wavelength ranges and developing a large-capacity, long-distance transmission system. The performance of the transmission system is strongly influenced by the gain and noise of used EDFAs. Therefore, detailed information about these EDFA characteristics is the key for advanced design of WDM systems. The gain and noise of EDFAs can be described by the propagation and rate equations of a homogeneous two-level laser medium modeling the interaction of the optical field with erbium ions. This approach is efficient for design of EDFAs, but requires accurate characteristics for all amplifier components [1, 2]. Another approach is the black-box model based upon input-output experimental data for a certain amplifier without requiring knowledge about the internal amplifier construction [3 - 7].

It is well-known that using EDFA as optical preamplifiers improves the receiver sensitivity of detection systems. The predominant noise in optically preamplified receivers is the amplified spontaneous emission (ASE). The present paper proposes modeling this type of EDFA based on propagation and rate equations.

This paper is organized as follows. Section II presents the rate equations of a homogeneous three-level Er^{3+} active medium and propagation equations for EDFA with single-pumped forward traveling configuration. The gain and ASE noise are numerically analyzed as functions of position for various signal and pump powers in Section III. Section IV summarizes the conclusions.

II. BASIC EQUATIONS

In a glass, the energy states of Er^{3+} are modified by local electric fields and by dynamical pertubation. That causes homogeneous and inhomogeneous broadening of the Stark-split levels. Fig. 1 shows the energy level diagram with $^4I_{11/2}$, $^4I_{13/2}$ and $^4I_{15/2}$ levels of Er^{3+}in a glass. The $^4I_{11/2}$–$^4I_{15/2}$ transition corresponds to the 980-nm pump band and the $^4I_{13/2}$–$^4I_{15/2}$ transition corresponds to the 1460–1500-nm resonant pumping band. No pump excited-state absorption (ESA) occurs for 980-nm or 1480-nm pumped EDFA [2]. For that reason, other energy levels of Er^{3+} were not included in the figure.

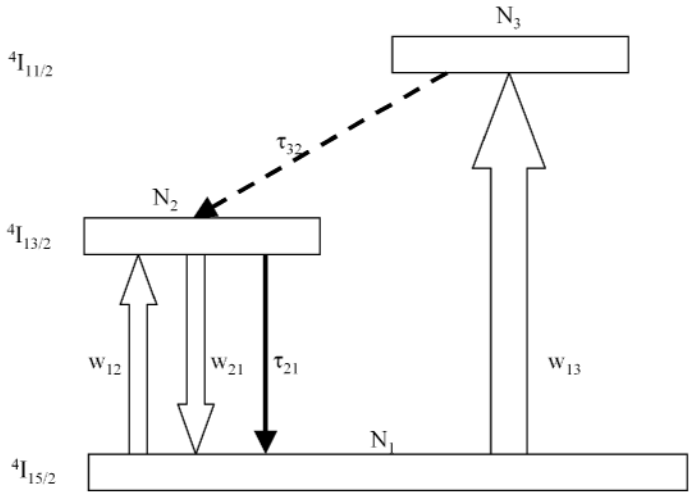

Fig. 1. Energy level transitions for Er^{+}

Let us denote the ${}^4I_{15/2}$, ${}^4I_{13/2}$ and ${}^4I_{11/2}$ as levels 1, 2 and 3 with their population densities at a longitudinal fiber coordinate z as $n_1(z,t)$, $n_2(z,t)$ and $n_3(z,t)$, respectively. Neglecting the upconversion in Er^{3+}, the rate equations describing the dynamics of such a EDFA are:

$$\begin{aligned}\frac{\partial n_1}{\partial t} &= -w_{13}n_1 - w_{12}n_1 + w_{21}n_2 + \frac{n_2}{\tau_{21}}\\ \frac{\partial n_2}{\partial t} &= w_{12}n_1 - w_{21}n_2 - \frac{n_2}{\tau_{21}} + \frac{n_3}{\tau_{32}}\\ \frac{\partial n_3}{\partial t} &= w_{13}n_1 - \frac{n_3}{\tau_{32}}\end{aligned} \tag{1}$$

In these equations, w_{ij} represents the transition rate between the i and j levels:

$$w_{ij} = \frac{\Gamma(\lambda)\lambda}{hcA_{core}}\sigma_{ij}(\lambda)\,p(z,t,\lambda) \tag{2}$$

where $\Gamma(\lambda)$ - wavelength-dependent overlap factor; h - Planck's constant; c - light velocity in vacuum; λ - radiation wavelength; A_{core} - effective core section of the fiber; $\sigma_{ij}(\lambda)$ - wavelength-dependent corresponding transition cross-sections; $p(z,t,\lambda)$ - optical power density at wavelength λ in a longitudinal fiber coordinate z. Finally, τ_{ij} denotes the spontaneous emission lifetime for the transition betweeen the i and j levels.

We can rewrite Eqs. (1) in terms of the population densities in a fiber volume unit by introducing:

$$N_i = \int n_i(z,t)\,dz \tag{3}$$

$$W_{ij} = \int w_{ij}(z,t,\lambda)\,dz \quad = \frac{\Gamma(\lambda)\lambda}{hcA_{core}}\sigma_{ij}(\lambda)\,P(t,\lambda) \tag{4}$$

with $P(t, \lambda)$ - the optical power density at wavelength λ. The integrals are taken over such a fiber length that the corresponding fiber volume is equal to a volume unit. The rate equations can then be cast into:

$$\begin{aligned} \frac{dN_1}{dt} &= -W_{13}N_1 - W_{12}N_1 + W_{21}N_2 + \frac{N_2}{\tau_{21}} \\ \frac{dN_2}{dt} &= W_{12}N_1 - W_{21}N_2 - \frac{N_2}{\tau_{21}} + \frac{N_3}{\tau_{32}} \\ \frac{dN_3}{dt} &= W_{13}N_1 - \frac{N_3}{\tau_{32}} \end{aligned} \tag{5}$$

Solving Eqs. (5) in steady-state regime, we have:

$$N_2^* = \frac{W_{12} + W_{13}}{W_{12} + W_{21} + W_{13} + \frac{1}{\tau_{21}}} N \tag{6}$$

where $N = N_1 + N_2 + N_3 \approx N_1 + N_2$ is the total population density by assuming an instantaneous decay of the excited state ${}^4\mathrm{I}_{11/2}$.

The population expression (6) needs to be modified if the ASE signal is taken into account. The ASE power at a coordinate z along the fiber is the sum of the ASE power from the previous fiber part and the local ASE power. The propagation equations for ASE powers can be written as follows:

$$\begin{aligned} \pm\frac{dP_{ASE}^{\pm}}{dz} &= \{\Gamma(\lambda)[\sigma_{21}(\lambda) N_2^* - \sigma_{12}(\lambda) N_1^*] - \alpha(\lambda)\} P_{ASE}^{\pm}(\lambda) \\ &+ \Gamma(\lambda)\sigma_{21}(\lambda) N_2^* P_0(\lambda) \end{aligned} \tag{7}$$

where $P_{ASE}^{+}(\lambda)$ and $P_{ASE}^{-}(\lambda)$ are the forward and backward propagating optical powers at wavelength λ in a wavelength interval $\Delta\lambda$ and $P_{ASE}(\lambda_j) = P_{ASE}^{+}(\lambda_j) + P_{ASE}^{-}(\lambda_j)$. The second term is the local ASE power defined by:

$$P_{ASE}^* = \Gamma(\lambda)\sigma_{21}(\lambda)N_2^* P_0(\lambda) \tag{8}$$

where the parameter $P_0(\lambda)$ represents the contribution of the spontaneous emission into the mode and is given by $P_0(\lambda) = 2hc^2/\lambda^3$ [2]. Due to the small signal power under consideration, Rayleigh back-scattering is omitted.

For the same argument reported in [2], with allowance for the ASE signal, the steady-state population density N_2^* can be expressed:

$$N_2^* = \frac{W_{12} + \frac{\Gamma(\lambda)\lambda\sigma_{12}(\lambda)P_{ASE}(\lambda)}{hcA_{core}} + W_{13}}{W_{12} + W_{21} + \frac{\Gamma(\lambda)\lambda}{hcA_{core}}[\sigma_{12}(\lambda) + \sigma_{21}(\lambda)]P_{ASE}(\lambda) + W_{13} + \frac{1}{\tau_{21}}} N \tag{9}$$

The propagation of the pump power along the active fiber is described by the following differential equation:

$$\frac{dP_p}{dz} = -N_1^*\sigma_{13}\Gamma(\lambda_p)P_p - \alpha_p P_p \tag{10}$$

The signal power is amplified along the active fiber according to:

$$\frac{dP_s}{dz} = [N_2^* \sigma_{21} - N_1^* \sigma_{12}]\Gamma(\lambda_s)P_s - \alpha_s P_s \tag{11}$$

where $\Gamma(\lambda_p)$, $\Gamma(\lambda_s)$ - overlap factors at the pump and signal wavelengths; α_p, α_s - possible intrinsic background losses in the fiber at the pump and signal wavelengths.

Special attention is paid to the gain and the amplified spontaneous emission (ASE) noise defined as [2]:

$$G = \frac{P_s - P_{ASE}}{P_{s0}}$$

$$P_{ASE} = P_{ASE}^{+} + P_{ASE}^{-}$$

where P_s is the output signal power, P_{s0} - the input signal power. The total ASE power P_{ASE} is the sum of the forward and backward propagating ASE powers denoted by P_{ASE}^{+} and P_{ASE}^{-}, respectively.

III. SIMULATION AND RESULTS

The investigated EDFA has a length of 18 m and a core effective section of 5.10^{-12}m^2. The Er^{3+} ions are concentrated in the core. One pump of 980 nm is coupled with the signal of 1550 nm in a forward-traveling unidirectional configuration. The fiber parameters are the Er^{3+} ion concentration $N = 1.35.10^{25}m^{-3}$ [8], the lifetime $\tau_{21} = 1.1.10^{-1}s$ [8], the overlap factors $\Gamma(\lambda_p) = 0.5$ [1] and $\Gamma(\lambda_s) = 0.7$[2], the cross sections $\sigma_{12} = 2.10^{25}m^{-3}$, $\sigma_{21} = 2.7.10^{25}m^{-3}$ and $\sigma_{13} = 0.6.10^{25}m^{-3}$[1], the background losses $\alpha_p = 2dB/m$ and $\alpha_s = 0.2.10^{-3}dB/m$ [1].

Using REA for the steady-state population densities, the propagation equations (7), (10) and (11) have been solved numerically by MatLab Simulink executed on Window platform. Gain and ASE power are considered with allowance for the influence of pump and signal powers. The related results are displayed in Figs. 2-5.

We start by considering the signal gain of a 18 m length of EDFA. The signal and pump are taken to be copropagating and injected at $z = 0$. The gains at 1550 nm are computed as a function of fiber length for various pump powers. By increasing the fiber length, the gain started growing before reaching to a plateau, then dropped. The higher the pump power, the wider the plateau and the slower the gain dropping. This might be explained as follows. Created at the beginning of the fiber, the higher population inversion makes the gain factor simply proportional to the emission cross section. By and by along the fiber, the population inversion decreases and the gain reaches its saturation. In the last section of the fiber, the population inversion is too low to enable the amplification.

The similar situation is observed when considering the signal gain as a function of fiber length for various signal powers. But the difference is that the gain behavior here is due to the depletion level of the population caused by signal power magnitude.

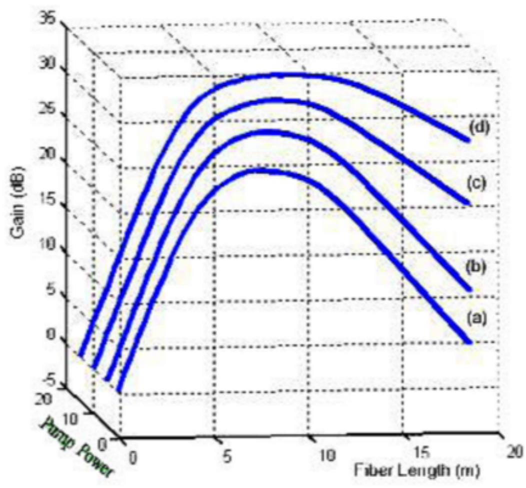

Fig. 2. Signal gain at 1550 nm as a function of EDF length for pump powers of $P_{p1} = 10mW$(a), $P_{p2} = 20mW$ (b), $P_{p3} = 30mW$ (c), $P_{p4} = 40mW$ (d).

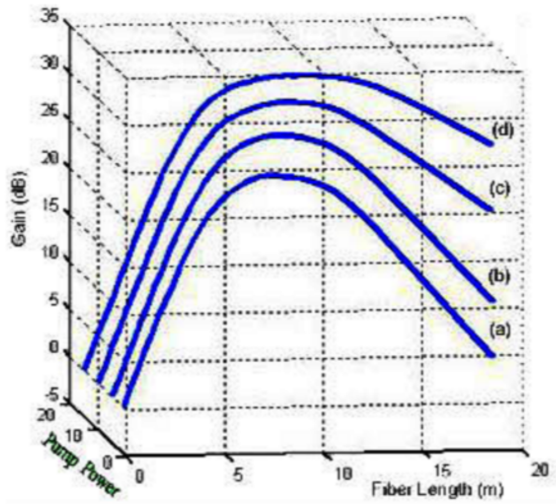

Fig. 3. Signal gain at 1550 nm as a function of EDF length for signal powers of $P_{s1} = 1mW$(a), $P_{s2} = 2.5mW$(b), $P_{s3} = 0.125mW$(c), $P_{s4} = 0.06mW$(d)

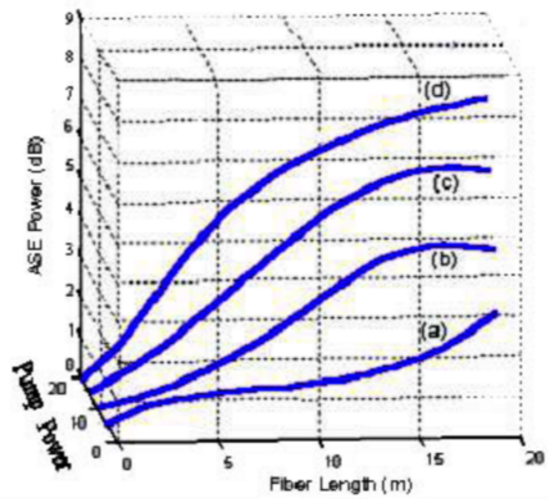

Fig. 4. ASE power as a function of EDF length for pump powers of $P_{p1} = 10mW$(a), $P_{p2} = 20mW$ (b), $P_{p3} = 30mW$ (c), $P_{p4} = 40mW$ (d).

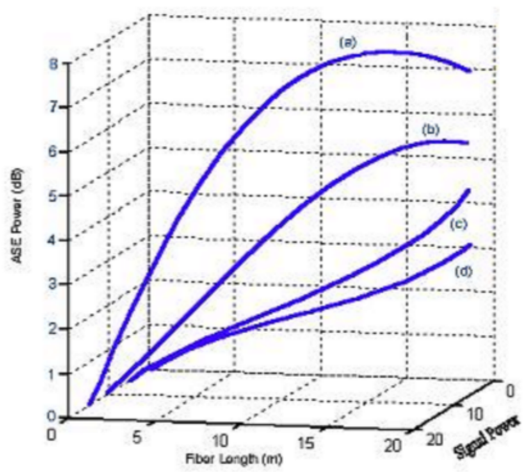

Fig. 5. ASE power as a function of EDF length for signal powers of $P_{s1} = 1mW$(a), $P_{s2} = 2.5mW$(b), $P_{s3} = 0.125mW$(c), $P_{s4} = 0.06mW$(d)

Let's consider the ASE power as a function of position for various pump/signal powers. The common feature is that the ASE power is created and grown when traveling over the high-inverted fiber section. The higher the pump/signal power, the larger the ASE power and its rate of change along the EDF. It is worth to be noted that lower pump powers is not enough to invert the entire fiber and therefore the ASE might grow to a value where the upper population should be significantly depleted (the plateau in Fig. 4). The same thing might occur for signal powers lower than the considered values (see Fig. 5).

IV. CONCLUSION

In this work, we study theoretically the signal gain and the ASE noise of EDFAs based on rate equation approach and propagation equations. A comprehensive model is presented using the experimental fiber parameters. The obtained results are in agreement with those reported in related papers as far as the used approximation holds. Further investigations on other characteristics of EDFA and sensitized EDFA on the same basis are in progress and will be published in the near future.

ACKNOWLEDGMENT

The authors wish to thank Assoc. Prof. Le Viet Du Khuong for his computational assitance and Prof. Dinh Van Hoang for his valuable comments.

REFERENCES

1. E. Desurvire, *Erbium-Doped Amplifiers Principles and Application.* New York, Wiley, 1994.
2. P. C. Becker, N. A. Olsson, J. R. Simpson, *Erbium-Doped Fiber Amplifiers Fundamentals and Technology.* Academic Press, 1999.
3. J. Burgmeier, A. Cords *et al.*, *J. Lightwave Technol.*, **16** (1998) 1271-1275.
4. C. R. Giles, E. Desurvire, *J. Lightwave Technol.*, **9** (1991) 271-283.
5. M. Movassaghi, M. K. Jackson *et al.*, *J. Lightwave Technol.*, **16** (1998) 812-817.
6. Nguyen Tuan Anh, Phung Quoc Bao, *Communications in Physics*, **13** (2003) 41-47.
7. Nguyen Tuan Anh, Phung Quoc Bao, *Journal of Science, VNU, Mathematics – Physics,* **T. XIX**, No. 2, (2003) 31-37.
8. M. Achtenhagen, R. J. Beeson *et al.*, *J. Lightwave Technol.*, **19** (2001) 1521-1526.

Received 14 January 2004