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DEPENDENCE OF THE COSMIC MUON FLUX ON ATMOSPHERIC PRESSURE AND TEMPERATURE

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Abstract. *The dependence upon atmospheric pressure of the cosmic muon flux in Hanoi has been measured to be -1.47± 0.50 permil per mbar in agreement with expectation from a naive estimate. The dependence over ground temperature is observed to largely reflect the correlation existing between temperature and pressure. Once the effect of this correlation is removed a smaller, barely significant temperature dependence of -0.57± 0.34 permil per ˚C subsists.*

I. INTRODUCTION

In a recent experiment the zenith angle distribution of the cosmic muon flux was measured in Hanoi using the telescope of the VATLY laboratory in the Institute of Nuclear Science and Technology [1]. Such a distribution is a useful input to the air shower simulation [2] used by the SuperKamiokande collaboration [3] for their studies of atmospheric neutrino oscillations. The very high value, 17GV, of the geomagnetic rigidity cutoff in Vietnam makes the Hanoi measurement of particular interest. The cosmic muons are decay products of secondary pions and kaons produced in nuclear interactions of the hadronic constituents of the cosmic air shower with the atmosphere. At sea level, where the measurement is performed, the shower, that contains 90% of muons, is already well beyond maximum development and the muon flux increases with altitude. It is therefore expected that it should decrease with atmospheric pressure, as higher pressures should be approximately equivalent to a thicker atmosphere. However, a possible dependence on atmospheric conditions was neglected in earlier publications [1]. The subject of the present study is to search for such a possible dependence.

II. MEASUREMENT OF THE MUON FLUX

The muon flux Φ is measured as the product of the detected muon rate R_μ and the detector acceptance A, the muon rate being itself the product of the total rate R and the relative muon abundance f, namely $\Phi = R_\mu\ A = R\ f\ A$. To a very good approximation the telescope acceptance and the muon abundance take constant values independent of the telescope orientation, $A = 22m^2msr$ and $f = 0.90$. It is therefore sufficient to search for a correlation between the total detected rate R and the atmospheric parameters under study.

The total detected rate R is obtained from the measurement of the time intervals T separating two successive events. Each set of measurements generally takes place over a full 24 hours day with a fixed setting of the telescope orientation. During that time a large number of events, of order of magnitude of 10^5, are collected. For each event the particular value of T is recorded. It is measured using a 10 kHz clock that is read into a fast scaler. The scaler is reset as soon as it is read out, the associated dead time being negligible.

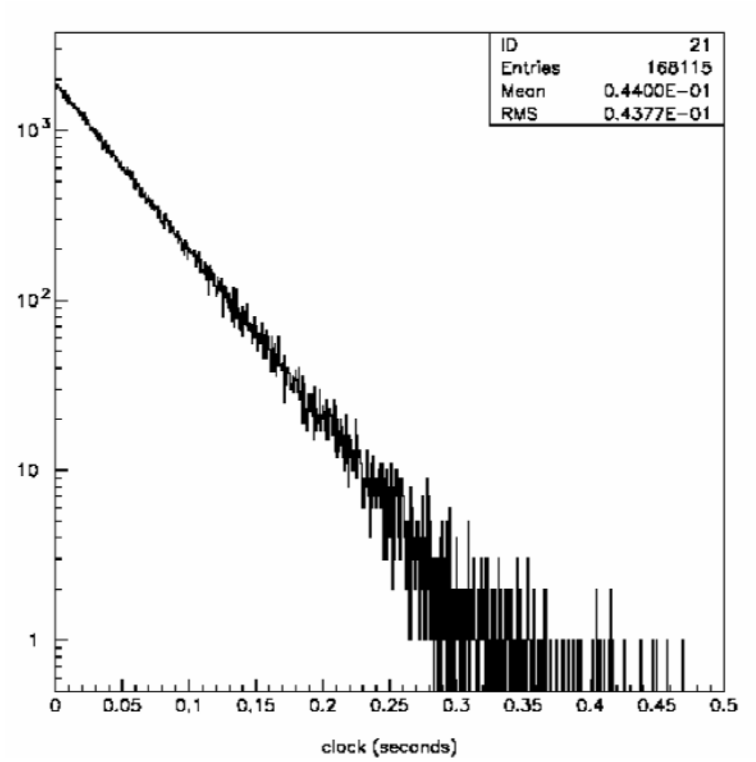

Fig. 1. A typical T distribution as measured for a given set of measurements.

The event rate varies from *2.3 Hz* at vertical incidence to *0.2 Hz* at the largest zenith angle setting (75^o). For each set of measurements T displays an exponential distribution characteristic of a constant density of probability of occurrence. Such a typical distribution is displayed on Fig. 1. Within statistics, the mean value, $\langle T\rangle$, and the root mean square deviation from the mean (rms), ΔT, are equal. Indeed, using the quantity ω_j=*1-* $\Delta_j T/\langle T\rangle_j$ to give a quantitative evaluation of their equality, we find on average over all sets of measurement $\langle\omega_j\rangle = 0.04 \pm 0.70$ permil. This is illustrated in Fig. 2 where the values taken by ω_j are displayed as a function of $1/\sqrt{N_j}$. The lines indicate the one and two standard deviation limits. The event rate is simply obtained from $\langle T\rangle$ as $R = 1/\langle T\rangle$. In the remainder of this work the search for correlation between R_μ and atmospheric conditions is therefore reduced to a search for correlation between the Tdistribution measured in a given set of measurements and the associated atmospheric conditions. The correlations measured for each individual set of measurement will then be combined in a single result.

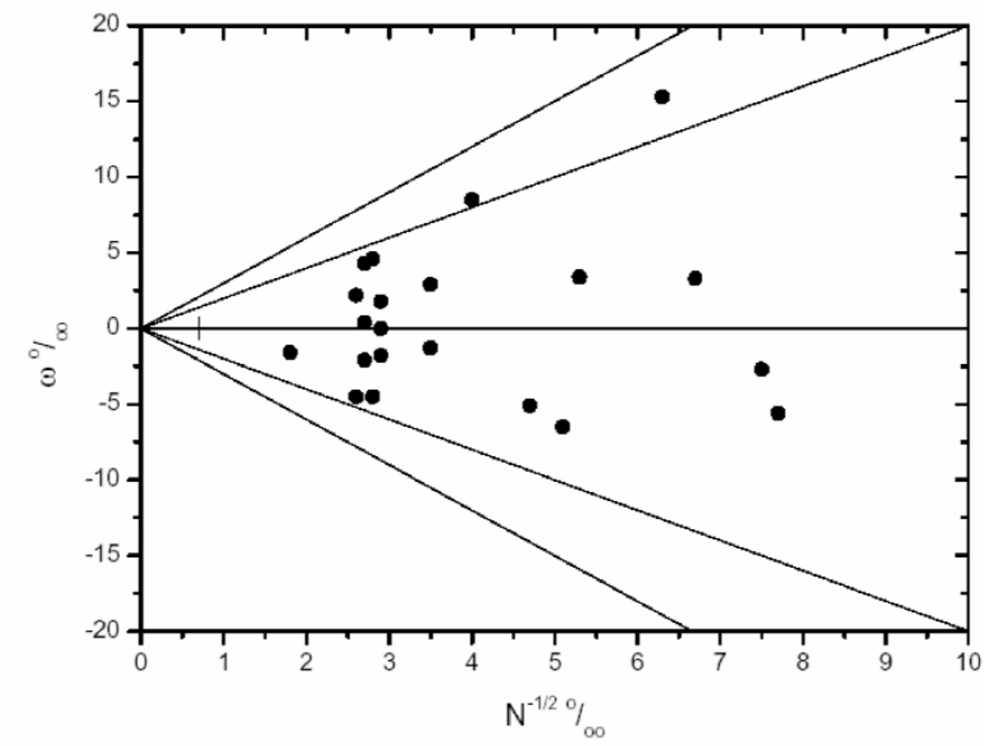

Fig. 2. The values taken by $\omega_j = 1 - \Delta_j T/\langle T\rangle_j$ as a function of $1/\sqrt{N_j}$, both in permil. The lines indicate the one and two standard deviation limits. The left most points corresponds to the value of ω_j averaged over all data sets.

III. ATMOSPHERIC CONDITIONS

The distribution of the air density over altitude, from ground level to, say, 20 km and over the whole Hanoi area influences the development of an air shower. However, in a static approximation, this distribution simply scales with the ground level atmospheric pressure. This latter quantity, p, is therefore expected to provide a sensible estimate of the effective air thickness (measured in g/cm^2) seen by the shower during its development. It is measured by the Meteorology Institute in Hanoi at regular intervals of 3 hours, and at a distance of about 3 km from the laboratory. Together with atmospheric pressure, and simultaneously, the ground temperature t is also measured. A possible correlation between t and the shower development cannot be excluded but its interpretation would be much less transparent than in the case of p as t is not *a priori* simply correlated to the effective air thickness. More will be said in the last section when the results will be discussed. For the time being it is sufficient to calculate, for each T measurement, the values taken by p and t at the time of the measurement. This is done by linear interpolation between the p and t measurements that bracket the T measurement.

The basic material available for the present study is therefore a set of over two millions triplets of numbers, T_{ij}, p_{ij} and t_{ij} where the index i refers to the event number within a given set of measurements and the index j to the set number. While i is reaching values of order 10^5, j varies between 1 and 21.

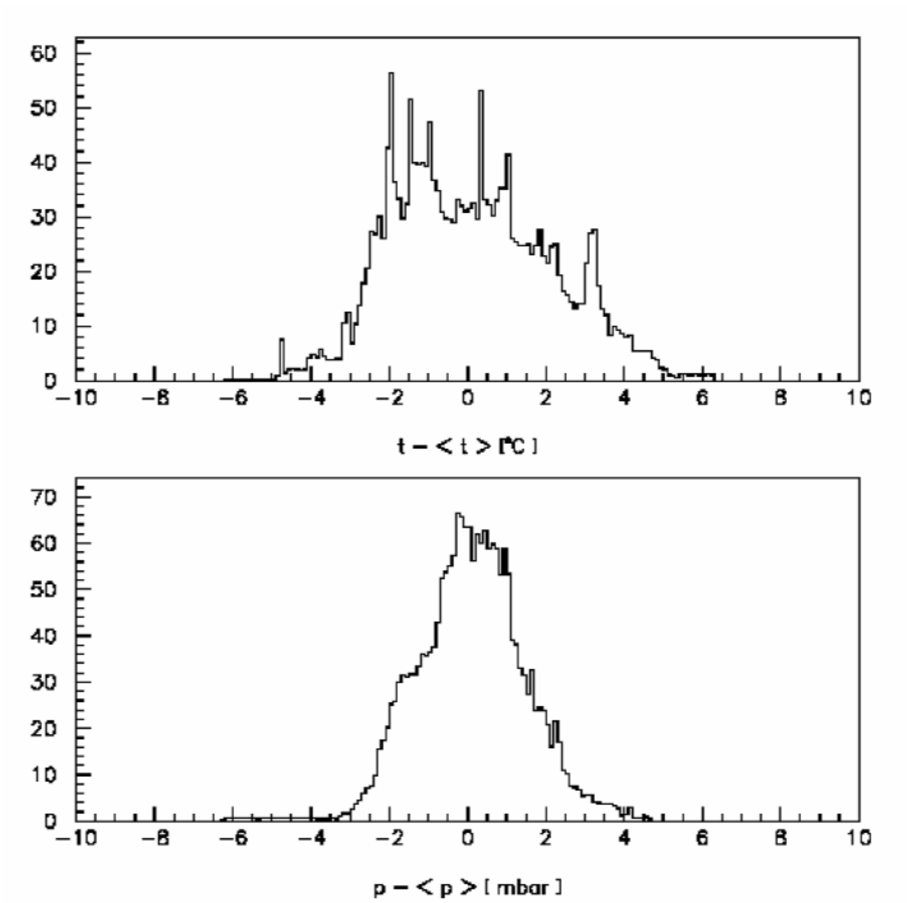

Fig. 3. Distributions of $t_{ij} - \langle t \rangle_j$ and $p_{ij} - \langle p \rangle_j$ for all sets of measurements together. Their rms values are *2.1 permil per* $^\circ C$ and *1.4 permil per mbar* respectively. The unit on the ordinate axes is 1000 events per bin.

Searching for correlation between T on the one hand and p or t on the other implies that the latter variables vary over broad enough a range during a same set of measurements. Fig. 3 displays the measured distributions of $t_{ij} - \langle t \rangle_j$ and $p_{ij} - \langle p \rangle_j$. Here $\langle t \rangle_j = \sum_i t_{ij}/N_j$, is the mean value of t over the set j, N_j being the number of data points in set j, and similarly for p. These distributions illustrate the span of t and p around their respective means within a same set, averaged over all sets. Their *rms* values are Δt=*2.1*$^o C$ and Δp = 1.4 mbar, respectively. The sensitivity of the experiment can be approximately estimated from these numbers by dividing *1/*Δt (resp *1/*Δp) by the statistical factor $\sqrt{N_{tot}}$ where N_{tot} is the total number of events in the experiment, in slight excess of 2 millions.

The resulting estimates are *0.34 permil per* $^\circ C$ for the correlation with temperature and *0.50 permil per mbar* for the correlation with pressure.

Fig. 4 is a two-dimensional plot of the quantities $\delta_t/\Delta t$ and $\delta_p/\Delta p$, where $\delta_t = t - \langle t \rangle$ and $\delta_p = p - \langle p \rangle$, as measured by the Meteorology Institute during the period of data taking that extended from September 9^{th} to October 31^{st}, 2002. Here $\langle t \rangle = 25.9^o$C and $\langle p \rangle = 1011.9$ *mbar* are averages over the 296 couples of measurements provided by the Meteorology Institute over the whole period. Similarly $\Delta t = 3.4^o$C and $\Delta p = 3.2$ *mbar* are *rms* values over the whole period. The plot gives evidence for a strong anticorrelation between the two quantities. Its strength is measured by the correlation coefficient $C = \langle (\delta_t/\Delta t)(\delta_p/\Delta p) \rangle = -0.62 \pm 0.03$ ($C = 0$ corresponds to no correlation and $C = 1$ to maximal correlation). The ellipses shown in the plot, having half-axes $C_+ = \sqrt{1+C}$ and $C_- = \sqrt{1-C}$, obey the equation:

$(\delta_t/\Delta t + \delta_p/\Delta p)^2/(2C_+^2) + (\delta_t/\Delta t - \delta_p/\Delta p)^2/(2C_-^2) = 1$ (or 4), corresponding to 1 (or 2) standard deviations, respectively. The implications of this correlation for the present study will be discussed later.

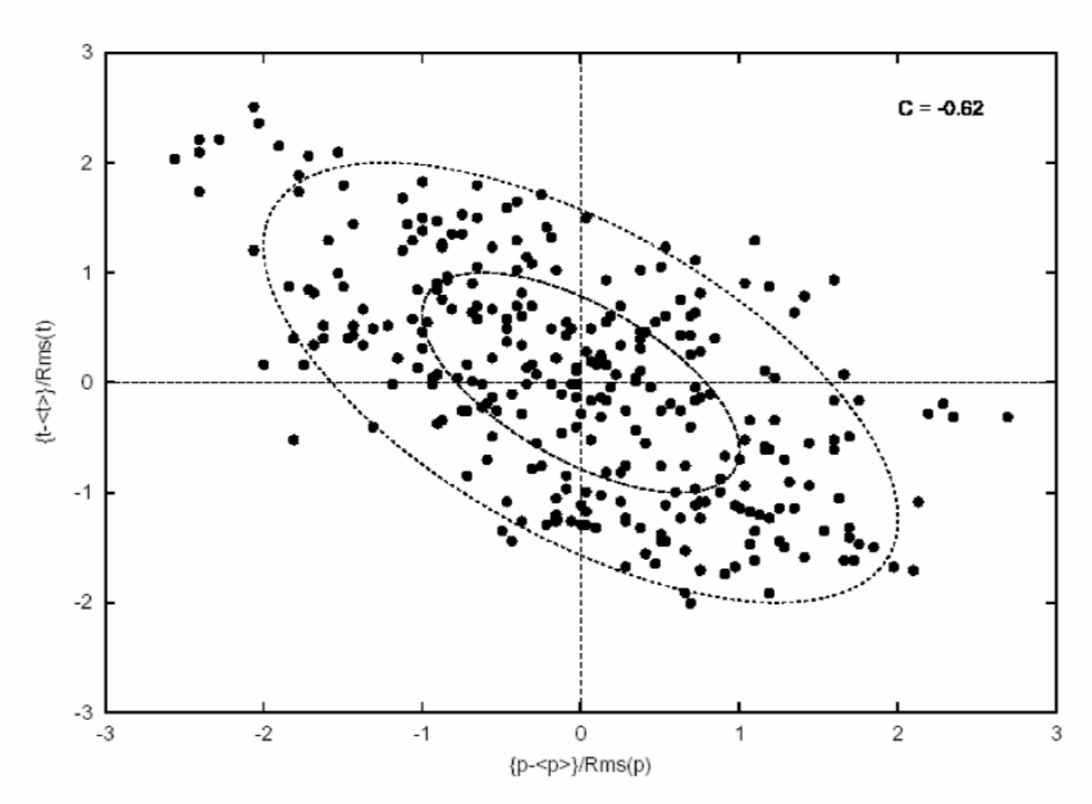

Fig. 4. The 296 couples of (p, t) measurements made by the Meteorology Institute during the period of data taking (25/09/02 to 31/10/02) are displayed in the reduced form $\delta_t/\Delta t$ *vs* $\delta_p/\Delta p$, giving evidence for a strong anti-correlation. Also shown are the one- and two-standard deviation ellipses.

IV. SEARCH FOR CORRELATION BETWEEN T AND (t, p)

For each set j of measurements we search for the best linear fit of T as a function of $x = t$ or p. Namely we minimize a *chi squared* defined as $\chi_j^2 = \sum_i (T_{ij} - \alpha_j x_{ij} - \beta_j)^2/\Delta_j T^2$. The minimization reduces, after having defined $\partial x_{ij} = x_{ij} - \langle x \rangle_j$ and $\partial T_{ij} = T_{ij} - \langle t \rangle_j$ to $\alpha_j = \sum_i (\partial x_{ij} \partial T_{ij}) / \sum_i (\partial x_{ij})^2$ and to $\beta_j = \langle t \rangle_j - \alpha_j \langle x \rangle_j$. It should be noted that $\Delta_j T$ is not a gaussian uncertainty but the *rms* value of an exponential distribution (equal to its mean value and common to all measurements of a same set). The uncertainty attached to the evaluation of α_j is measured by the shift in α_j that produces an increase of χ_j^2 by one unit, namely $\Delta\alpha_j = \Delta_j T/(\Delta_j x \sqrt{N_j})$ where $\Delta_j x$ is the *rms* value of x over set j. It is in practice preferable to work with $\lambda_j = \alpha_j / \langle T \rangle_j$ that measures the relative change of rate rather than the absolute change of rate as α_j does.

Usually it is important to subject such analyses to a χ^2 test, checking that χ^2/N does not deviate from its expected value of unity by more than what corresponds to some previously agreed confidence level, say 95%. In the present case, however, where the experimental uncertainty is defined as the *rms* value of the quantity measured, such a test is meaningless. Indeed one can readily see that the best fit value of χ_j^2/N reads $1 - (\lambda_j \Delta_j x)^2$ implying that if $\lambda_j = 0$, χ_j^2/N is exactly equal to 1. In general, if λ_j differs from zero, χ_j^2/N is smaller than 1 by an amount $(\lambda_j \Delta_j x)^2$ that measures how much the fit has improved by introducing a dependence of T upon x but this quantity depends only upon λ_j and $\Delta_j x$ and therefore contains no additional information on the quality of the fit. All such information is contained in the relative values of λ_j and $\Delta\lambda_j$ that tell whether λ_j does or does not significantly deviate from zero. Indeed, for the best fit, $(\lambda_j/\Delta\lambda_j)^2 = N_j - \chi_j^2$.

Table 1 lists for each set j the mean and *rms* values of T_{ij}, t_{ij} and p_{ij} together with the values taken by N_j, λ_j and $\Delta\lambda_j$ for each of the two best fits (temperature and pressure). Fig. 5 displays the dependence of λ_j over z_j, the zenith angle of the telescope axis corresponding to set j, for both temperature and pressure. In principle, there is no reason for λ to be independent of zenith angle. However, as no clear dependence is visible from Fig. 5, attempting to calculate weighted averages over all sets is justified. This gives:

$$\langle \lambda_t \rangle = -1.11 \pm 0.34 \ \text{ permil per }\ ^\circ\mathrm{C}$$

$$\langle \lambda_p \rangle = 1.47 \pm 0.50 \ \text{ permil per mbar}$$

with χ^2 values of 0.98 and respectively 1.03 per degree of freedom (there are 20 degress of freedom), that demonstrates the consistency of the different data sets and justifies the assumption of independence over zenith angle. The uncertainties are exactly the same as had been estimated earlier.

V. RESULTS AND CONCLUSIONS

The results presented in the preceding section give evidence for a significant correlation between T and p, meaning an anti-correlation between the muon rate and p as qualitatively expected from the arguments presented earlier. A more quantitative estimate of the expected correlation is obtained by noticing that an approximate dependence of the form $R = R_0 cos^2 z$ of the rate R over zenith angle z is obeyed over the whole z range of the experiment, indicating that $R/R_0 = (L_0/L)^2, L$ being the atmosphere effective thickness at zenith angle z and L_0 its vertical incidence value. The shower development being dominated by the value of the nuclear interaction length rather than of the muon decay length, it is reasonable to assume that the relations $dT/T= -dR/R=2dL/L=2dp/p$, corresponding to $\lambda_p = 2/\langle p\rangle$ over the whole z range, are approximately verified. The predicted value of λ_p is therefore *2.0 permil per mbar*, in good qualitative agreement with the measured value. A more accurate prediction would imply giving up the two simplifying assumptions made here: static atmospheric regime and negligible decay length effect, both of which would require complicated simulations that are well beyond the scope of the present study. It would also require a better experimental accuracy of the measurement of λ_p in order to match that of the prediction and allow for a critical discussion of the result.

Turning now to the temperature dependence, the result of the preceding section gives again evidence for a significant effect. However at least part of it is expected to result from the anti-correlation that exists between p and t. As we have a good qualitative understanding of the p dependence, it is legitimate to unfold it from the determination of $\lambda_t$and search for a possible remaining correlation, this time genuine, described by a lower value, $\lambda^*{}_t$, of λ_t. Indeed, to an excellent approximation $\lambda^*{}_t$ is given by the relation $\lambda^*{}_t = \lambda_t - \lambda_p \langle \partial p \partial t\rangle / \langle \partial t^2\rangle$, that reduces on average, after having introduced the average (t, p) correlation coefficient $C = -0.62$, to $\lambda_t^* = \lambda_t - \lambda_p C\Delta p/\Delta t$. Replacing λ_p by 1.47 *permil per mbar*, λ_t by *−1.11 permil per* $^\circ C$ and Δp and Δt by their average values of *1.4 mbar* and *2.1* $^\circ C$, respectively, one obtains an approximate evaluation of λ^*, $\lambda^*{}_t = -1.11 + 0.91\Delta p/\Delta t = -0.50$ *per mil per* $^\circ C$. The fits were therefore repeated with ∂T_{ij} replaced by $\partial T_{ij} /(1+ \lambda_p\, \partial p_{ij})$, the pressure corrected values, and λ_p fixed at *1.47 permil per mbar* for all sets of measurements. The best fit results are displayed in Fig. 5 and listed in Table 1. The new average value of the best fit values of λ_t is $\lambda^*{}_t = -0.57 \pm 0.34$ *permil per degree*, in excellent agreement with the estimate made above. The values taken by $\Delta\lambda_j$ are essentially unchanged in the new fits. The remaining temperature correlation corresponds to less than 1.7 standard deviations, a barely significant effect.

In summary, we found evidence for an anti-correlation of the muon cosmic rate in Hanoi (equal and opposite to the correlation measured for T) with atmospheric pressure of *-1.47± 0.50 permil per mbar* in good qualitative agreement with the crude prediction, *-2 permil per mbar*, of a naive model neglecting dynamical atmospheric effects and muon decay length effects. The correlation with ground temperature, *+1.11± 0.34 permil per* $^\circ C$, is partly explained by the anti-correlation existing between temperature and pressure. Once the data are corrected for their dependence over atmospheric pressure, a smaller, barely significant correlation of *+0.57± 0.34 permil per* $^\circ C$ remains. The results of the present study retrospectively justify having ignored such effects in earlier publications [1]. Moreover, they illustrate the good quality of the experimental data from which small effects at the permil level can be revealed without suffering from systematic biases.

Table 1. *The table lists for each set of measurement the zenith angle z ($^\circ$), the mean and rms values of p (mbar), t ($^\circ C$) and T (ms), the number of events in the set, N (in units of 10^3) and the results of the fits, $\lambda \pm \Delta\lambda$, in permil per mbar or per $^\circ C$.*

z	$\langle p\rangle$	$\langle t\rangle$	$\langle T\rangle$	Δp	Δt	ΔT	N	λ_p	$\Delta\lambda_p$	λ_t	λ^*_t	$\Delta\lambda_t$
0	1009.7	28.1	439	0.80	2.01	437	129	−0.28	3.45	−3.58	−3.52	1.38
15	1006.0	31.0	469	1.57	1.31	465	62	−1.31	2.54	−1.43	−0.11	3.04
45	1011.1	28.3	912	1.22	1.74	899	142	1.66	2.14	−0.14	0.55	1.50
5	1010.6	27.7	444	0.47	1.98	446	129	7.82	2.88	−2.08	−1.85	1.41
20	1011.5	27.8	517	1.14	1.37	517	117	−0.26	2.57	−1.26	−0.39	2.14
35	1010.8	26.5	696	0.97	0.95	694	82	0.99	3.57	−3.17	−2.30	3.66
50	1013.5	25.7	1106	2.62	3.19	1108	119	0.38	1.11	−0.47	0.54	0.91
65	1016.4	24.8	2411	1.57	3.05	2374	25	−1.76	3.96	−0.07	0.44	2.04
10	1018.6	26.3	465	1.39	2.45	468	39	7.37	3.69	−4.18	−3.45	2.09
25	1016.3	24.8	562	1.39	2.66	561	117	−1.38	2.09	0.79	1.11	1.10
40	1015.1	24.4	786	1.59	2.88	787	83	0.99	2.19	−1.15	−1.28	1.21
55	1011.2	24.2	1365	1.71	2.62	1372	46	1.93	2.73	−1.67	−1.44	1.78
70	1012.3	28.3	3336	1.25	2.29	3345	18	3.31	5.93	−0.62	−0.02	3.23
5	1012.8	27.9	449	1.15	2.18	448	143	3.42	2.28	−0.56	−0.27	1.21
30	1007.0	29.2	618	1.50	2.28	619	306	3.39	1.21	−2.24	−1.62	0.80
45	1012.5	21.4	931	1.35	1.40	927	139	0.46	1.97	−0.41	0.77	1.90
60	1015.6	21.4	1793	0.84	0.39	1803	17	6.23	9.31	−15.76	−13.8	13.19
60	1015.6	25.5	1779	1.10	1.15	1773	36	2.15	4.78	6.55	7.48	4.58
75	1009.1	25.5	5101	1.30	1.83	5084	22	0.16	5.14	−3.31	−2.84	3.66
15	1013.3	21.4	479	0.91	0.77	480	133	−3.72	3.01	3.53	4.84	3.58
0	1014.7	24.2	440	1.44	1.70	442	145	3.10	1.83	0.29	0.92	1.55

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