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COMPETITION BETWEEN STOKES AND ANTI-STOKES WAVES IN RAMAN FIBER LASER

CHU VAN LANH AND DINH XUAN KHOA
Faculty of Physics, Vinh University

Abstract. The set of intracavity field equations describing the evolution of intra-cavity pumping, Stokes and anti-Stokes powers of Raman laser is presented. Most attention is given to intra-cavity competition of two Raman waves, which depends on cavity properties, frequency shift and phase mismatch. The calculated results are based on Ge-doped and D_2-gas-in glass fiber lasers at CW-regime and pulse-pumped regime.

I. INTRODUCTION

One of the bright achievements of laser physics in the 1990s was the creation of highly efficient medium-power continuous-wave (CW) single-mode Raman fiber lasers for the near infrared. The lasers differed mainly by the type of fibers which has various Stokes (consequently anti-Stokes) frequency shifts, by the design of Stokes (anti-Stokes) cavities, and by the pumping sources [1]. It is based on stimulated Raman scattering of Raman material placed inside of optical resonator. The classical theory of Raman lasers is improved to describe the operation of them [2, 3]. In previous works, when investigate the unstationary regime of Raman laser operating at Stokes wave, it is clear that the power of Stokes wave can be transfered to anti-Stokes wave, in many ways the opposite process can be occur [4].

In this paper we present the competition between Stokes and anti-Stokes waves depending on the phase mismatch, frequency shift and properties of optical resonator.

II. THE RATIO OF STOKES AND ANTI-STOKES POWERS FOR CW-RAMAN LASER

We consider three intracavity fields: pump (mode-p), Stokes (mode-s) and anti-Stokes (mode-a) as shown in Fig. 1.

In this situation, beside the two-photon Raman interaction, there also exists a four-wave mixing process, by which the Stokes and anti-Stokes fields can be strongly coupled. We assume a triple-resonance condition, i.e., all the three fields are resonant with the cavity, although in reality this condition will be experimentally difficult due to dispersion effects [3]. As shown in previous works [3, 4], the set of rate equations for intracavity

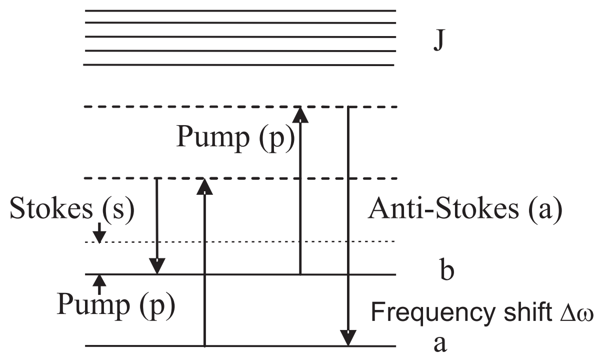


Fig. 1. Energy level diagram of the far-off resonance Raman process

powers is

$$\begin{aligned}
\dot{P}_p + \gamma_p P_p &= \gamma_{ep}\sqrt{P_p P_{ep}} - \frac{\omega_p}{\omega_s}\frac{k_p}{k_s} G(\delta)\,\frac{8\mu_0}{\pi b}\left[\omega_s P_s - \frac{k_p + k_s}{k_p + k_a}\omega_a P_a\right] \\
\dot{P}_s + \gamma_s P_s &= \frac{8\mu_0\omega_p}{\pi b\omega_s} G(\delta)\, P_p\left[\omega_s P_s - C\omega_a P_a\right] \\
\dot{P}_a + \gamma_a P_a &= -\frac{\omega_p}{\omega_s}\frac{8\mu_0}{\pi b} G(\delta)\, P_p\left[\frac{k_p + k_s}{k_p + k_a}\omega_a P_a \; - C\omega_s P_s\right]
\end{aligned} \quad (1)$$

where $\gamma_q = (c/n_q L)\ln\sqrt{R_{1q}R_{2q}}$ is the intra-cavity lifetime of photon q-mode, $\gamma_{ep} = (2c/n_p L)\sqrt{T_{1p}}$ is the intracavity lifetime of external field, n_q is the refractive index of fields q, R is the cavity mirror's reflectance, T is the transmittance, the subscript "1" presents the front mirror that couples the external field E_{ep}, and "2" means back mirror, $G(\delta) \approx (-\omega_s/4)\left(N\hbar d_0^2 D/\varepsilon_0\right)\left(\gamma_{ab}/\left(\gamma_{ab}^2 + \delta^2\right)\right)\left(\lambda_p/\left(\lambda_p + \lambda_s\right)\right)$ is the gain factor,N is the number density, D is the population difference between levels a and b,γ_{ab} is the coherence dephasing rate, δ is the two-photon detuning for $a \leftrightarrow b$transition, d_0 is the coupling constant relating to third-order susceptibility $\chi^{(3)}$, $C = \left[(k_p + k_s)/\sum k\right]\sin(\Delta kL/2)/(\Delta kL/2)$ is the coupling coefficient relating to phase mismatch, $\sum k = 2k_p + k_s + k_a$, $\Delta k = 2k_p - k_s - k_a$ is the phase mismatch, L is the cavity length, $\Delta\phi = 2\phi_p - \phi_s - \phi_a$ is the phase difference between the three waves (relating to imaginary part of the field), and

$$P_q(t) = \frac{\pi\varpi_{0q}^2 n_q}{4}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left|E_q(t)\right|^2 = \frac{\pi b_q}{4\omega_q\mu_0}\left|E_q(t)\right|^2 \quad , \quad (2)$$

with considering $b_p \approx b_s \approx b_a = b$.

The field amplitude equations at steady state (means for CW-Raman laser) have the form

$$\begin{aligned}\gamma_p P_p &= \gamma_{ep}\sqrt{P_p P_{ep}} - \frac{\omega_p}{\omega_s}\frac{k_p}{k_s}G\left(\delta\right)\frac{8\mu_0}{\pi b}\left[\omega_s P_s - \frac{k_p+k_s}{k_p+k_a}\omega_a P_a\right]\\ \gamma_s P_s &= \frac{8\mu_0\omega_p}{\pi b\omega_s}G\left(\delta\right)P_p\left[\omega_s P_s - C\omega_a P_a\right]\\ \gamma_a P_a &= -\frac{\omega_p}{\omega_s}\frac{8\mu_0}{\pi b}G\left(\delta\right)P_p\left[\frac{k_p+k_s}{k_p+k_a}\omega_a P_a \; - C\omega_s P_s\right]\end{aligned} \tag{3}$$

Let ξ denote the ratioP_a/P_s, from two last Eqs.(3) we can eliminate P_p to obtain

$$C\omega_a\gamma_a\xi^2 - \left(\gamma_a\omega_s + \frac{k_p+k_s}{k_p+k_a}\gamma_s\omega_a\right)\xi + C\gamma_s\omega_s = 0, \tag{4}$$

which has two roots

$$\xi_\pm = \frac{\frac{k_p+k_s}{k_p+k_a}\omega_a\frac{\gamma_s}{\gamma_a} + \omega_s}{2C\omega_a} \pm \frac{\sqrt{\left(\frac{k_p+k_s}{k_p+k_a}\omega_a\frac{\gamma_s}{\gamma_a} + \omega_s\right)^2 - 4C^2\omega_s\omega_a\frac{\gamma_s}{\gamma_a}}}{2C\omega_a} \tag{5}$$

We discard the root ξ_+ since it corresponds to $P_a/P_s > 1$ which is physically impossible. We find that ξ_- is a constant that is not dependent on the pumping rate and that describes the competition between Stokes and anti-Stokes waves.

From (5), we can see that the competition between two waves depends not only on the cavity properties (γ_s, γ_a), phase mismatch (C), their wavelengths (λ_q) (or exactly frequency shift $\Delta\omega$ shown in Fig.1), but on the confocal parameters (b).

III. COMPETITION IN TWO KINDS OF CW-RAMAN FIBER LASERS

In this calculation, we want to discuss a competition between anti-Stokes and Stokes waves by finding dependence of their power ratio on laser cavity's properties, phase mismatch for four-wave-mixing, and two fibers with different frequency shifts. To simply, we plot the dependence of ξ_- on ratio γ_s/γ_a changing from 0% to 400% and phase mismatch $\Delta kL/2$ changing from 0 to 2π. The wavelengths are chosen to be 1.06 μm of Diode-pumped Nd-doped fiber for pump wave, 1.3 μm for Stokes wave and 0.82 μm for anti-Stokes wave of Ge-doped fiber [5,6], and 1.55 μm for Stokes wave and 0.58 μm for anti-Stokes wave of D_2-gas-in-glass fiber [7].

The dependence of ratio ξ_- on γ_s/γ_a for two above mentioned fiber lasers are illustrated in Fig. 2 and Fig. 3. From two figures one can see that the ratio ξ_- increases when the ratio γ_s/γ_a increases. That means when reflectivity of mirrors for Stokes wave increases the intracavity power of Stokes wave increases. Since the strongly coupling between Stokes and anti-Stokes waves in four-waves-mixing, the intracavity power of anti-Stokes consequently increases too. But, the saturation will be happen for anti-Stokes power, although reflectivity for Stokes wave increases some time more than one for anti-Stokes.

Moreover, one can see too the intracavity power of anti-Stokes of D_2-gas-in glass laser ($\xi_- \geq 60\%$ at $\gamma_s/\gamma_a > 400\%$) is more than one of Ge-doped laser ($\xi_- \leq 60\%$ at $\gamma_s/\gamma_a > 400\%$). This behavior can be explained by transition probability between two Raman levels, as illustrated in Fig. 1. When frequency shift $\Delta\omega$ is larger the transition

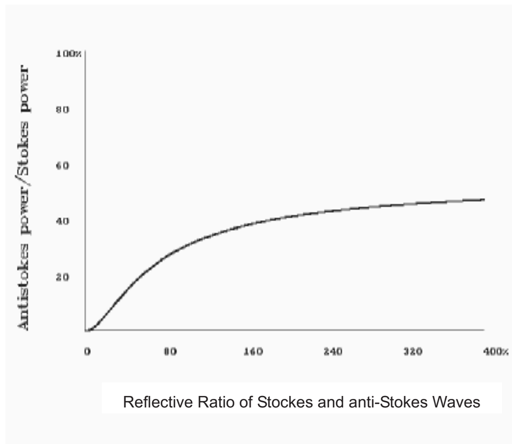


Fig. 2. ξ_{-} *vs* γ_s/γ_a for wavelengths 1.06 μm, 1.3 μm and 0.82 μm of Raman Ge-doped fiber laser

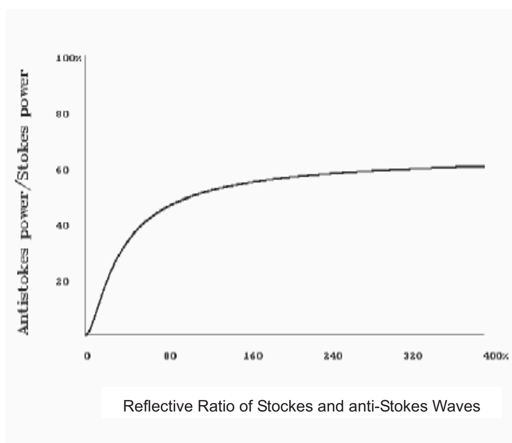


Fig. 3. ξ *vs* γ_s/γ_a for wavelengths 1.06μm, 1.55 μm and 0.58 μm of D_2-gas-in-glass Raman laser

probability is smaller, so the energy transfer from pump and Stokes fields to anti-Stokes field is more difficult, i.e., the coupling constant is limited. It is clear that the frequency shift in Raman fiber influences on the competition between Stokes and anti-Stokes waves.

The dependence of ξ_{-} on the phase mismatch is calculated and presented in Fig. 4 and Fig. 5 for two lasers. From two figures it is clear that the ratio ξ_{-} reaches a maximum at phase matching condition ($\Delta k = 0$) and decreases with increasing of phase mismatch (ΔkL). The shape of this characteristic is absolutely similar to one of phase-efficiency characteristic of Second Harmonic Generation and Sum Frequency Generation in three-wave-mixing [9].

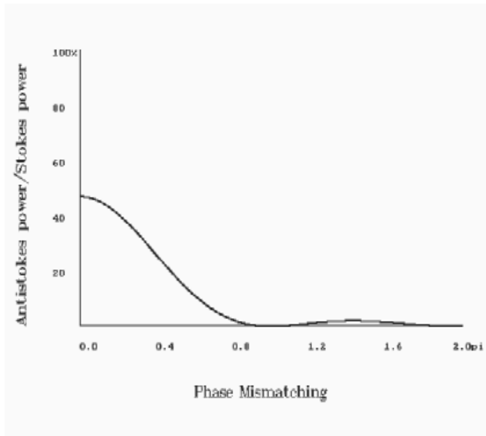


Fig. 4. ξ_- *vs* $\Delta kL/2$ for wavelengths 1.06 μm, 1.3 μm and 0.82 μm of Ge-doped laser

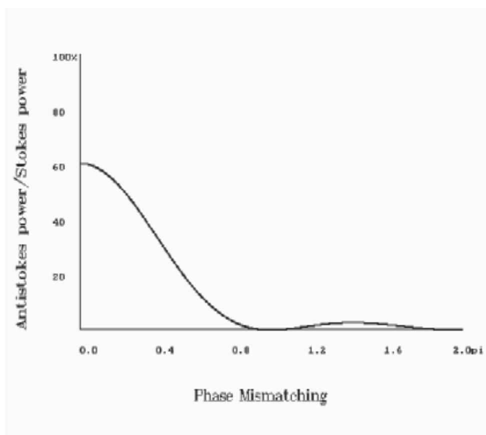


Fig. 5. ξ_- *vs* $\Delta kL/2$ for wavelengths 1.06 μm, 1.55 μm and 0.58 μm of D_2–gas-in-glass laser

IV. COMPETITION IN PULSE-PUMPED D_2-GAS-IN -GLASS LASER

Consider an external pump pulse is Gaussian given by

$$P_{ep}\left(t\right)=P_{\max}\exp\left(-\left(\frac{\sqrt{\ln 2}t}{\tau}\right)^2\right), \tag{6}$$

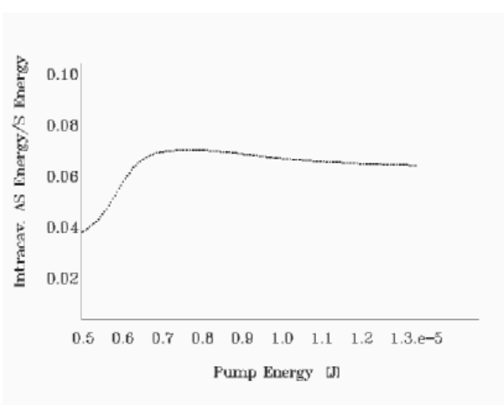
where $P_{\max}$ is the peak, τ is the half of duration, and a sample of Raman fiber laser consists of the pump pulse, resonant cavity and Raman medium, which are chosen with parameters given following. The pump pulse at wavelength 1.06μm (of Diode-pumped Neodym laser for example) has an energy $W=\left(0\div 4.510^{-5}\right)J$ and half duration time $\tau=10\ ps$, which is focused in center of laser cavity with beam waist to be $w=\left(0.05\div 0.45\right)\mu m$.

The properties of resonant cavity are chosen as: reflectivities $R_{1p} = 0.5$, $R_{2p} = 0.999$, $R_{1s} = 0.999$, $R_{2s} = 0.95$, $R_{1a} = 0.999$, $R_{2a} = 0.95$(in this case $\xi_{loss} = 1$), and length $L = (200 \div 1000)\,\mu m$. The Raman medium is a sample of D_2-gas-in-glass fiber [7, 8] with $\alpha(\delta) \approx 1.510^{-9} cm^2/W$[3], so $G\,(\delta)$ is calculated to be$\approx 1.310^{-4} cm^2/W$. Consequently, the Raman wavelengts are 1.55 μm for Stokes wave and 0.57μm for anti-Stokes one [7, 8]. Using above given parameters and substituting (6) to (1), and by numerical four-order integrated Runger-Kuta method, we find dependence of ratio of Stokes power and anti-Stokes power on some parameters as illustrated in Fig. 6 to Fig. 9.

From Fig. 6 can see that anti-Stokes increases with great rate in comparison with Stokes until reaches maximum value. After that the rate decreases. If one considers the ratio of confocal parameters b_a/b_s changes from 0.2 to 1, which influences on ξ_{-}, and $\xi_{-} - b_a/b_s$ characteristic is presented in Fig. 7. It can bee seen that the anti-Stokes power will be more intense if $b_a < b_s$. But it is well known $k_a > k_s$ always, so beam waist w_a must be much larger than w_s. This condition is satisfied difficultly in trio-cavity.

Dependence of ξ_{-} on intracavity loss ratio γ_s/γ_a and on Δk are plotted in Fig. 8 and Fig. 9, respectively. As well as in CW- regime, two quantities in Fig. 8 are proportional one to other, which explains the coupling of Stokes and anti-Stokes waves. From Fig. 9, ξ_{-} decreases speedily when mismatch Δk increases, that is in good agreement with phase-matching condition in nonlinear optical interaction.

In summary, we can control the competition of Stokes and anti-Stokes waves in Raman laser by the way to change cavity properties and phase mismatch. Moreover, to generate anti-Stokes wave with high power it is necessary to make a phase-matching condition, to use cavity mirrors with reflective ratio between Stokes and anti-Stokes waves more than 100%, to choice an optimal pump power and finally to use Raman medium having shorter frequency shift.



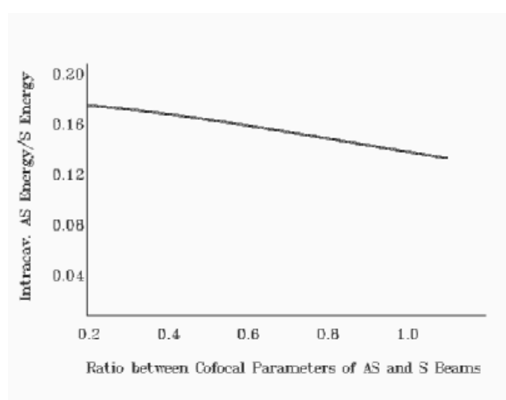


Fig. 6. ξ_{-} *vs* W for D_2-gas-in-glass laser. **Fig. 7** ξ_{-} *vs* b_a/b_s for D_2 - gas-in-glass laser

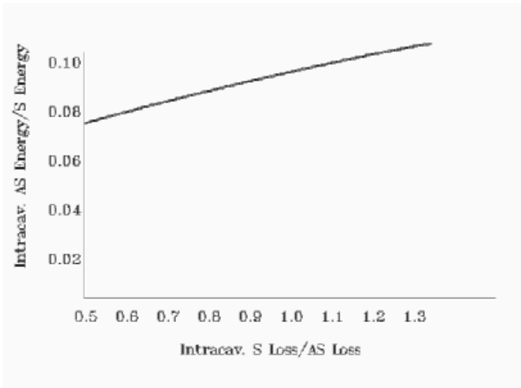


Fig. 8 ξ_- *vs* γ_s/γ_a for D_2-gas-in-glass laser.

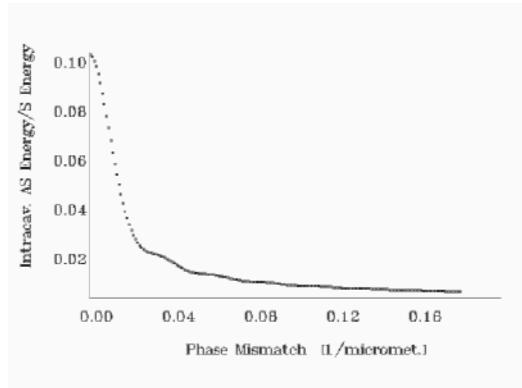


Fig. 9. ξ_- *vs* Δk for D_2-gas-in-glass laser.

V. CONCLUSION

The ratio of intracavity powers of Stokes and anti-Stokes waves for CW- and pulse-pumped- Raman laser is derived by using semi-classical set of intracavity field equations and term of measurable power. The competition between two Raman fields is calculated and discussed by investigating the dependence of ratio of powers on pump energy, confocal parameters, cavity properties, phase mismatch and frequency shift in two Raman fiber lasers.

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