Communications in Physics, Vol. 17, No. 4 (2007), pp. 213-220

HIGHER-DIMENSIONAL STRING COSMOLOGICAL MODEL WITH BULK VISCOUS FLUID IN LYRA MANIFOLD

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Abstract. *Locally rotationally symmetric five dimensional string cosmological model with bulk viscous fluid in Lyra manifold is constructed. This model is obtained for time dependent displacement field, i.e.* $\beta = \beta(t)$ *and constant bulk viscous coefficient. Some physical and geometrical properties of the model are discussed.*

I. INTRODUCTION

Nowadays relativists are interested in theories with more than four dimensional space-times. Alvarez et al. [1], Randjbar-Daemi et al. [2], Marciano [3] suggested that the experimental detection of time variation of fundamental constants could provide strong evidence for the existence of extra dimensions. The extra dimensions in the space-time contracted to a very small size of Planck length or remain invariant. Further, during contraction process, extra dimensions produce large amount of entropy which provides an alternative resolution to the flatness and horizon problem [4]. Misner [5] pointed out that viscosity plays an important role in the formation of galaxy. Viscosity also accounts for the large entropy per baryon observed in the present universe [6]. Banerjee et al. [7] constructed Bianchi type I cosmological models with viscous fluid in higher dimensional space-time. Chatterjee and Bhui [8] obtained exact solutions of the field equations in a five dimensional space time with viscous fluid. Singh et al. [9] obtained exact solutions of the field equations for a five dimensional cosmological model with variable G and bulk viscosity in Lyra geometry.

The study of string theory is important in the early stages of the evolution of the universe before the particle creation. Cosmic strings have received considerable attention in cosmology as they are believed to give rise to density perturbations leading to the formation of galaxies [10]. Chatterjee [11] constructed massive string cosmological model in higher dimensional homogeneous space time. Krori et al. [12] constructed Bianchi type I string cosmological model in higher dimension and obtained that matter and strings coexist through out the evolution of the universe. Rahaman et al.[13] obtained exact solutions of the field equations for a five dimensional space time in Lyra Manifold when the source of gravitation is massive strings.

Lyra [14] modified the Riemannian geometry by introducing a gauge function into the structure less manifold as a result of which the cosmological constant arises naturally from the geometry.

The analog of Einstein's field equations based on Lyra's geometry in normal gauge as obtained by Sen [15] and Sen and Dunn [16] are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -\chi T_{ij} \tag{1}$$

where ϕ_i is the displacement vector and other symbols have their usual meanings as in the Riemannian geometry.

So far the study of string with bulk viscosity in higher dimensional Lyra manifold is not yet found in the literature. Therefore we have taken an attempt to construct LRS Bianchi I five dimensional string cosmological model with bulk viscous fluid in Lyra manifold. Earlier Bali and Upadhaya [17] constructed four dimensional LRS Bianchi type I string cosmological model with constant bulk viscous coefficient in general relativity. In this paper the energy momentum tensor is assumed to be the simple extension of usual four dimensional cases.

II. THE METRIC AND THE FIELD EQUATIONS

Here we consider a five dimensional LRS Bianchi type I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) + C^2d\Psi^2 \tag{2}$$

where A, B and C are functions of cosmic time t only.

We assume here that the co-ordinates to be commoving i.e.

$$u^0 = 1 \quad and \quad u^1 = u^2 = u^3 = u^4 = 0 \tag{3}$$

Further we consider here the displacement vector ϕ_i in the form

$$\phi_i = (\beta(t), 0, 0, 0, 0) \tag{4}$$

The energy momentum tensor for a cloud of string dust with a bulk viscous fluid given by Landau and Lifshitz [18], Letelier [19] and Bali and Dave [20] is

$$T_{ij} = \rho u_i u_j - \lambda w_i w_j - \xi u^l_{;l}(g_{ij} + u_i u_j), \tag{5}$$

where ξ is the bulk coefficient of viscosity, ρ is the proper density for a cloud of strings with particles attached to them , λ is the string tension density, $u^i$is the five velocity of the particles, g_{ij} is the covariant fundamental tensor , w^i is the unit space like vector representing the direction of the string satisfying

$$u^i u_i = -w^i w_i = -1 \tag{6}$$

and

$$u^i w_i = 0 \tag{7}$$

Further the expansion scalar is given by

$$\theta = u^l_{;l} \tag{8}$$

Using equations (4), (5) and (6) the explicit form of field equations (1) for the line element (2) are obtained as

$$-\left(\frac{B'}{B}\right)^2 - 2\frac{A'B'}{AB} - 2\frac{B'C'}{BC} - \frac{A'C'}{AC} + \frac{3}{4}\beta^2 = -\chi\rho \tag{9}$$

$$2\frac{B''}{B} + \frac{C''}{C} + \left(\frac{B'}{B}\right)^2 + 2\frac{B'C'}{BC} + \frac{3}{4}\beta^2 = \chi(\lambda + \xi\theta) \tag{10}$$

$$\frac{A''}{A} + \frac{B''}{B} + \frac{C''}{C} + \frac{A'B'}{AB} + \frac{B'C'}{BC} + \frac{A'C'}{AC} + \frac{3}{4}\beta^2 = \chi\xi\theta \tag{11}$$

$$\frac{A''}{A} + 2\frac{B''}{B} + \left(\frac{B'}{B}\right)^2 + 2\frac{A'B'}{AB} + \frac{3}{4}\beta^2 = \chi\xi\theta \tag{12}$$

where dash denotes differentiation with respect to t. In the following section we intend to derive the exact solutions of the field equations using$\beta = \beta(t)$ and $\xi = \xi_0$ (constant)(Mohanty and Pattanaik [21], Bali and Upadhaya [17], Bali and Yadhav [22]) in order to overcome the difficulties due to non linear nature of the field equations.

III. COSMOLOGICAL SOLUTIONS

Here there are six unknowns viz. A, B, C, β, ρ and λ involved in four field equations (9)-(12). In order to avoid the insufficiency of field equations for solving six unknowns through four field equations here we consider the power law i.e.

$$C = B^n \tag{13}$$

where n is a constant.

Subtracting equation (11) from (12) we find

$$\frac{B''}{B} - \frac{C''}{C} + \left(\frac{B'}{B}\right)^2 + \frac{A'B'}{AB} - \frac{A'C'}{AC} - \frac{B'C'}{BC} = 0 \tag{14}$$

Substituting equation (13) in equation (14) we get

$$\frac{B'}{B}\left(\frac{B''}{B'} + (n+1)\frac{B'}{B} + \frac{A'}{A}\right) = 0, \tag{15}$$

which yields following three cases

Case I: $\frac{B''}{B'} + (n+1)\frac{B'}{B} + \frac{A'}{A} = 0$

Case II: $B' = 0$

Case III: $B' = 0$ and $\frac{B''}{B'} + (n+1)\frac{B'}{B} + \frac{A'}{A}$ =0.

Now in the following subsections we intend to derive the exact solutions of the field equations for the above mentioned cases.

III.1. Case I

$$\frac{B''}{B'} + (n+1)\frac{B'}{B} + \frac{A'}{A} = 0 \tag{16}$$

In this case we find

$$B(t) = \left[(n+2)\left(\int \frac{dt}{A(t)} + k\right)\right]^{\frac{1}{n+2}}$$

from which it is clear given any function $A(t)$we can find a $B(t)$. Therefore the solutions are unique. However for further studies here we consider

$$\frac{B''}{B'}+(n+1)\frac{B'}{B}=-\frac{A'}{A}=k(>0\ cons\tan t)$$

which yields

$$B=\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^{\frac{1}{n+2}} \tag{17}$$

and

$$A=k_3e^{-kt} \tag{18}$$

where $k_1\neq 0$, k_2 and $k_3\neq 0$ are constants of integration.
Now equation (13) yields

$$C=\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^{\overline{n+2}^{\,n}}. \tag{19}$$

Substituting equations (17) - (19) in equation (11) we get

$$\frac{3}{4}\beta^2=\chi\xi_0\left[-k+\frac{k_1e^{kt}}{\frac{k_1}{k}e^{kt}+k_2}\right]+\frac{(2n+1)k_1^2e^{2kt}}{(n+2)^2\left(\frac{k_1}{k}e^{kt}+k_2\right)^2}-k^2. \tag{20}$$

Putting equations (17)-(20) in equations (9) and (10) we find

$$\rho=k\xi_0-\frac{k_1e^{kt}}{\chi\left(\frac{k_1}{k}e^{kt}+k_2\right)}(k+\chi\xi_0) \tag{21}$$

and

$$\lambda=\frac{kk_1e^{kt}}{\chi\left(\frac{k_1}{k}e^{kt}+k_2\right)}-\frac{k^2}{\chi}. \tag{22}$$

The particle density is obtained as

$$\rho_p=\rho-\lambda=k\xi_0+\frac{k^2}{\chi}-\frac{k_1e^{kt}}{\chi\left(\frac{k_1}{k}e^{kt}+k_2\right)}(2k+\chi\xi_0). \tag{23}$$

In this case metric (2) takes the form

$$\begin{aligned}ds^2=&-dt^2+k_3^2e^{-2kt}dx^2\\&+\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^{\frac{2}{n+2}}\left(dy^2+dz^2\right)\\&+\left[(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)\right]^{\frac{2n}{n+2}}d\Psi^2.\end{aligned} \tag{24}$$

III.2. Case II

$$B' = 0 \tag{25}$$

In this case we find

$$B = a \quad (constant). \tag{26}$$

Now equation (13) yields

$$C = a^n \quad (constant). \tag{27}$$

Substituting equations (26) and (27) in the field equations (9) - (12) we obtain

$$\frac{3}{4}\beta^2 = -\chi\rho \tag{28}$$

$$\frac{3}{3}\beta^2 = \chi\lambda + \chi\xi\frac{A'}{A} \tag{29}$$

$$\frac{A''}{A} + \frac{3}{4}\beta^2 = \chi\xi\frac{A'}{A} \tag{30}$$

Here there are four unknowns involved in three equations (28) - (30). In order to get explicit solutions we have to assume a physical or a mathematical condition. Therefore in this case we consider the following cases:

$$\rho + \lambda = 0 \quad (Reddy\ [23, 24]) \tag{31}$$

$$\rho = \lambda \quad (geometric\ strings) \tag{32}$$

and

$$\rho = (1 + \omega)\lambda \quad (Takabayasi\ string\ or\ p - string). \tag{33}$$

III.2.1. $\rho + \lambda = 0$

Subtracting equation (28) from equation (29) and using relation (31) we obtain

$$A = constant \tag{34}$$

Substituting equation (34) in equation (30) we get

$$\beta^2 = 0 \tag{35}$$

Therefore in this case the model (20) reduces to empty flat model of Einstein's theory.

III.2.2. $\rho = \lambda$*(Letelier [25])*

Adding equations (28) and (29) and using relation (32) we find

$$\frac{3}{2}\beta^2 = \chi\xi_0\frac{A'}{A} \tag{36}$$

Subtracting equation (36) from two times of equation (30) we obtain

$$\frac{A'}{A}\left(\frac{A''}{A'} - \chi\xi_0\right) = 0 \tag{37}$$

which yields

$$A' = 0 \tag{38}$$

or

$$\frac{A''}{A'} - \chi\xi_0 = 0 \tag{39}$$

Using equation (38) we get the same model as obtained in section 3.2.1. However from equation (39) we obtain

$$A = \frac{a_1}{\chi\xi_0} e^{\chi\xi_0 t} + a_2 \tag{40}$$

where a_1 and a_2 are constants of integration.

Substituting equation (40) in equation (36) we find

$$\frac{3}{2}\beta^2 = \frac{\chi a_1 e^{\chi\xi_0 t}}{\frac{a_1}{\chi\xi_0} e^{\chi\xi_0 t} + a_2} \tag{41}$$

Equation (28) with the help of equation (41) yields

$$\rho(=\lambda) = \frac{-a_1 e^{\chi\xi_0 t}}{2\left(\frac{a_1}{\chi\xi_0} e^{\chi\xi_0 t} + a_2\right)} \tag{42}$$

Now the solutions given by (26), (27), (40), (41) and (42) satisfy the field equations(9)-(12) only when $a_1 = 0$. This immediately yields

$$\rho(=\lambda) = 0$$

$$\beta^2 = 0$$

and

$$A = constant.$$

Hence in this case the model is same as obtained earlier in section 3.2.1. Here we mention that in case II the model for p-string could not be obtained due to highly non linear nature of the field equations.

III.3. Case III

$B' = 0$ and $\dfrac{B''}{B'} + (n+1)\dfrac{B'}{B} + \dfrac{A'}{A} = 0$

This case is not acceptable due to indeterminacy of B.

IV. PHYSICAL AND GEOMETRICAL PROPERTIES

In the preceding section the metric (24) represents a string cosmological model with bulk viscous fluid in Lyra manifold. At the initial epoch $t = 0$, the metric becomes flat. As time increases the rate of expansion in the model along y and z axes are faster in comparison to the contraction of the model along x axes when $-2 < n < 0$.The extra dimension contracts if $-2 < n < 0$.The parameters involved in the model behave as follows:

(a) The rest energy density for the model (24) given by equation (21) satisfies the reality condition $\rho > 0$ when

$$k\xi_0\chi\left(\frac{k_1}{k}e^{kt} + k_2\right) > k_1 e^{kt}(k + \chi\xi_0).$$

(b) The scalar of expansion for the model is obtained as

$$\theta = \frac{k_1 e^{kt}}{\frac{k_1}{k} e^{kt} + k_2} - k.$$

At the initial epoch $t = 0$, θ is finite and $\theta \to 0$ when $t \to \infty$. The expansion in the model stops at infinite time. Thus there is finite expansion in the model.

(c) The shear scalar σ^2 for the model (24) is

$$\sigma^2 = \frac{1}{2}\left[\frac{k_1^2 e^{2kt}(n^2+2)}{(n+2)^2\left(\frac{k_1}{k}e^{kt}+k_2\right)^2} + \frac{k_1 e^{kt}(n+1)}{(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)} + k^2 + k\right].$$

Since $\lim\limits_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, the universe remains anisotropic throught the evolution.

(d) The spatial volume of the universe is

$$V = \frac{(n+2)\left(\frac{k_1}{k}e^{kt}+k_2\right)}{k_3 e^{kt}}, \quad k_3 > 0$$

Thus the volume of the universe is finite throught the evolution.

(e) The deceleration parameter (Fienstein et al, [26])

$$q = -3\theta^{-2}\left(\theta_{;i}u^i + \frac{1}{3}\theta^2\right) = -\left(\frac{3k_1 e^{kt}}{kk_2} + 1\right).$$

For k, k_1, $k_2 > 0$, the value of the deceleration parameter is negative, which indicates inflation in the model.

V. CONCLUSION

In this paper we have constructed a five dimensional string cosmological model with bulk viscous fluid in Lyra manifold. The model obtained is free from initial singularity, which supports the analysis of Murphy [27] that the introduction of bulk viscous fluid avoids the initial singularity.

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Received 06 October 2007.