Communications in Physics, Vol. 17, No. 4 (2007), pp. 221-226

ELECTRON SPIN EVOLUTION INDUCED BY HYPERFINE INTERACTION WITH NUCLEAR SPINS IN A 2D QUANTUM DOT

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Abstract. *We study the electron spin dynamics in 2D - quantum dots due to the hyperfine interaction with surrounding nuclear spins. The study shows that electron spin dynamics depends strongly on the initial state and polarization of the nuclear spin system. Obtained results has shown that electron spin dynamics is reproducible with initial randomly correlated states but depends on the individual initial tensor product state. The decay of electron spin does not have an exponential character, instead of it a power is given.*

I. INTRODUCTION

In the last decade, the spin dynamics of electrons in semiconducting nanostructures has attracted great attention from both theoretical and experimental points of view [1, 2]. The controlled manipulation of the spin, and in particular of its phase, is the primary prerequisite needed for novel applications in future quantum information processing. It is thus desirable to understand the mechanisms which limit the spin phase coherence of electrons. It was shown that in GaAs - semiconductors the spin decoherence times T_2 is unusually long exceeding 100 ns [3]. In the other hand, in GaAs each nucleus carries spin, the hyperfine interaction between electron and nuclear spins is unavoidable, and it is therefore important to understand its effect on electron spin dynamics. This is particularly interesting in the case, when the electrons are confined in closed systems such as a quantum dot with a spin 1/2 ground state. Besides the fundamental interest, these systems are potential candinates for spin qubits [4–11]. The spin dynamics in GaAs nanostructures is extensively studied in Refs. [7, 8, 12, 13].

In this work, we investigate the spin dynamics of a single electron confined to a 2D - quantum dot at low temperature. Neglecting both spin-obit coupling and eletron-phonon interaction [7], the hyperfine interation between electron and nulear spins becomes the dominante mechanism controling the spin dynamics of electrons [11]. Obtained results show that (i) the spin dynamics of electrons depends strongly on the initial state and the polarization of nulear spins, (ii) not randomly correlated initial states but the individual tensor product initial state affects the spin dynamics and (iii) the decay of electron spin doesnot have an exponential character, instead of it a power is given.

II. THE MODEL

The hyperfine interaction. We consider an electron confined in a 2D-*GaAs* quantum dot by the parabolic potential in the s-style state. We assume the electron to be

in some orbital eigenstates according to the confinning potential, e.g. the orbital ground state in the quantum dot. The remaining spin degree of freedom is coupling to the spins of surrounding nuclei via the hyperfine interaction. The Hamiltonian reads:

$$\hat{H} = \vec{S}\sum_i A_i \vec{I}_i. \tag{1}$$

Here the subscript i labels the nuclei, and the interaction constants A_i depend on the position of nuclear spin $\vec{I}_i$ in the quantum dot [11]:

$$A_i = A\nu_0|\Psi(\vec{r}_i)|^2,$$

where $n_0 = 1/\nu_0$ is the density of nuclei, A is the coupling constant [$\approx 10^{-5}$eV $\div$ 10^{-4}eV], $\Psi(\vec{r}_i)$ is the electron envelope wave function at nuclear site $\vec{r}_i$.

In the $GaAs$ - quantum dot, the nulear spin I equals to 1/2. The electron envelope wave function in the ground state in the quantum dot with a parabolic confinement potential has the well-known Fock-Darwin form [14]:

$$|\Psi(\vec{r}_i)|^2 = \frac{1}{2\pi}\frac{1}{(R/a)^2}\exp\left(-\rho_i/\left(R/a\right)^2\right)\chi_0^2\left(z\right),$$

where R is the effective radius of quantum dot, a is the characterizing length of confinement potential. The electron envelope wave function in the z - axis is given by:

$$\chi_0^2\left(z\right) = \theta\left(\frac{1}{2} - |z|\right)$$

The initial state. In the numerical simulations to be described below, the electron spin is initially in a single tensor product state with the nuclear spin system.

$$|\psi(t=0)\rangle = |\psi_{el}\rangle \otimes |\psi_{nuc}\rangle \tag{2}$$

i.e. the electron spin described by $|\psi_{el}\rangle$ is initially uncorrelated with the nuclear spins. However, there is still a large variety of possibilities for the initial nuclear spin state $|\psi_{nuc}\rangle$. Since the nuclear spins are 1/2, it is convenient to choose $|\uparrow\rangle$ and $|\downarrow\rangle$ as the basic states for electron spin and each nuclear spin. A simple choice is that the nuclear spin state is a tensor product of single states:

$$|\psi_{\rm nuc}\rangle = |\uparrow\rangle_1 \otimes |\uparrow\rangle_2 \otimes |\downarrow\rangle_3 \otimes ... \otimes |\uparrow\rangle_{\rm N}. \tag{3}$$

A nuclear spin state close to the above form can be generated experimentally by cooling down the nuclear spins in a strong external magnetic field. The strong magnetic field provides a quantization axis and suppresses dipolar interactions changing the spin projection along the field direction. Then, due to spin-lattice relaxation processes, the nuclear spin system will end up in a state of the type (3).

A more general state of the nuclear spins is the superposition of tensor product states,

$$|\psi_{\rm nuc}\rangle = \sum_{\rm T}\alpha_{\rm T}|T\rangle, \tag{4}$$

where the summation runs over all tensor product states of the form (3), i.e. over a complete basis of the underlying Hilbert space. The complex amplitudes $\alpha_{\rm T}$ satisfy the

normalization condition $\sum_{\mathrm{T}} |\alpha_{\mathrm{T}}|^2 = 1$. If the complex amplitudes α_{T} are chosen randomly, the nuclear spin state is called randomly correlated state.

In our simulations, we consider the GaAs - quantum dot with a given number N of nuclear spins contained in a plane dish of radius $R = \sqrt{N/n_0\pi}$. For the two dimensional GaAs - quantum dot, typically, $N \approx 10^3$ nuclei with the density $n_0 \approx 12.8\, nm^{-2}$ and the distribution of nuclear spins is regular. Thus, the radial coordinate ρ_i of i-th nuclear spin is defined as $\rho_i = \sqrt{(i-1/2)/\pi n_0}$ with i ranging from 1 to N.

Since the Hamiltonian (1) conserves J_z of the total momentum $\vec{J} = \vec{S} + \sum_i \vec{I}_i$, it is convenient to study the time evolution of electron spin in a given J_z subspace defined by eq.(2). The dimension of this subspace is given by:

$$C^{N+1}_{\frac{N+1}{2}-J_z} = \frac{(N+1)!}{\left(\frac{N+1}{2}-J_z\right)!\left(\frac{N+1}{2}+J_z\right)!} \tag{5}$$

To study the time evolution of the electron spin, we diagonalize the Hamiltonian for a subspace with a given value J_z and calculate the time evolution of the expectation value of S_z.

III. NUMERICAL RESULTS AND DISCUSSION

As we have discussed in the previous section, the dimension of the computational subspace given by Eq. (5) should increase so quickly with increasing N. For example, in the case of the fully unpolarized initial nuclear system the dimension of the computational subspace is $C = 1716$ and 6435 for $N = 12$ and 14, respectively. Because of this difficulty, in this work, we are only able to study for some small values of N. However, obtained results together with previous works [5, 10, 13] have provided a better understanding of the decohence of the electron spin in the quantum information processing.

Fig. 1 shows simulation results for the system of $N = 12$ nuclear spins with different values of the degree of polarization J_z. The upper left panel shows the expectation value $< S_z(t) >$ as a function of time for fully polarized initial nuclear system with electron spin pointing opposite to the nuclear spins. In the following panels the polarization of the nuclear system is successively reduced by lowering the value of J_z in the initial state. The case of a fully unpolarized nuclear spin system is shown in the bottom left panel with $J_z = -0.5$. Since the value of the z-component of the total spin J is fixed the expectation values of the transversal components S_x and S_y vanish. In all cases, $< S(t) >=< S_z(t) >$ decreases in magnitude. With decreasing the polarization the decay becomes more pronounced, and the accompanied oscillations suppresse. It is the decay of the envelope in these graphs but not the fast oscillations themselves that signals the spin decay. In our calculations, because of the limit of computer power, the system of $N = 12$ nuclear spins is the largest system in which we could investigate all degrees of polarization.

Next, we investigate the dependence of the electron spin evolution on the type of initial state. The Fig. 2 shows the dependence of electron spin on time with $N = 18$ and $J_z = 7.5$. The two left panels show the electron spin dynamics with randomly correlated initial states and the two right ones show the corresponding data for the tensor product

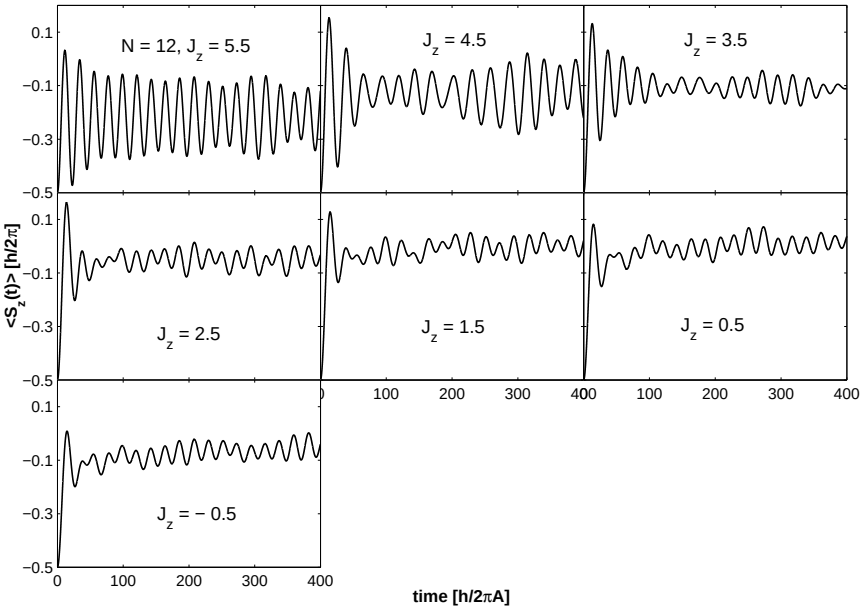

Fig. 1. The time evolution of the electron spin in the system of $N = 12$ nuclei. In the top left panel the nuclear spins are fully polarized in the initial state with the electron spin pointing opposite to the nuclear spins [$J_z = 5.5$]. In the following panels the number of flipped nuclear spins in the initial state is gradually increased (J_z decreases). The case, when the limiting of the fully unpolarized initial state is reached, is shown in the bottom left panel [$J_z = -0.5$].

initial states. In both cases, $< S_z(t) >$ decreases in magnitude. Results show that the electron spin dynamics doesn't depend on the randomly correlated initial state, but on the tensor product initial state. In the latter case, the time evolution depends significantly on the individual initial state. By comparing the data in the left panels with that in the right ones we see that the decay of electron spin in the case of the tensor product state is considerably slower than in the case of the randomly correlated state. In addition, the obtained results [un-presented] also show that the time evolution for the randomly correlated nuclear spin system closely mimics the average over all tensor product initial states [15]. These conclusions are relevant with different numbers of nuclear spins and different values of the degree of polarization J_z.

Finally, we estimated the decay law of the magnitude of $< S_z(t) >$. As shown in Fig. 3, the decay of $< S_z(t) >$ follows well the power law $t^{-\alpha}$. A rough estimation in different systems studied shows that for a given polarization α increases with increasing N. These results are in qualitatively agreement with results of the perturbative approach calculation [7, 8] for the large enough system. Khaetskii *et al.* [7, 8] have shown the $< S_z(t) >$ decays as $t^{-3/2}$ for the case of large external magnetic field and the tensor product initial states.

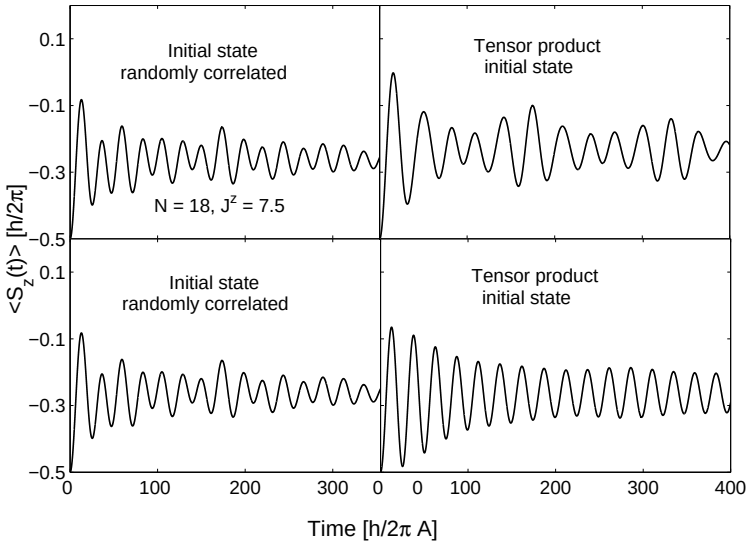

Fig. 2. Electron spin dynamics for $N = 18$ nuclei and $J_z = 7.5$. The left two panels show the results for two randomly correlated initial state of the nuclear system. The right two panels show the corresponding data for the two tensor product initial states.

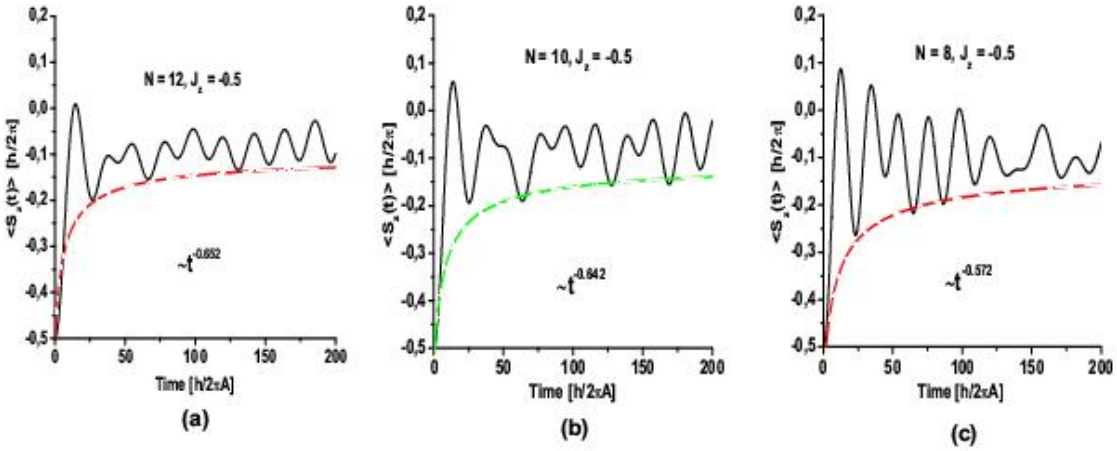

Fig. 3. The decay law of the electron spin dynamics for some cases with the randomly correlated initial state. The dashed lines describe the law $\propto t^{\alpha}$ with α given in the figure.

IV. CONCLUSION

In summary we have studied electron spin dynamics via its hyperfine interaction with nuclear spins in a 2D $GaAs$ - quantum dot. Obtained results show that the time

evolution of electron spin depends on the type of initial state and the polarization of the system. Electron spin dynamics doesn't depend on the randomly correlated initial states but on the tensor product initial states individually. The decay of the electron spin evolution is given by a power. The study provides a better understanding of the decohence of the electron spin in the quantum information processing.

ACKNOWLEDGMENT

The authors thank Nguyen Van Lien for suggesting the problem and reading the manuscript. This work is supported by the Ministry of Science and Technology (Vietnam) via the Fundamental Research Program (project No. 402306).

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Received 06 October 2007.